



Optimal Attack and Defense for Reinforcement Learning

Jeremy McMahan, Young Wu, Xiaojin Zhu, Qiaomin Xie

RL Basics

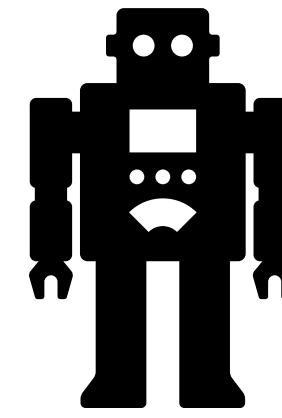
RL Interaction Protocol

RL Interaction Protocol

Models sequential decision making in **uncertain** environments

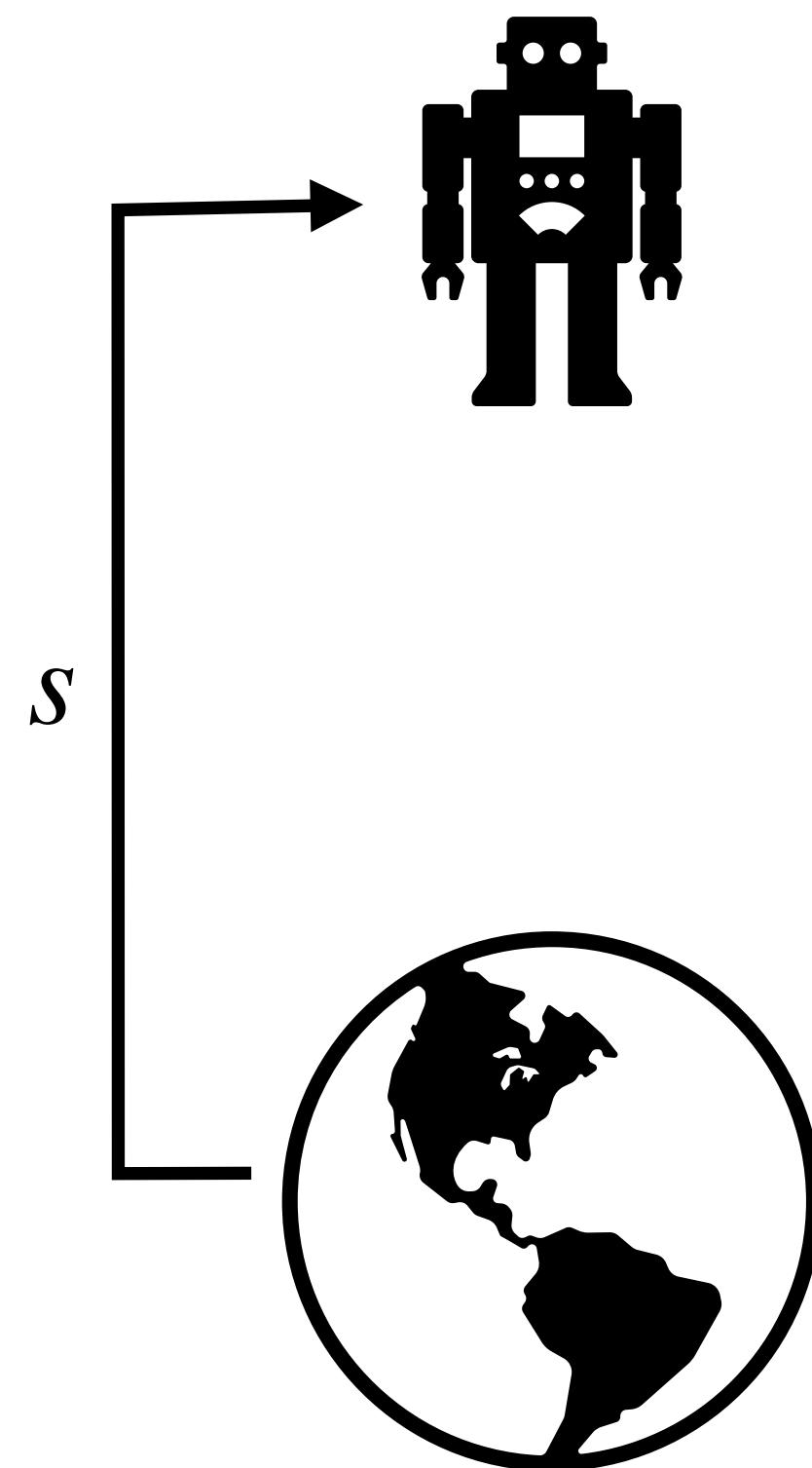
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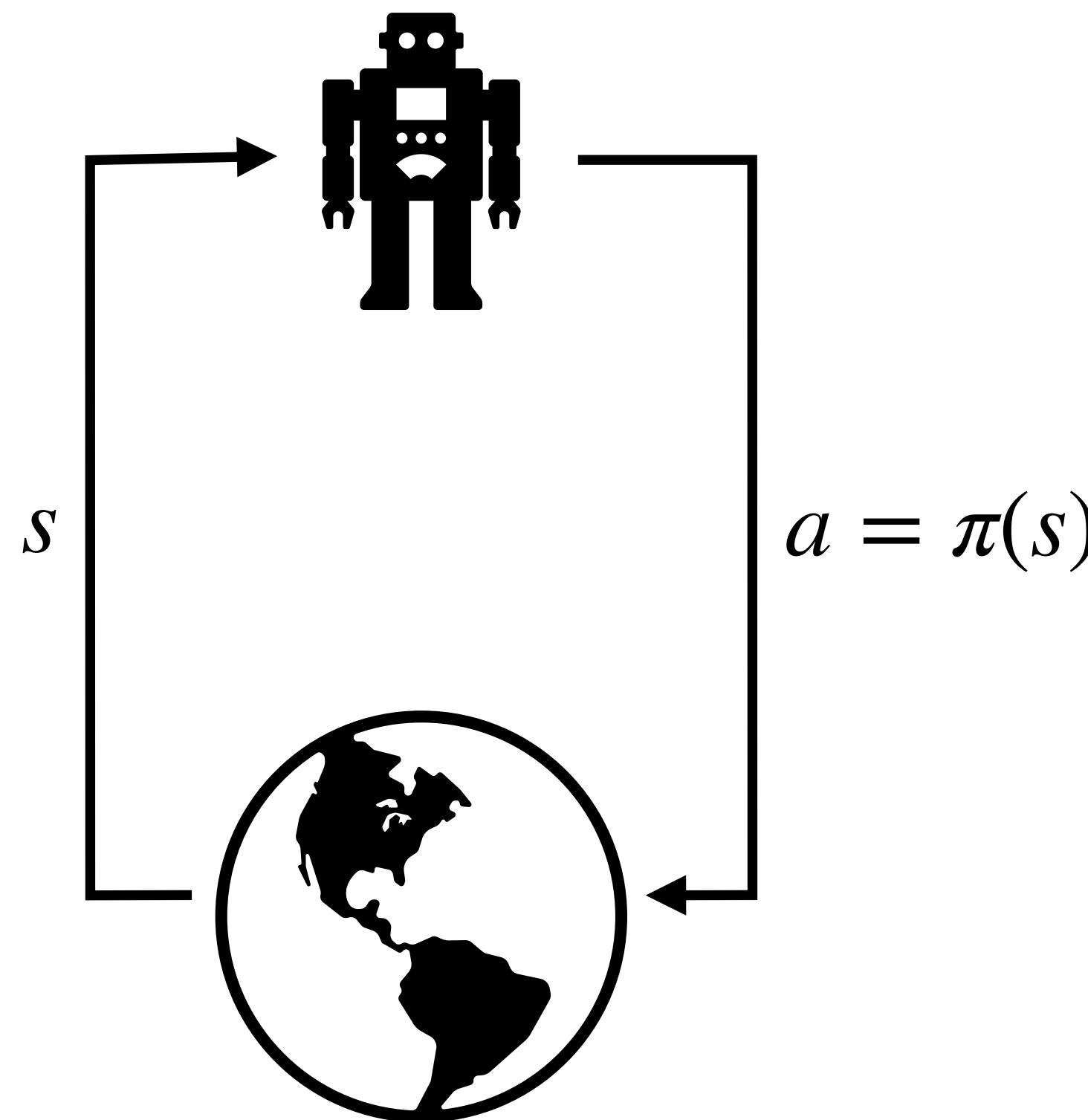
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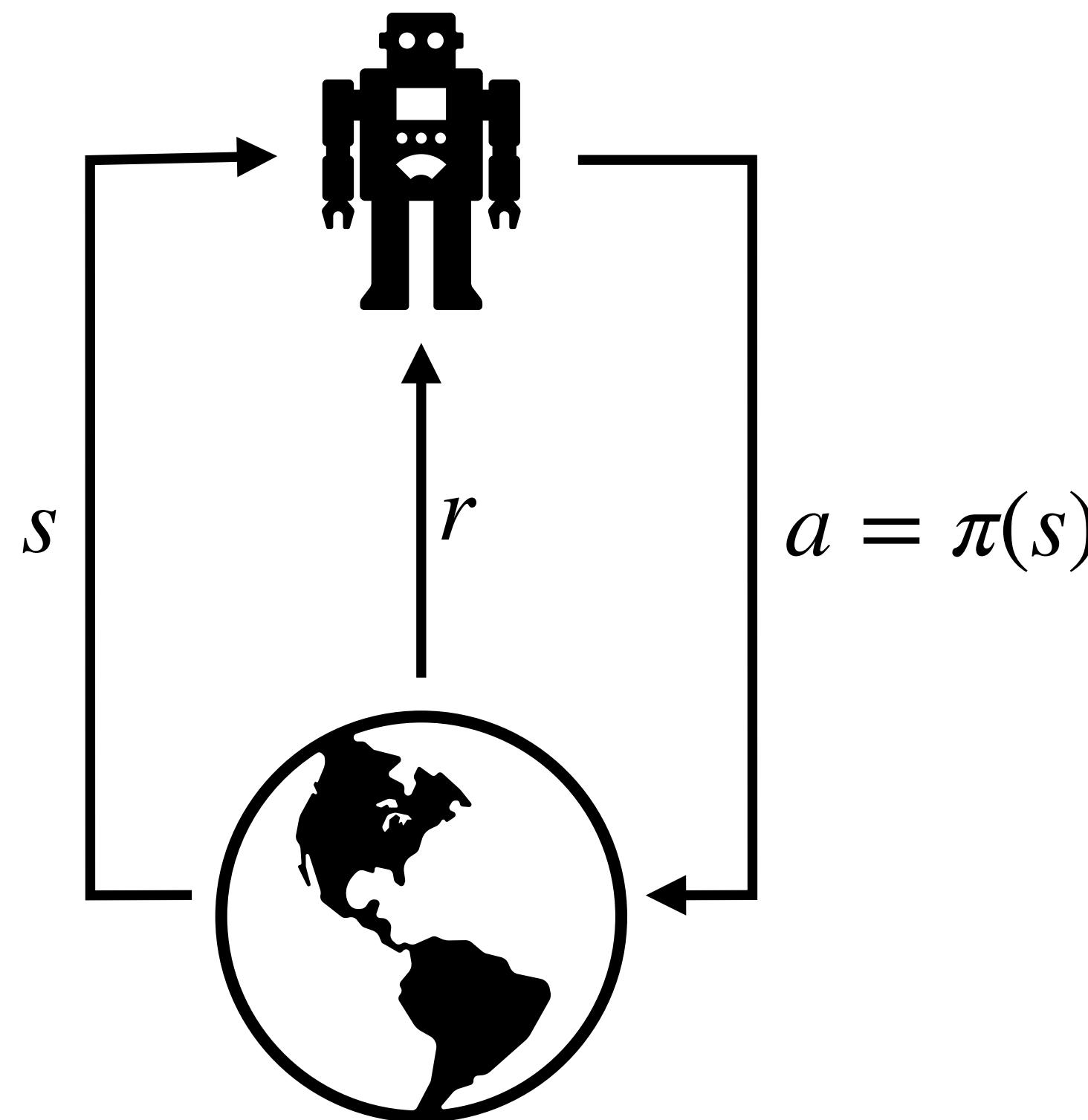
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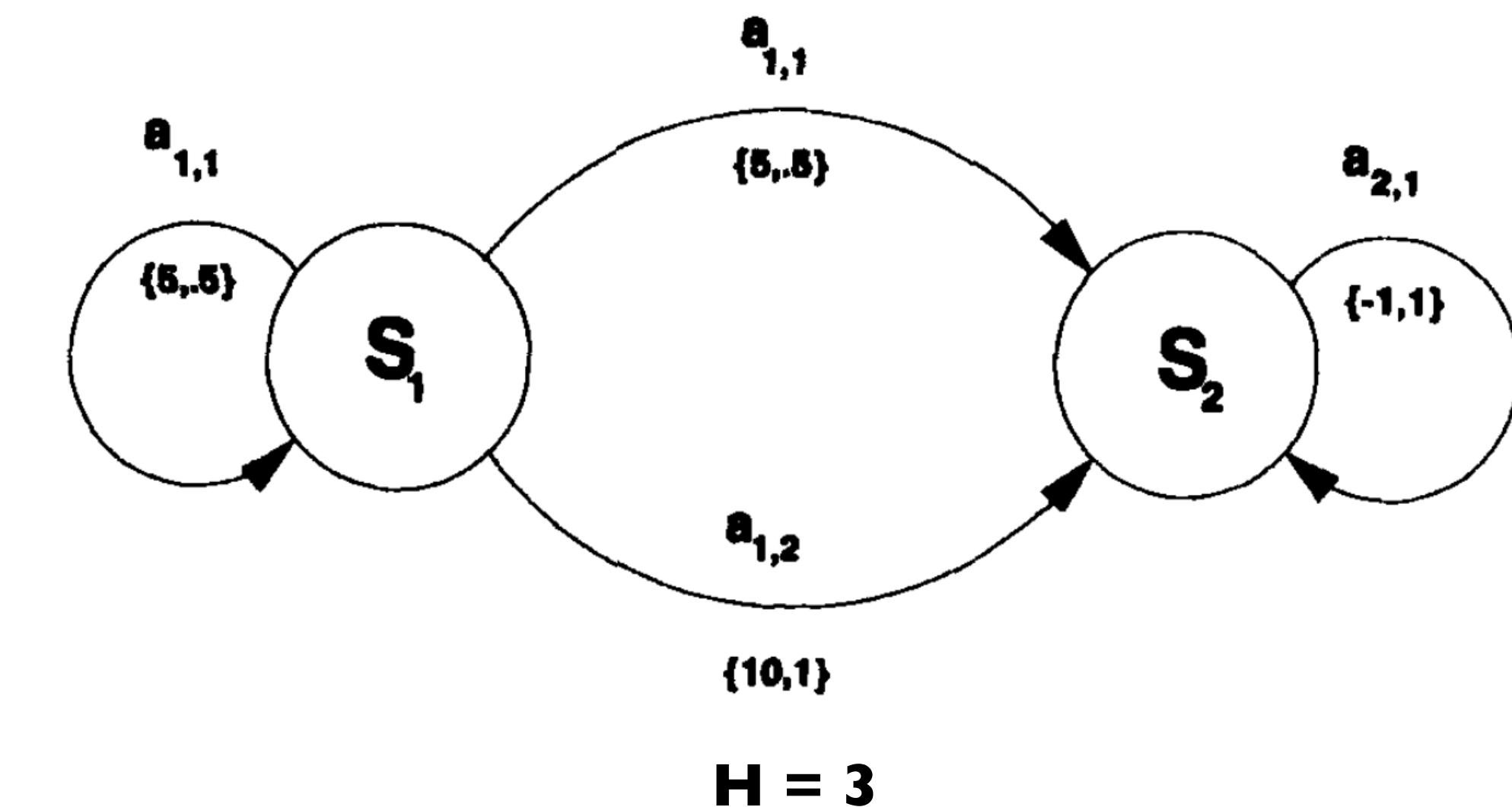
Models sequential decision making in **uncertain** environments



Model: MDPs

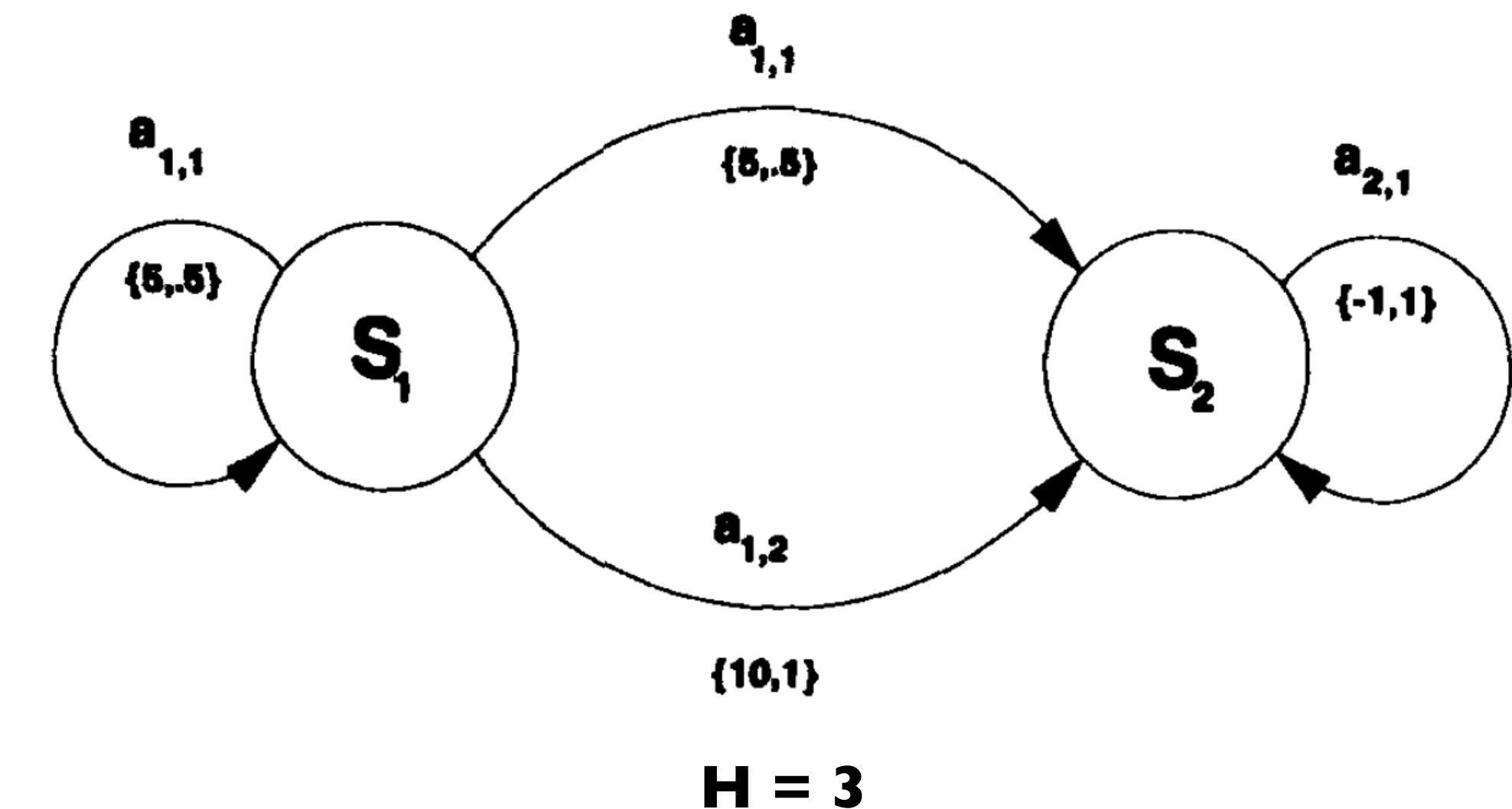
H = 3

Model: MDPs



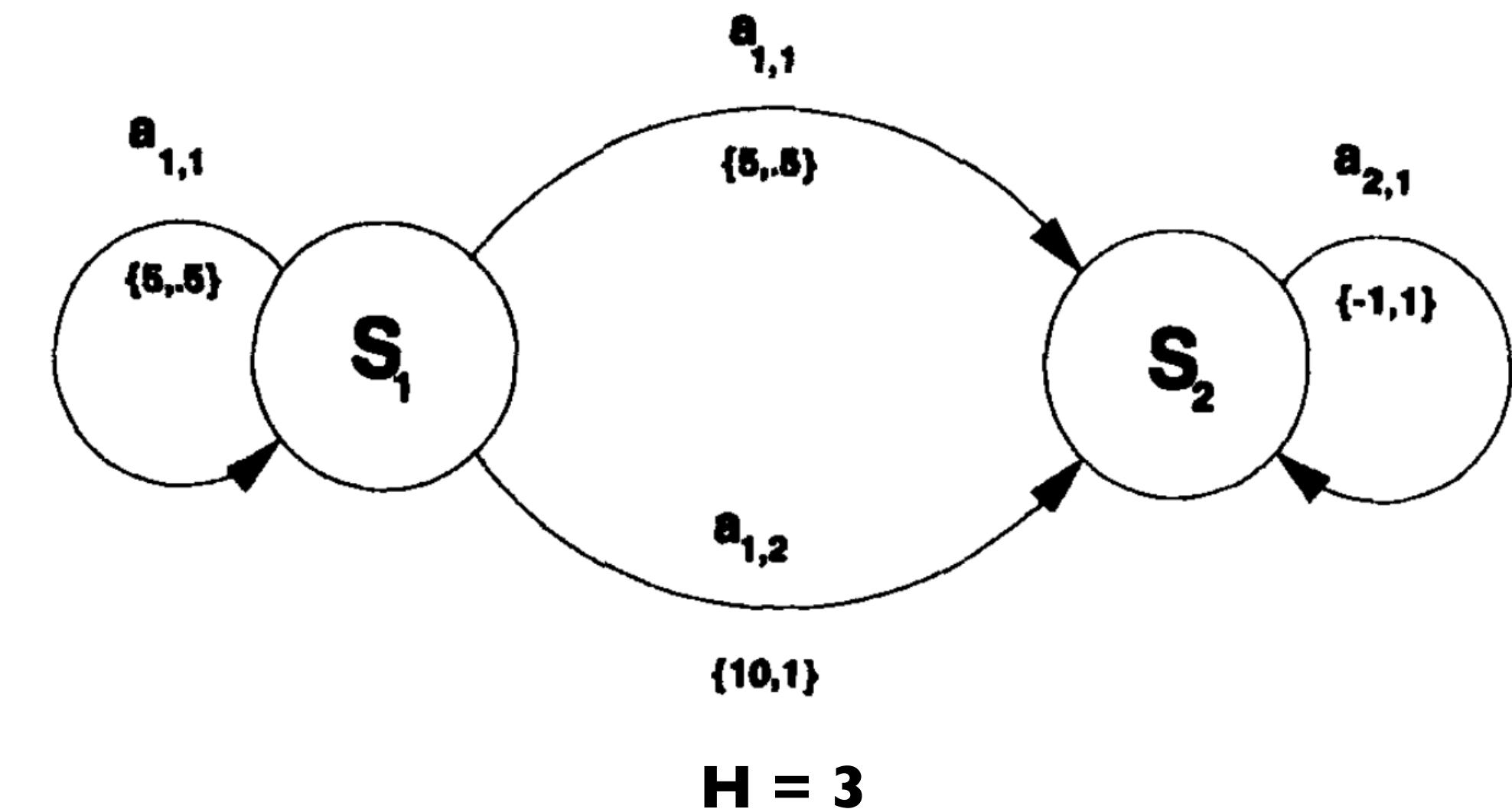
Model: MDPs

- States, S



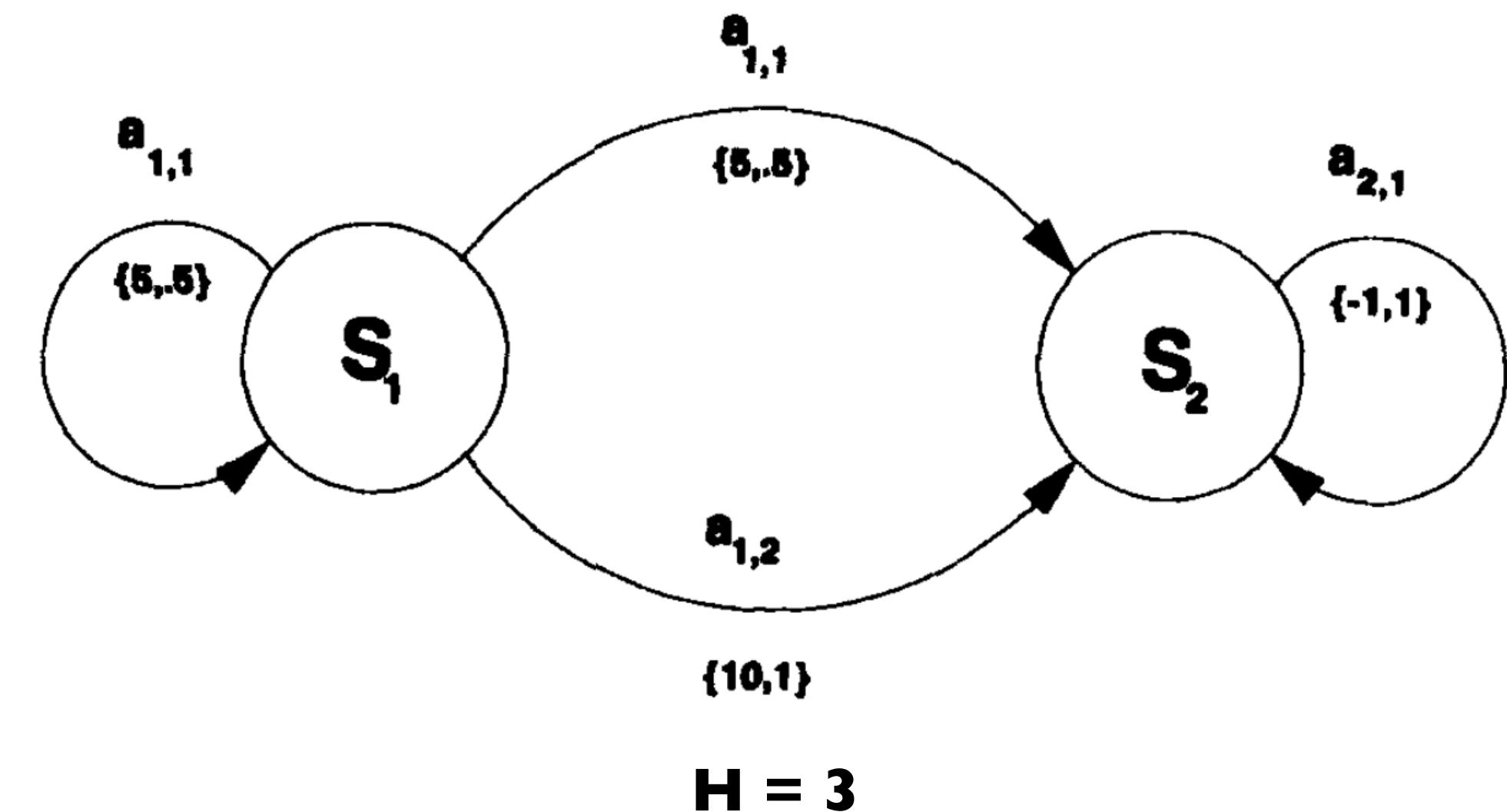
Model: MDPs

- States, S
- Actions, A



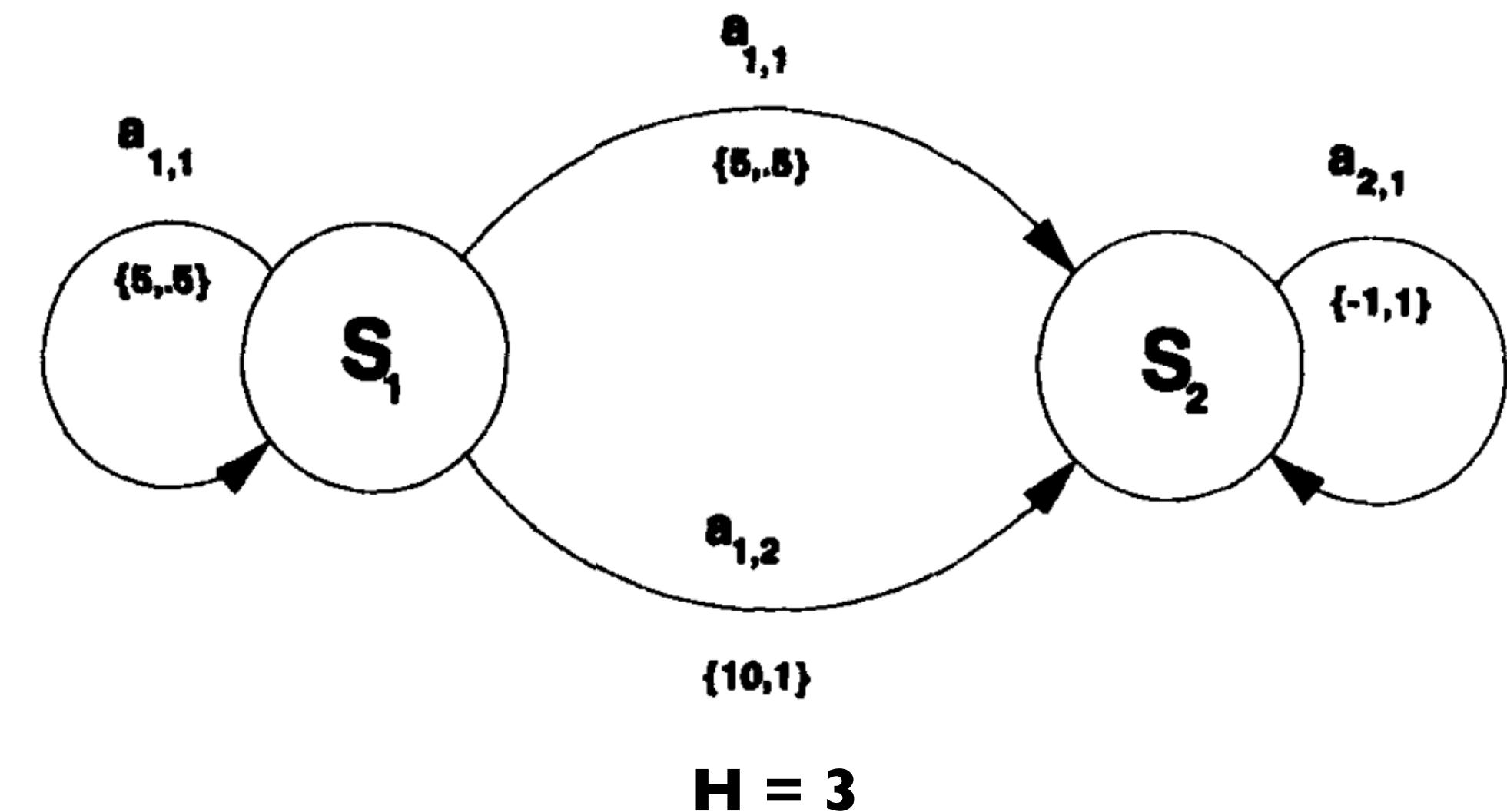
Model: MDPs

- States, S
- Actions, A
- Rewards, $r(s, a)$



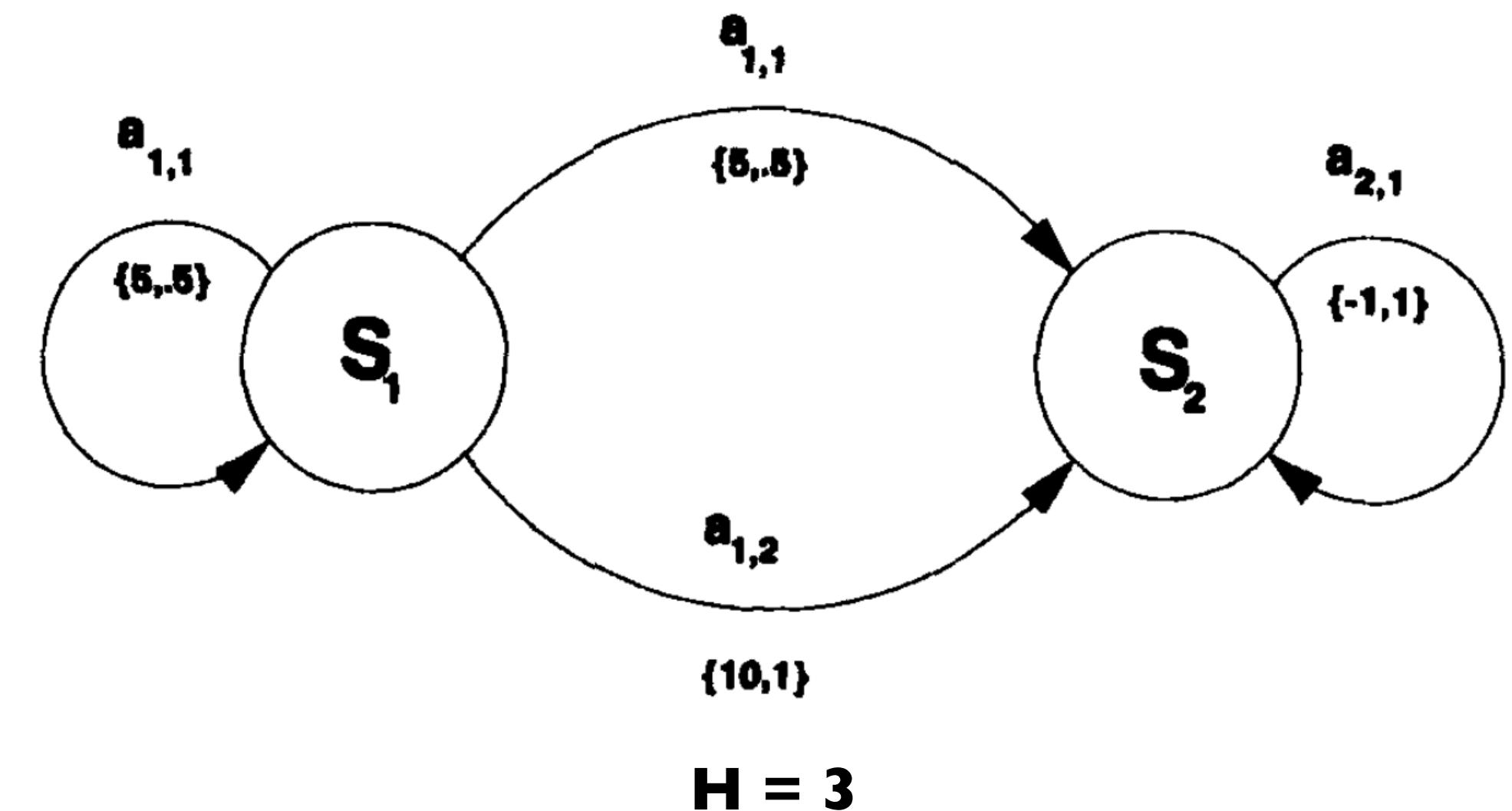
Model: MDPs

- States, S
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- Rewards, $r(s, a)$
- Transition Probabilities, $P(s' | s, a)$



Model: MDPs

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- Time Horizon, H



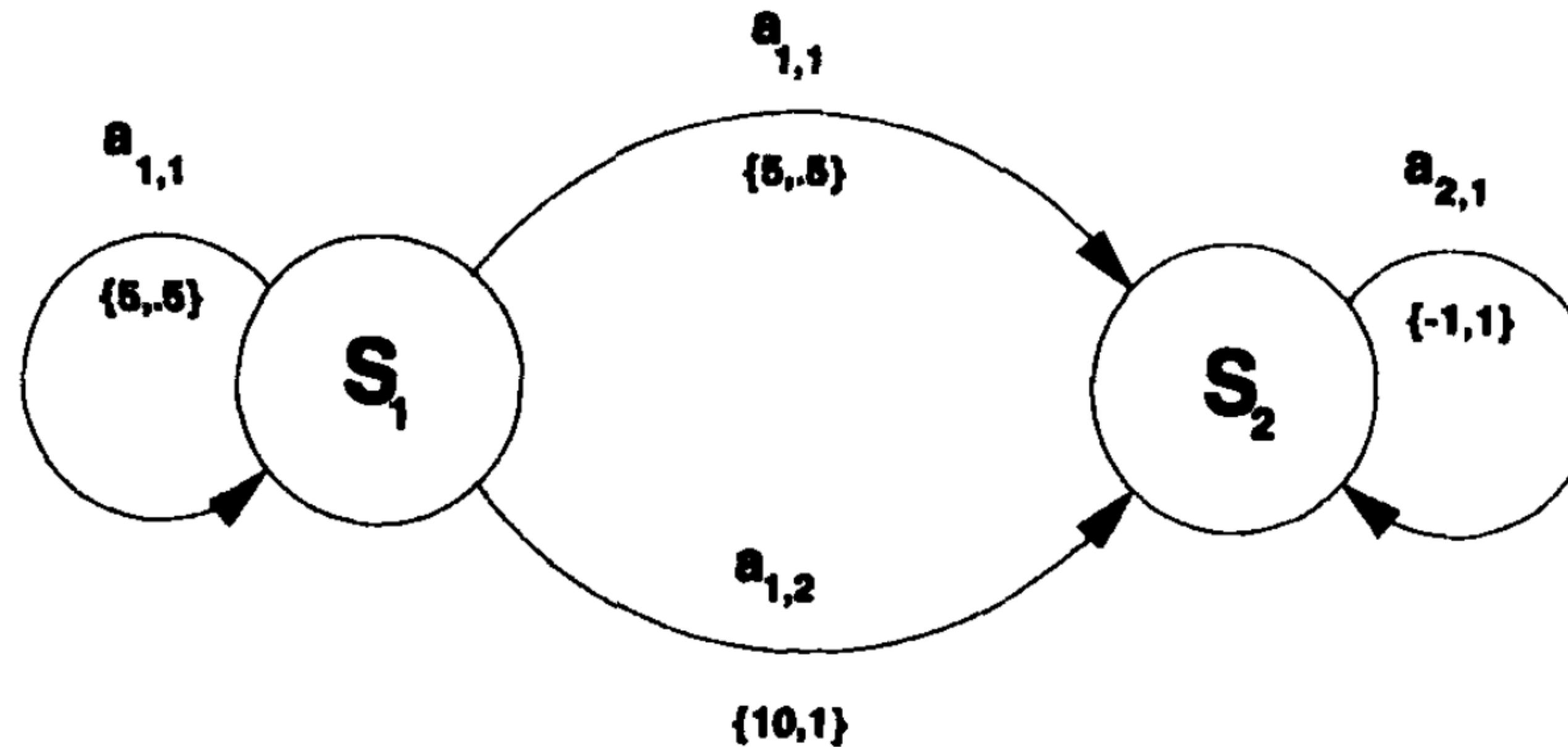
Policies

Policies

A *policy* is a plan of what action to take in each state.

Policies

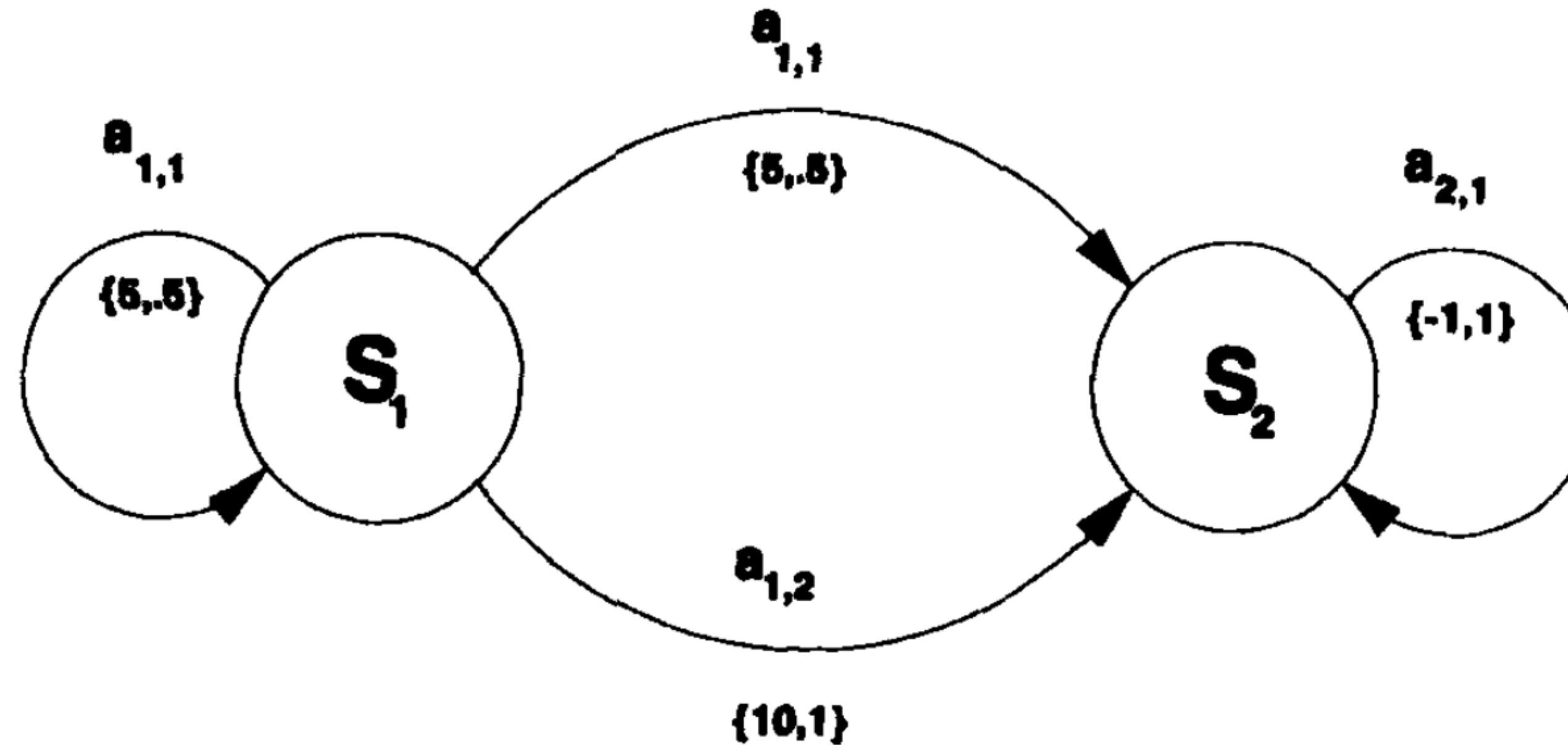
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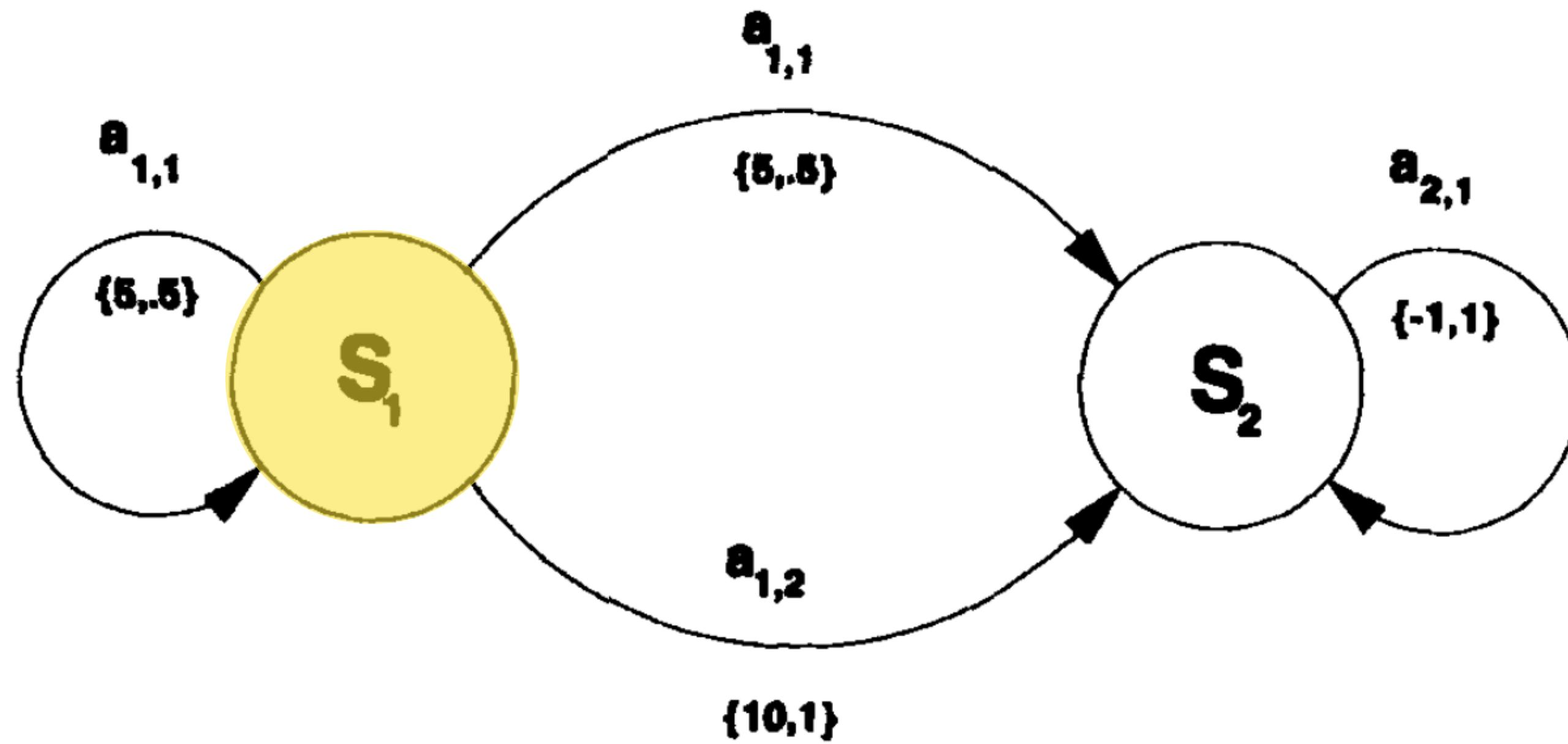
$$\pi(s_1) = a_{1,2} \quad \pi(s_2) = a_{2,1}$$



Value

The *value* of M under π is: $V^\pi(s) = E_\pi \left[\sum_{h=1}^H r_h(s, a) \mid s_0 = s \right]$.

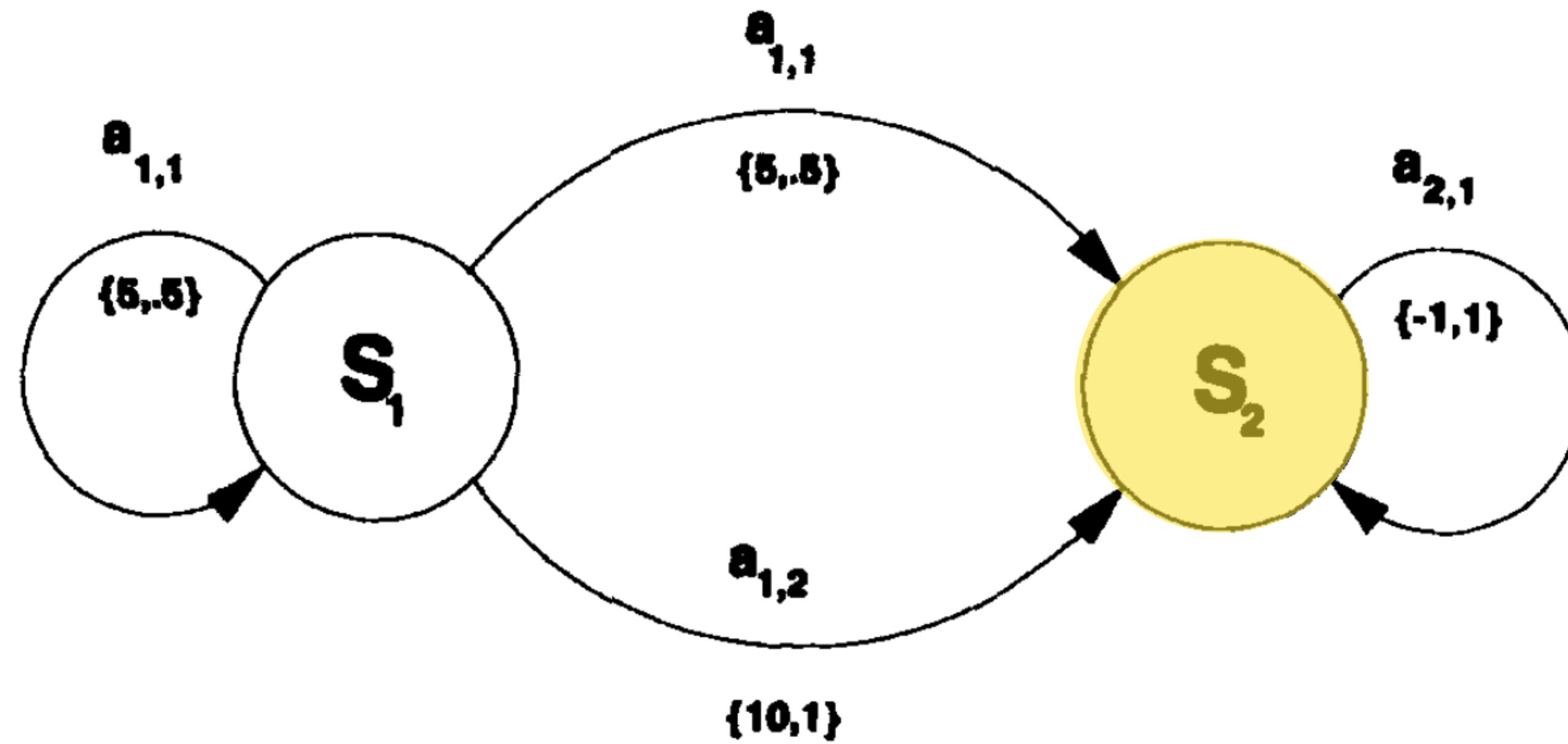
Value



$$\pi(S_1) = a_{1,2}$$

Reward = 10

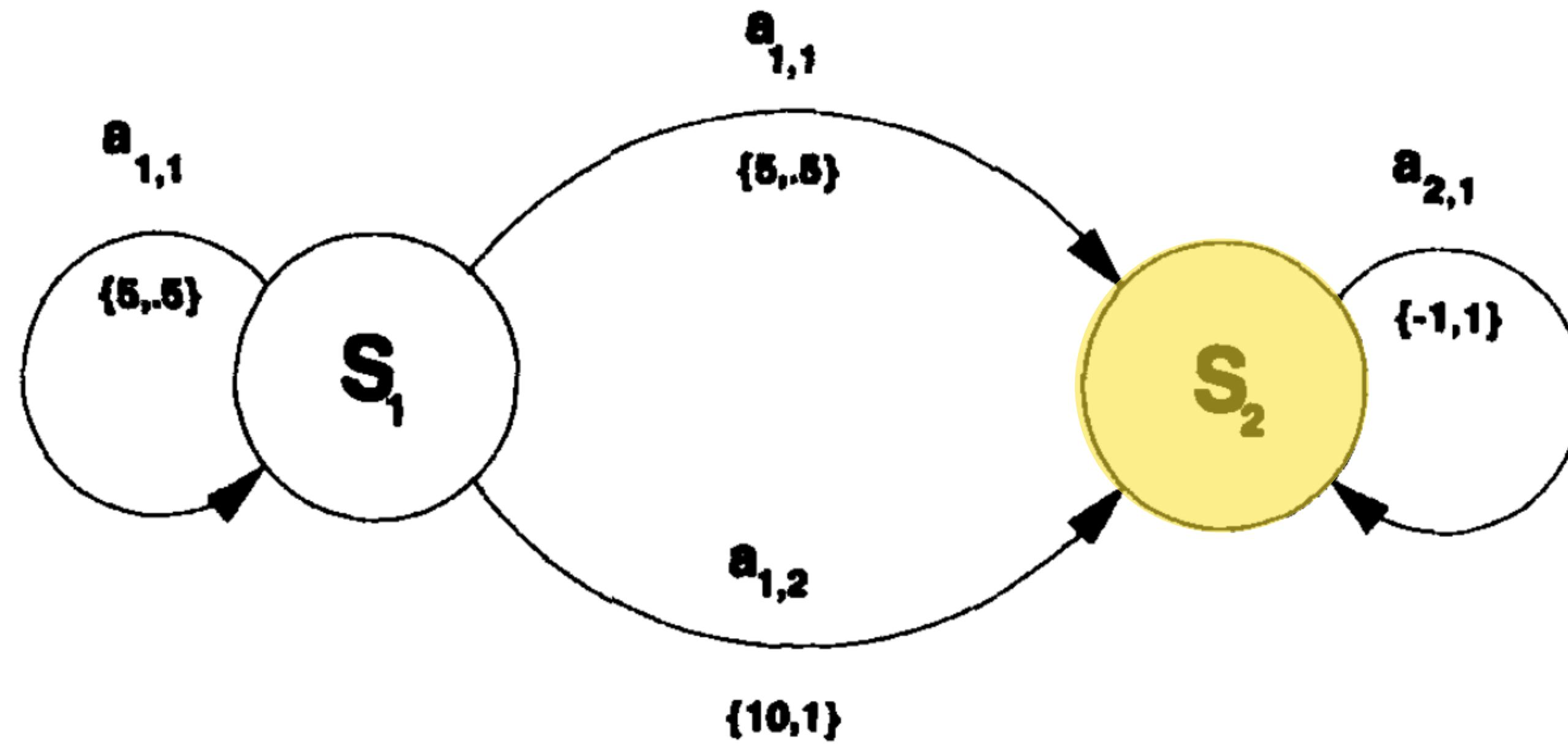
Value



$$\pi(S_2) = a_{2,1}$$

Reward = -1

Value



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Reward = -1

Value

$$V^\pi(s_1) = 10 - 1 - 1 = 8$$

Optimal Policies

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$$\pi^* = \sup_{\pi} V^\pi(s_0)$$

Example MDP

Example MDP



Example MDP

Disaster Relief with Autonomous Vehicles

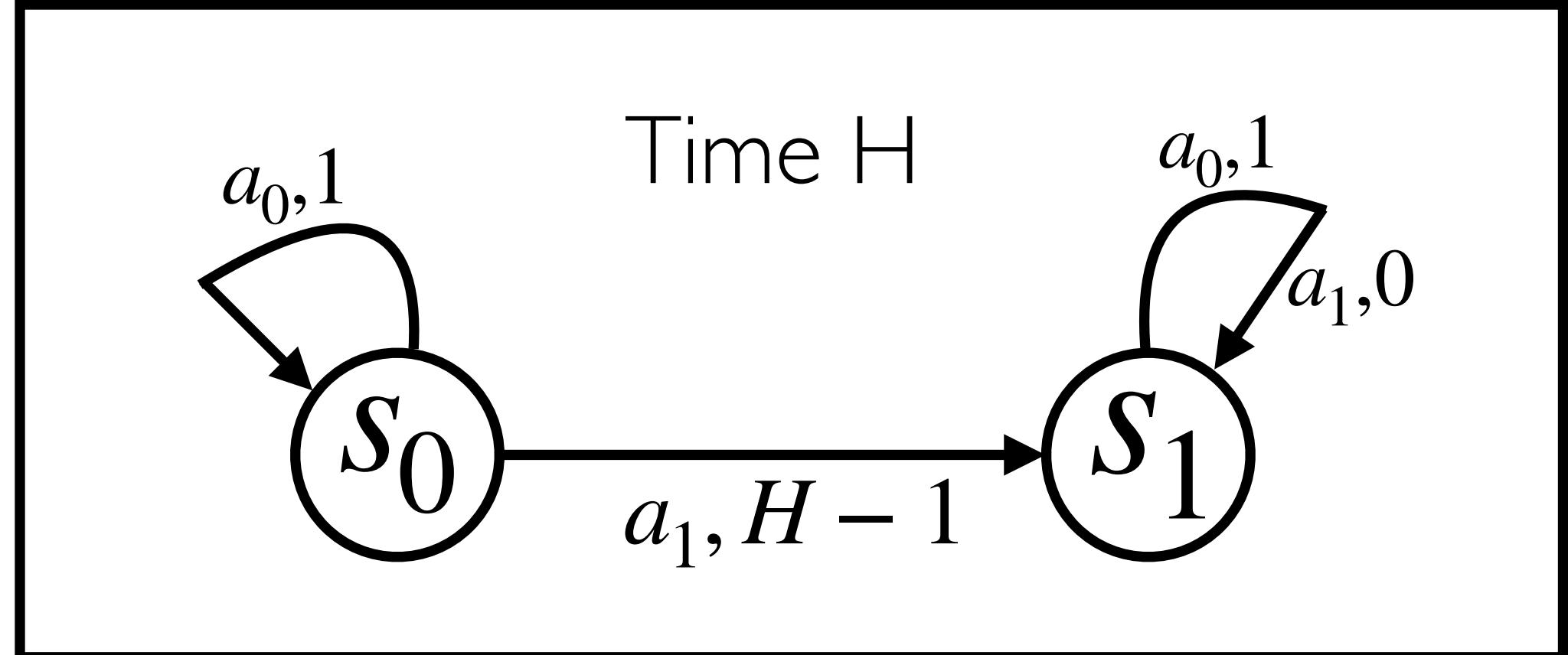
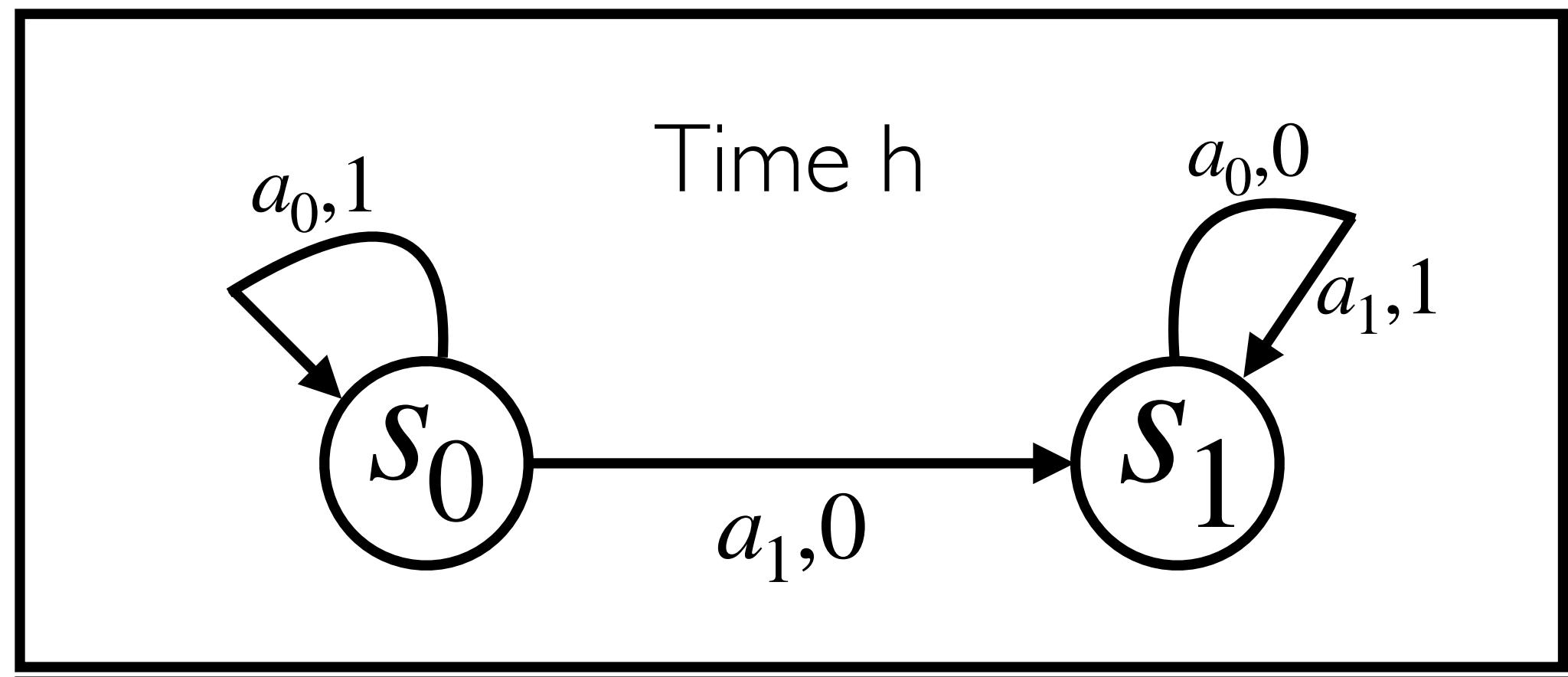
- State Space is \mathbb{R}^2
- Action Space is $[-1,1]^2$
- New location is $s + a$
- Reward for finding people in need.



Performance of Optimal Policies

Performance of Optimal Policies

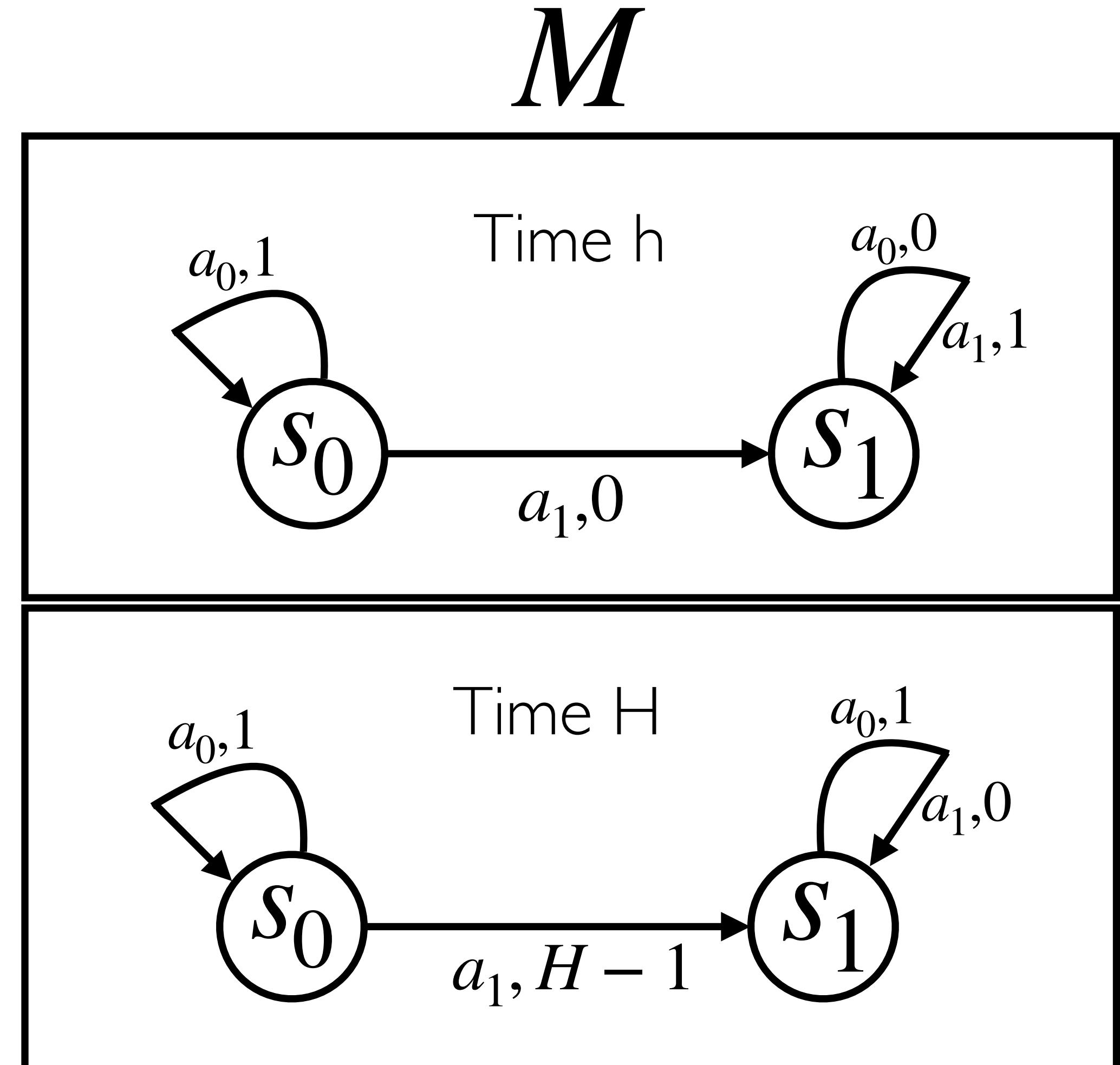
M



Performance of Optimal Policies

Unique optimal policy π^* is:

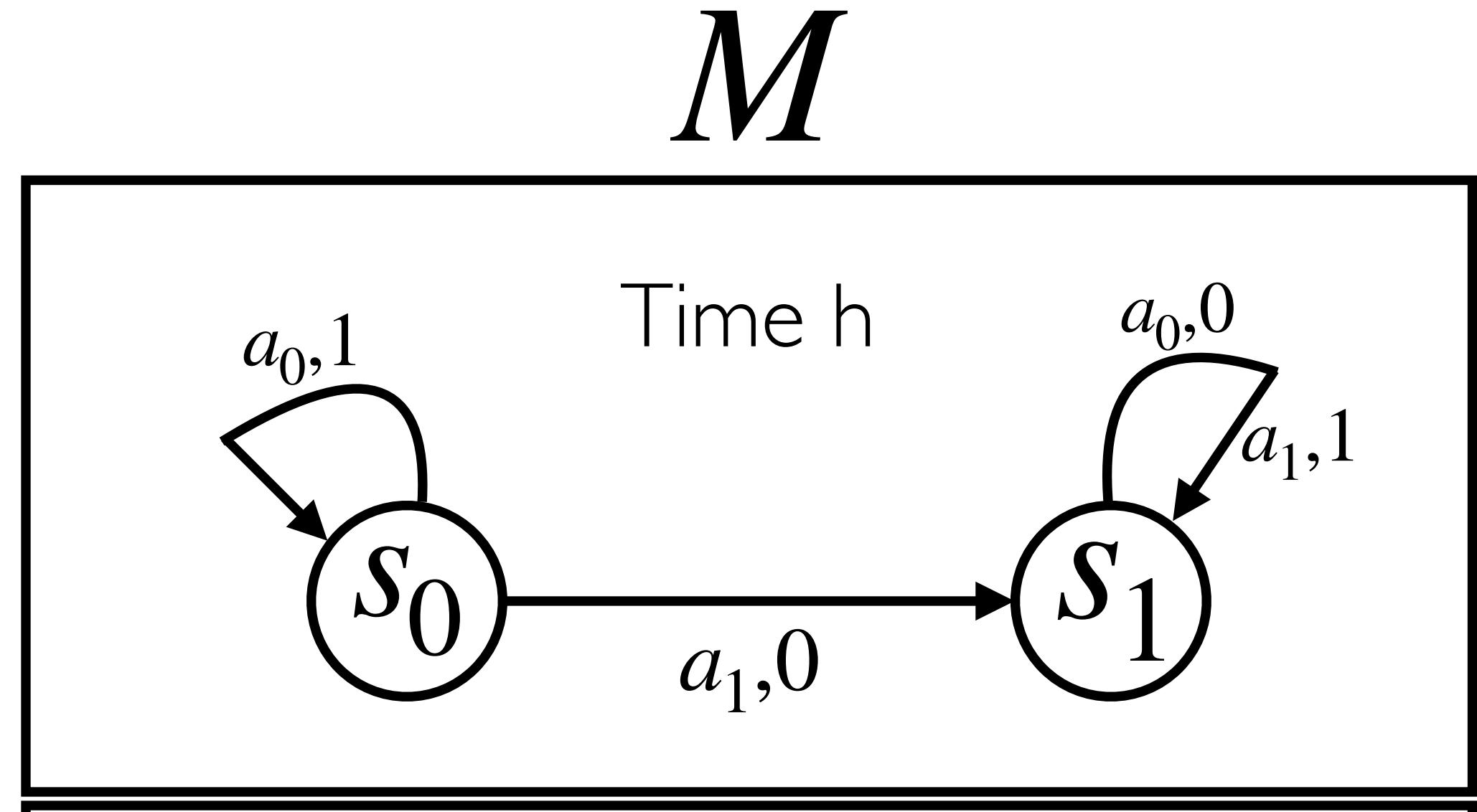
π^*		
t/S	s_0	s_1
h	a_0	a_1
H	a_1	a_0



Performance of Optimal Policies

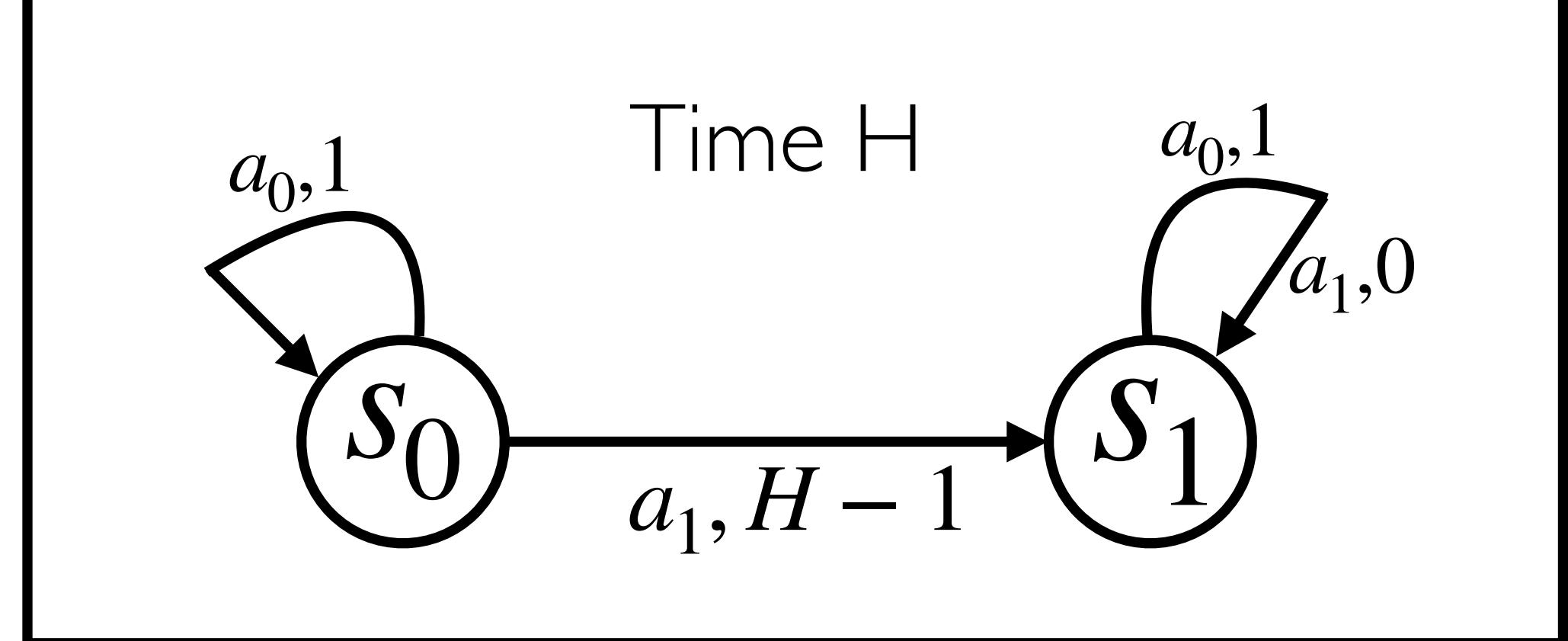
Unique optimal policy π^* is:

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The optimal policy achieves value:

$$V_M^{\pi^*} = 2(H - 1)$$



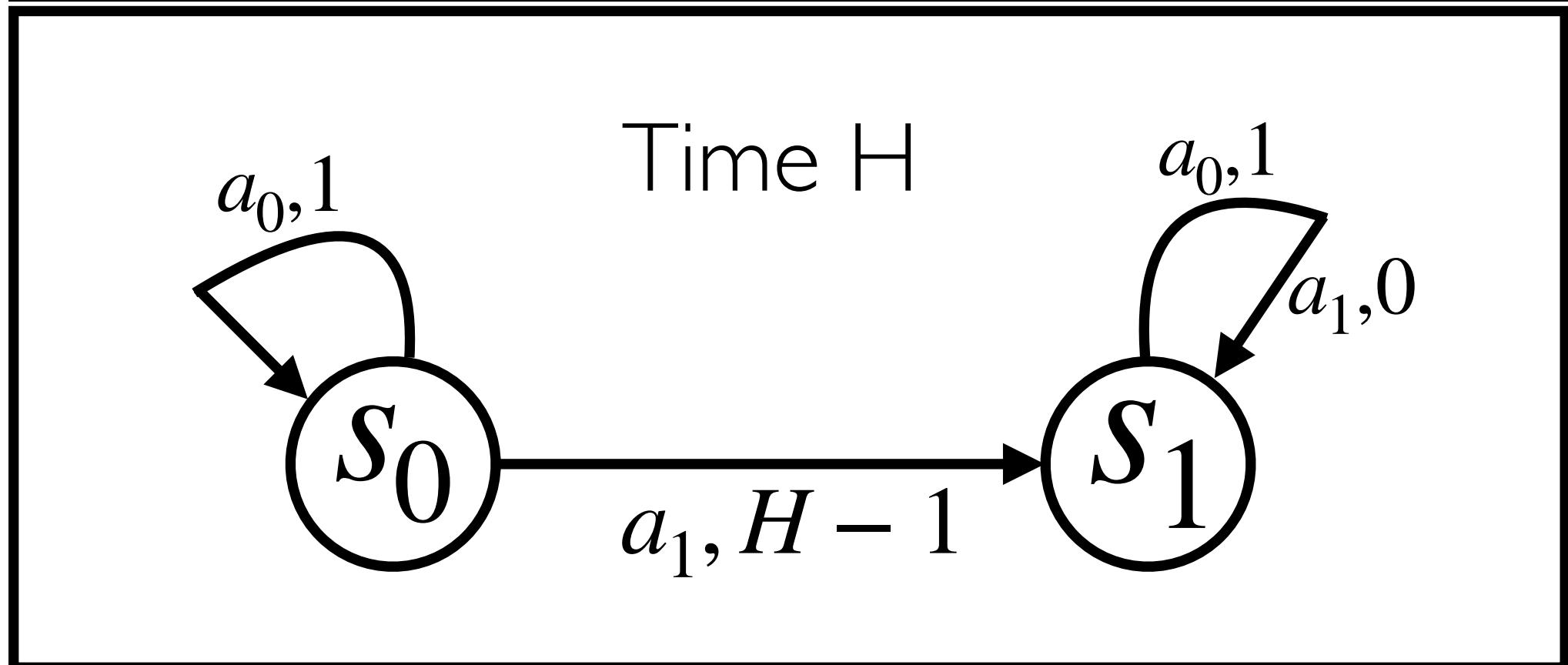
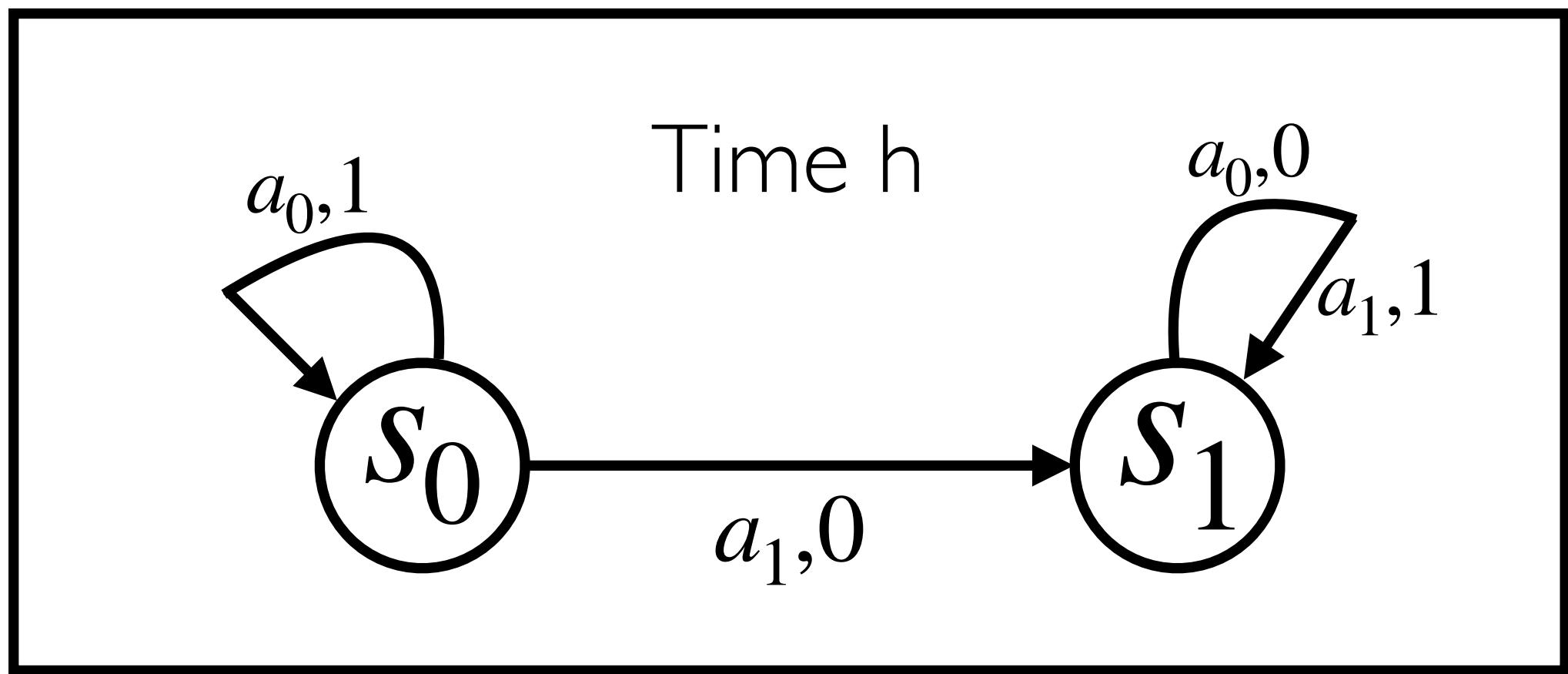
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Optimal Policies may behave poorly under measurement noise or adversarial manipulations!

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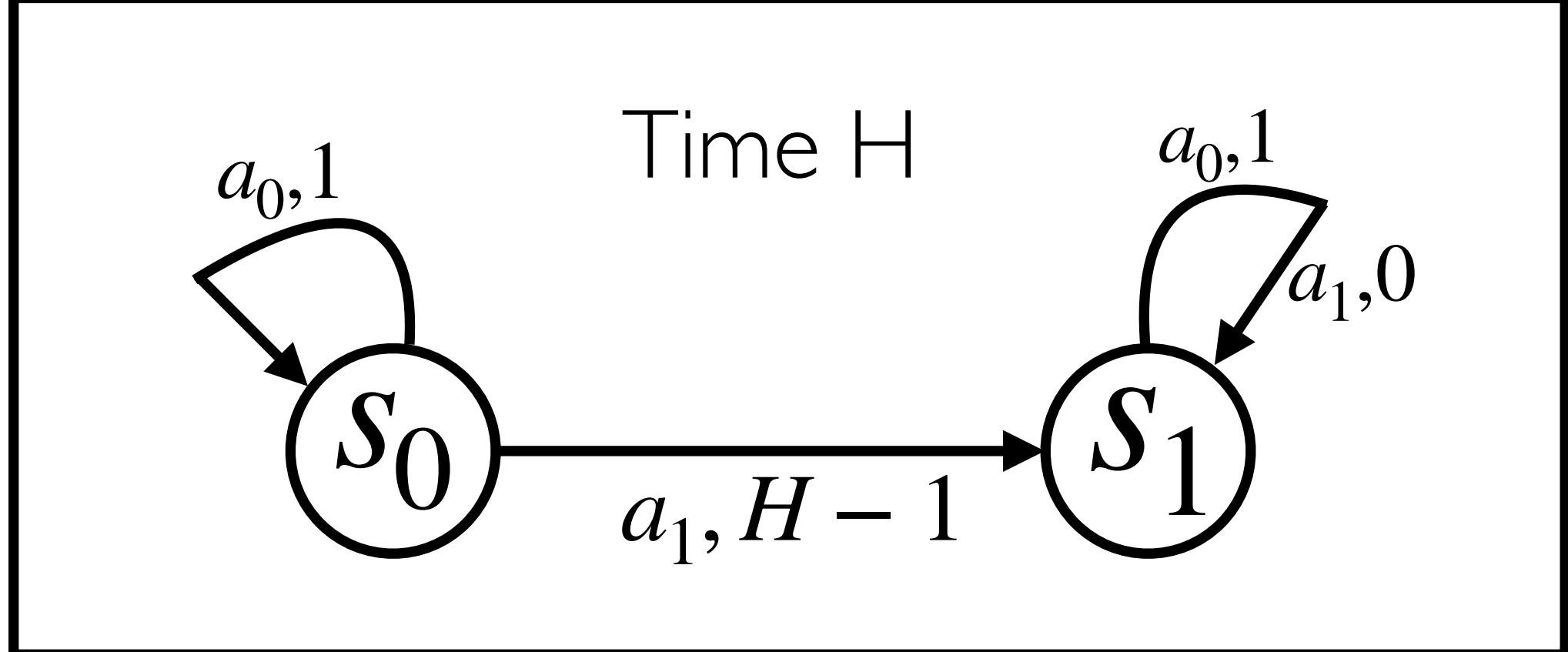
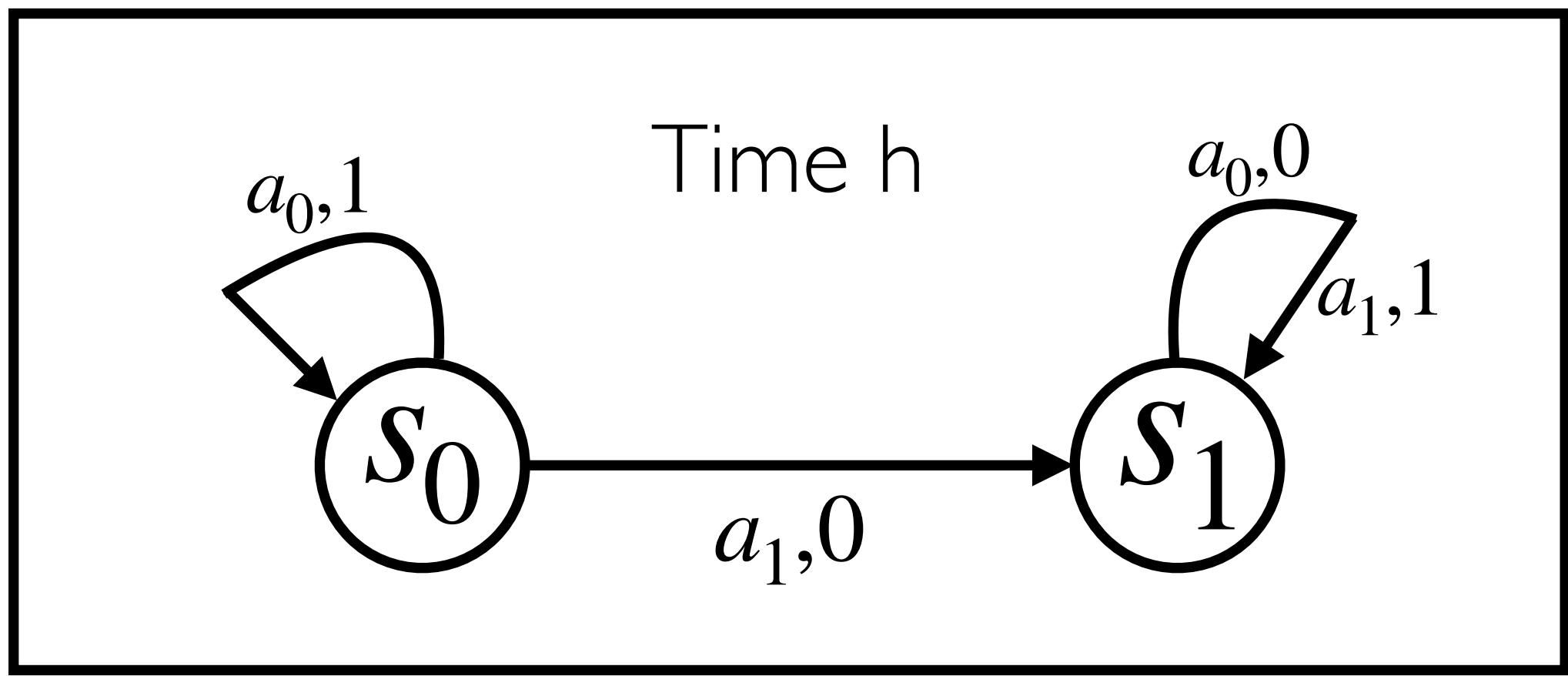
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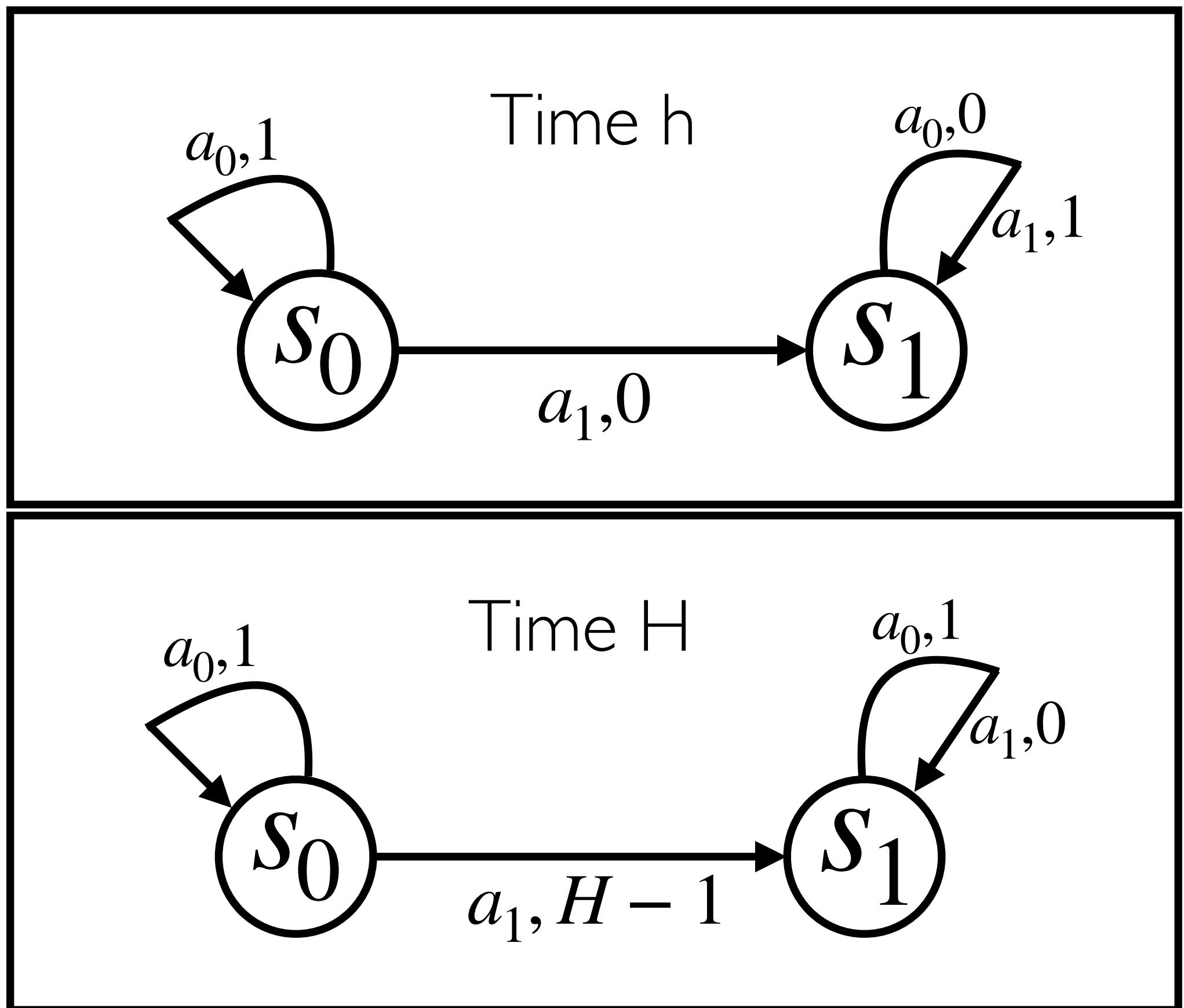
- If first state is actually s_1 or M receives a_1 instead, π^* at best gets $1/2$ of its value.



Optimal Policies are NOT Robust

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- If first state is actually s_1 or M receives a_1 instead, π^* at best gets $1/2$ of its value.
- If states are swapped consistently, π^* gets no value!



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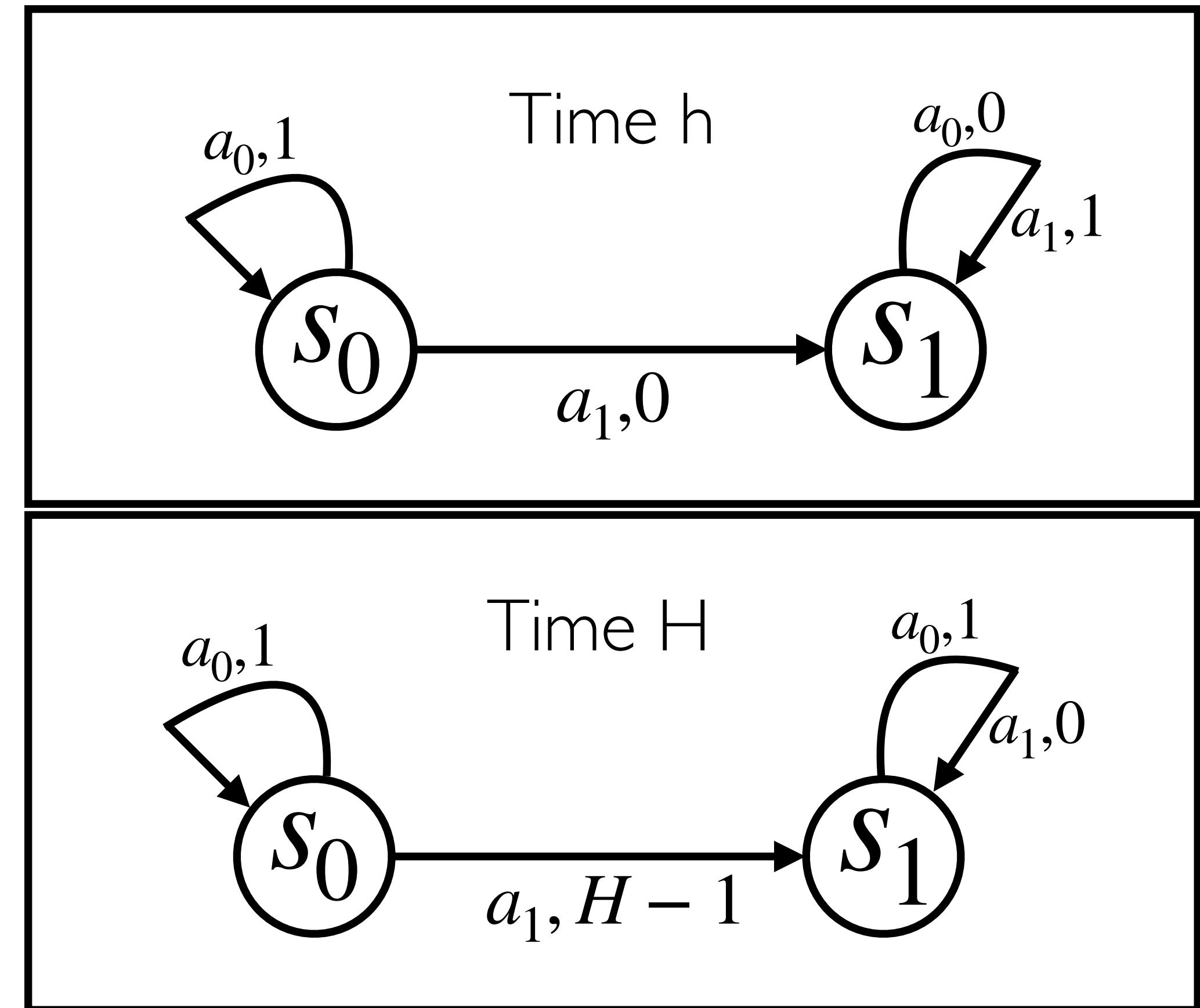
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$$\pi^* \circ \nu$$

t/S	s_0	s_1
h	a_1	a_0
H	a_0	a_1

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Security Threats to RL

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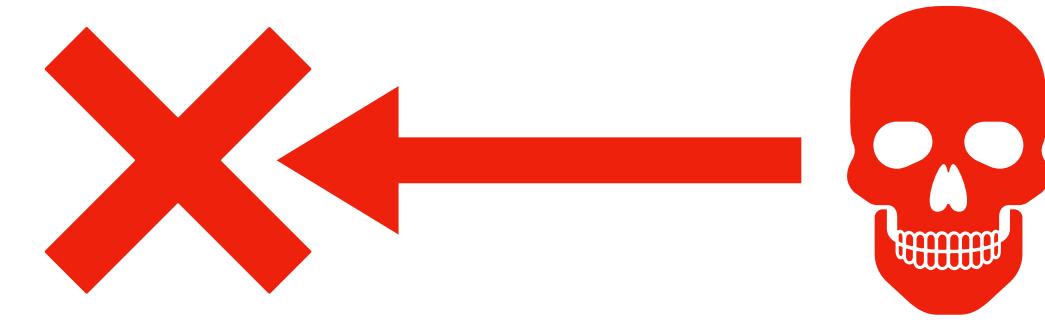
- Playing an optimal policy for the **ideal** environment is not always optimal for the **real** environment!
- Strategies to compute **robust** policies are needed.
- Inspiration for field of **adversarial** RL.

Adversarial RL

Adversarial RL

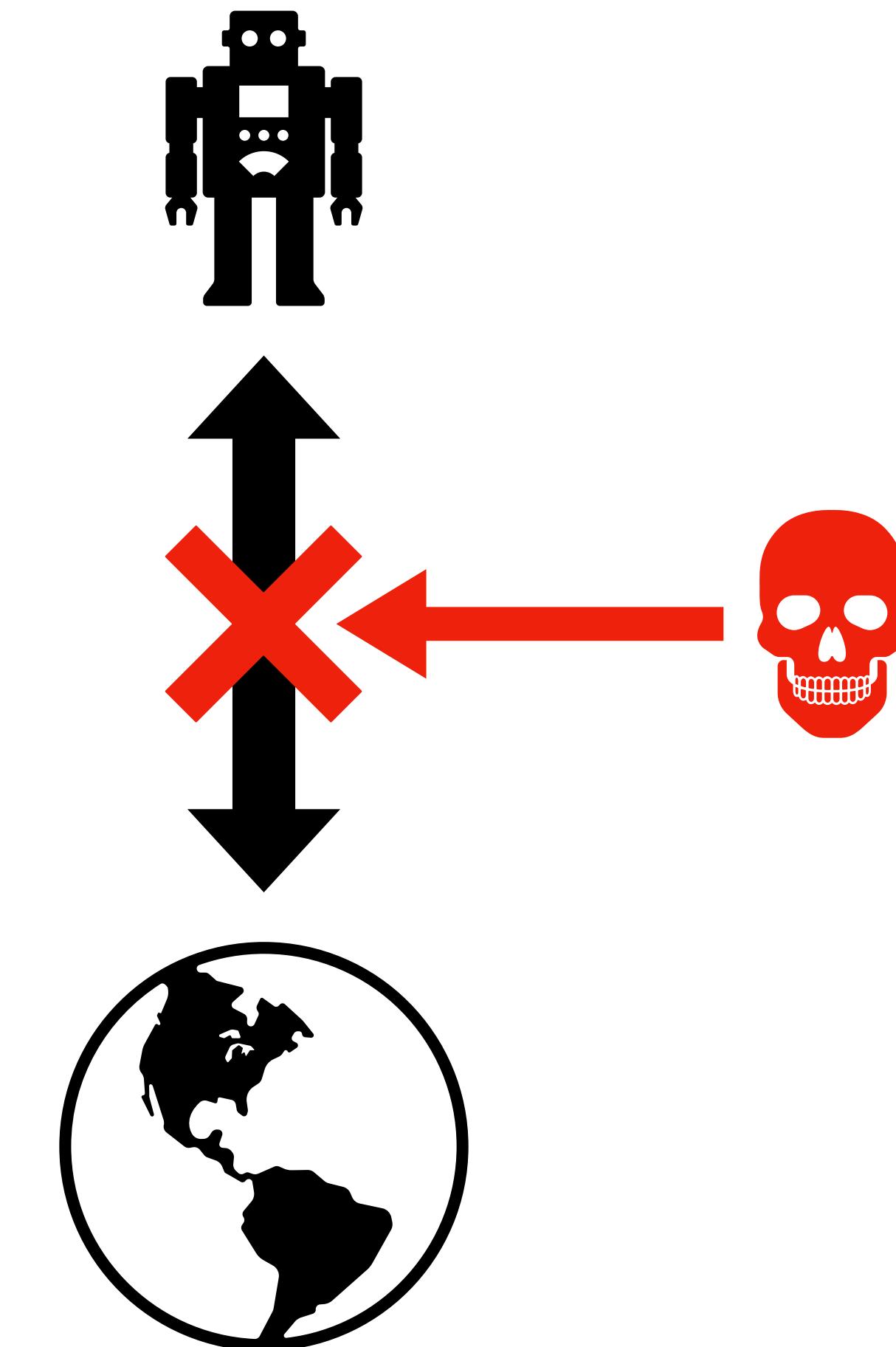
Adversarial RL

An external attacker can
manipulate the interaction.



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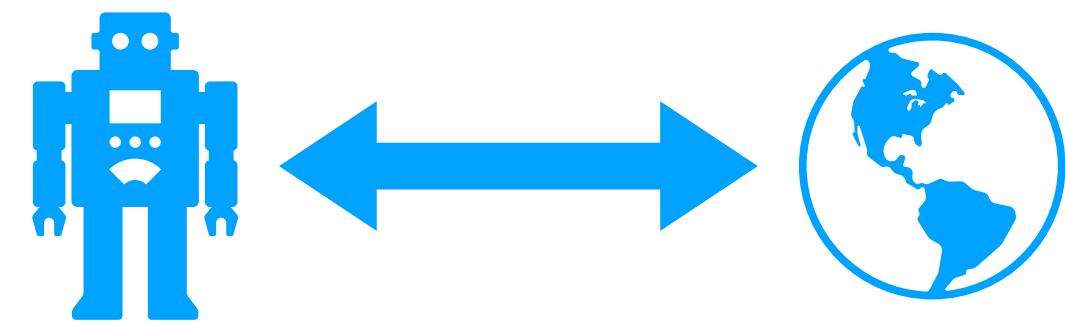
Attack Paradigms

Attack Paradigms

Training Time

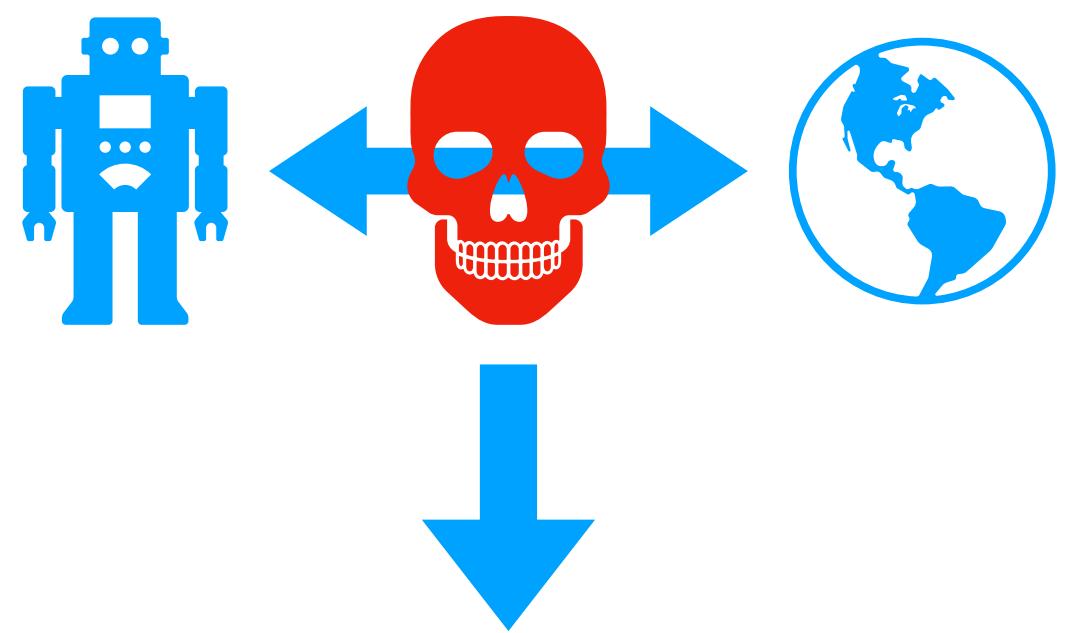
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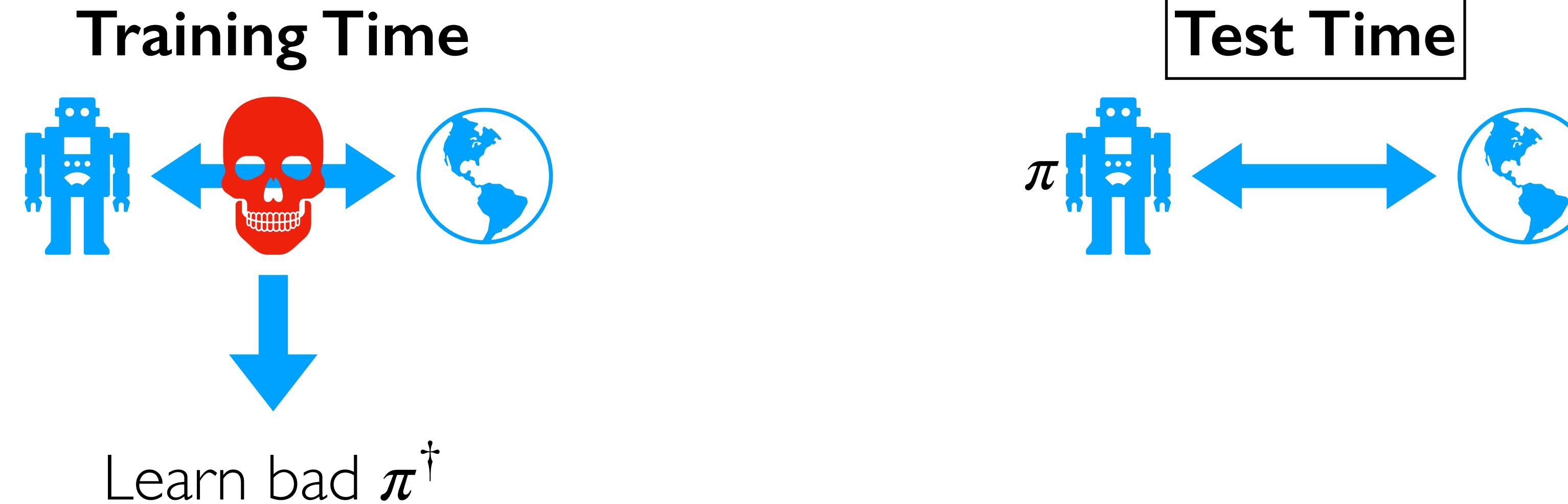


Learn bad π^\dagger

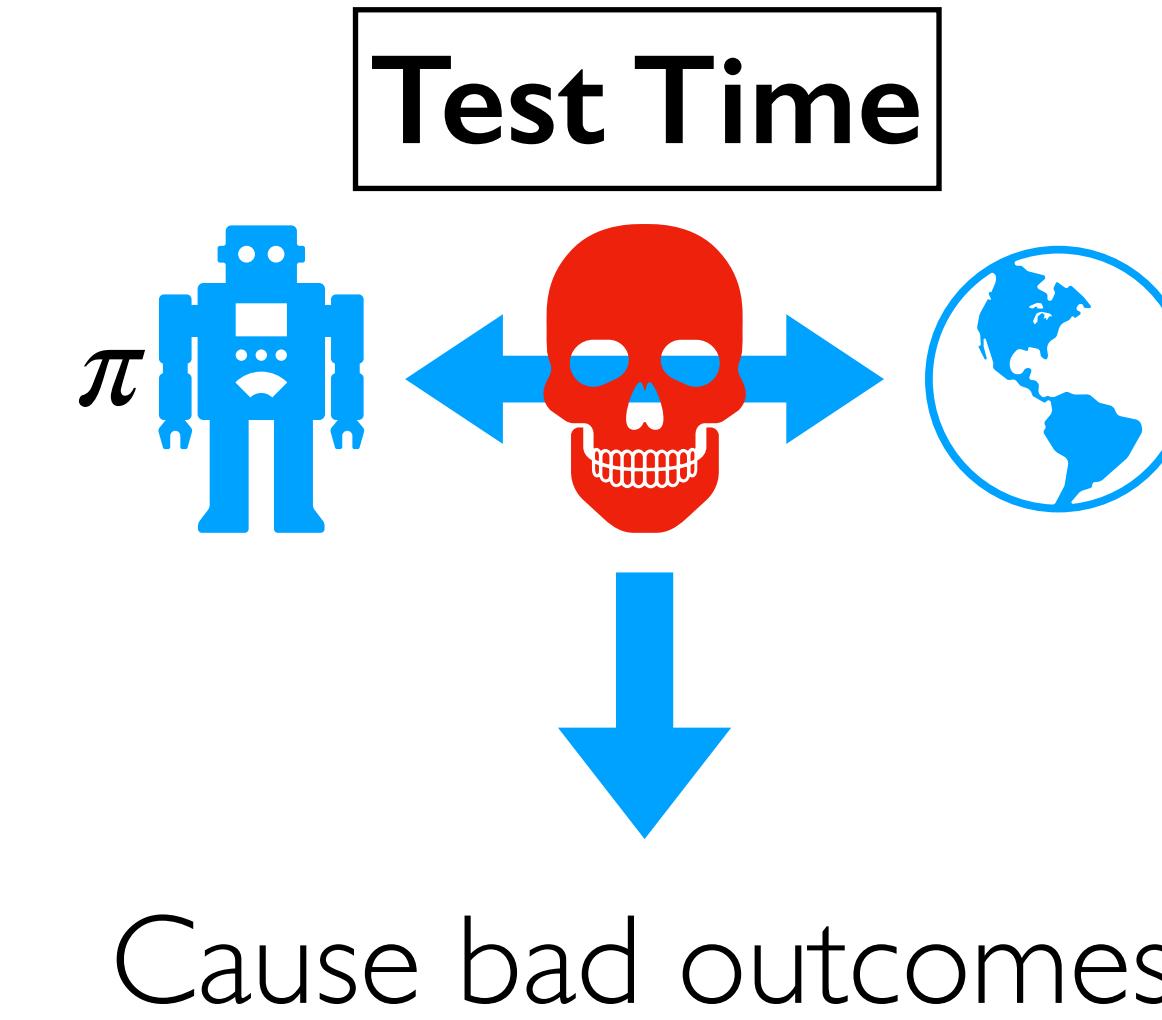
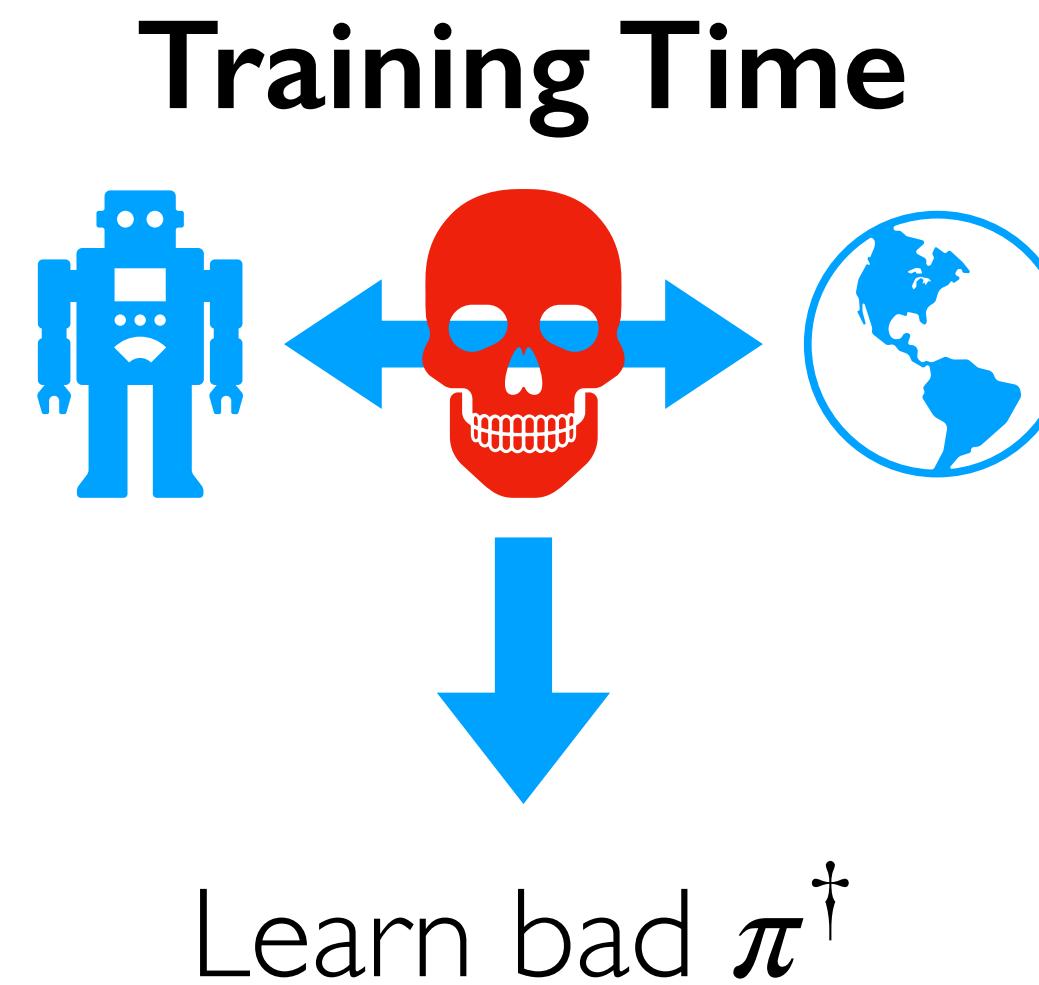
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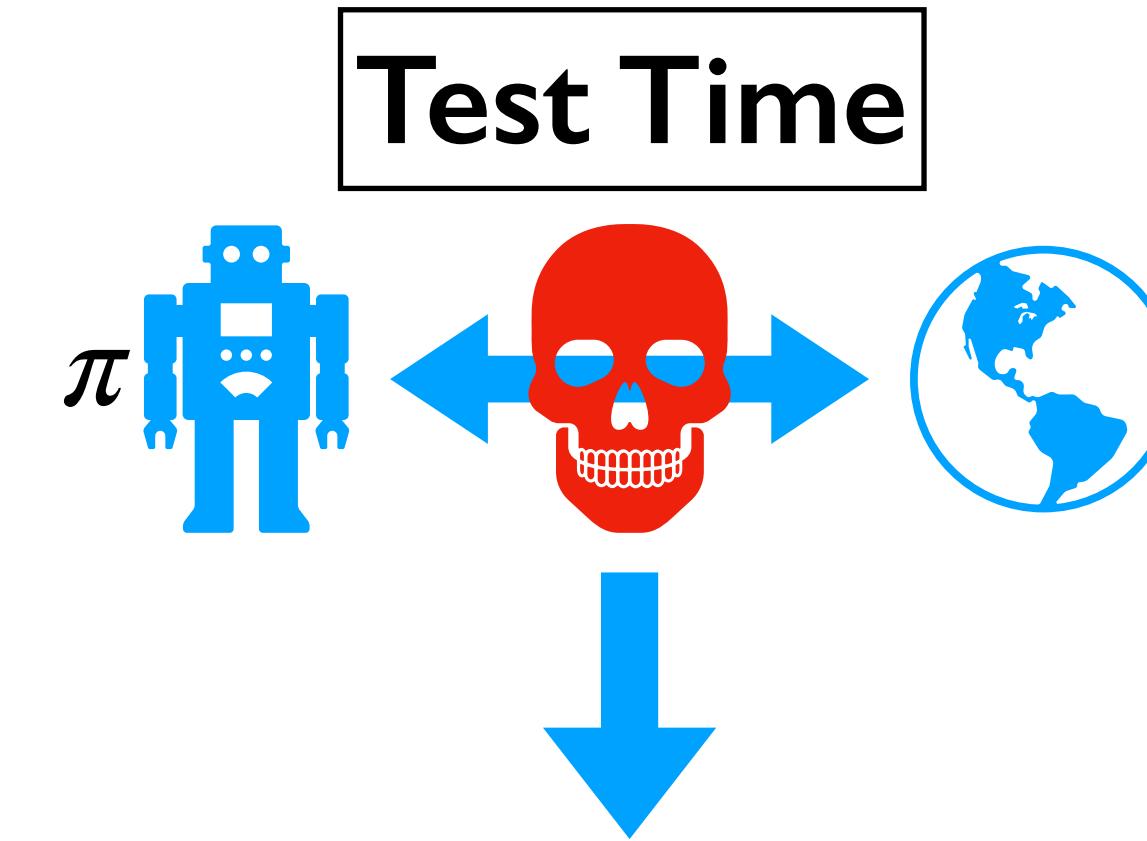
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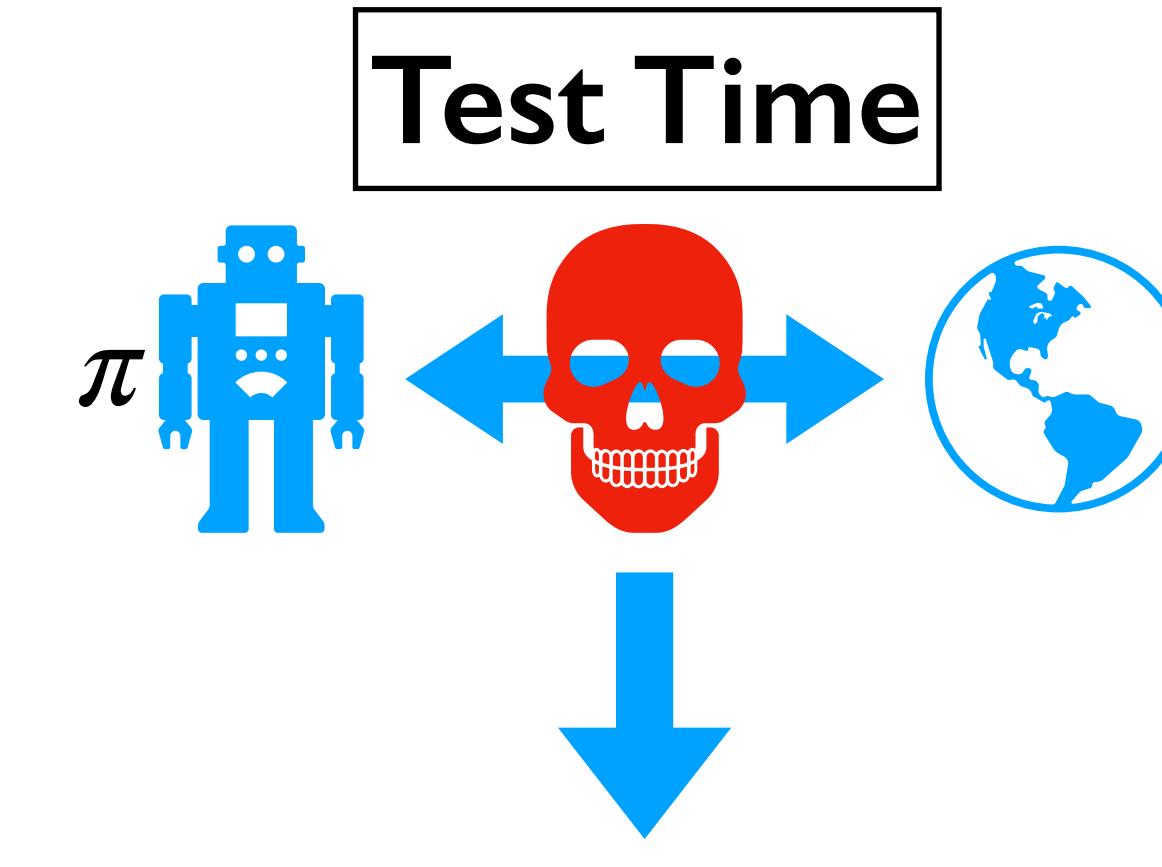
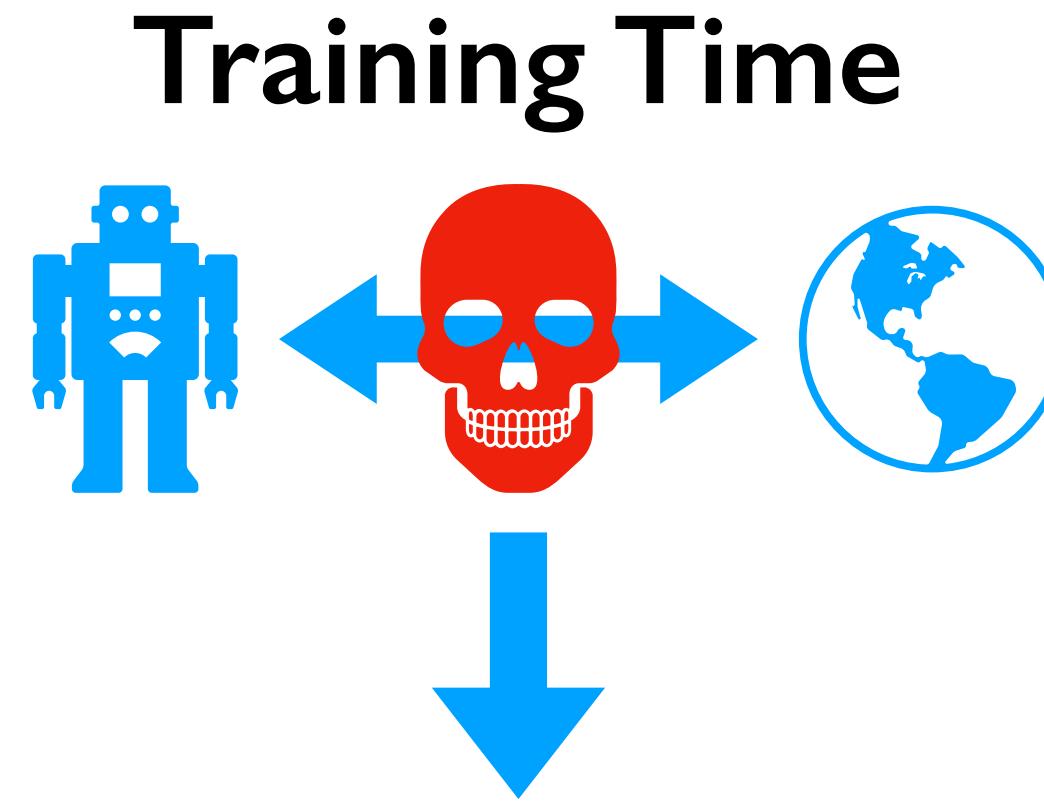
Learn bad $\pi^{'}$



Cause bad outcomes

Trojan

Attack Paradigms



Trojan

Hybrid: poison training to make policy easily test-time attackable

Panda Example

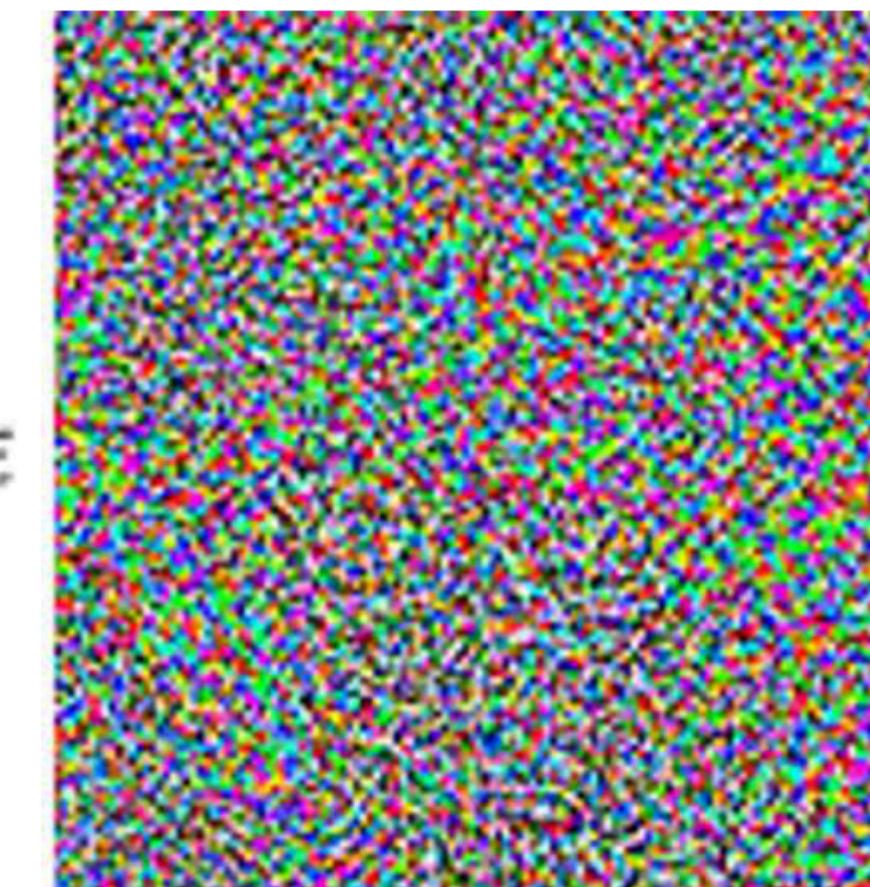
Panda Example

In [Explaining and Harnessing Adversarial Examples](#), Goodfellow and his team added a small perturbation to the image of a panda, as seen below. The result was surprising. Not only did the classifier mark the panda as a gibbon, but did so with high confidence.

As you can see, a barely noticeable disturbance that appears normal to us can easily deceive an ML model into predicting an incorrect class.



$+\epsilon$



=



“panda”

57.7% confidence

“gibbon”

99.3% confidence

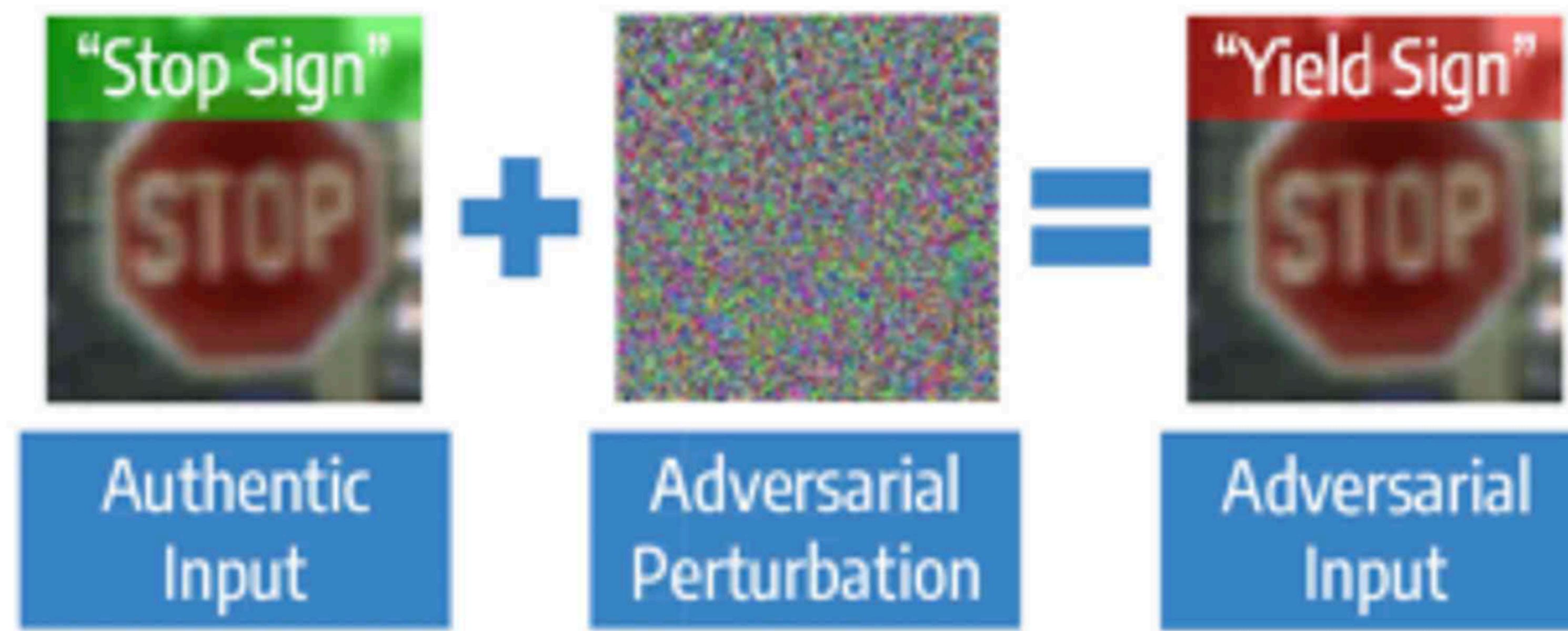
Source: [Goodfellow et al, 2014](#)

Car Crashing

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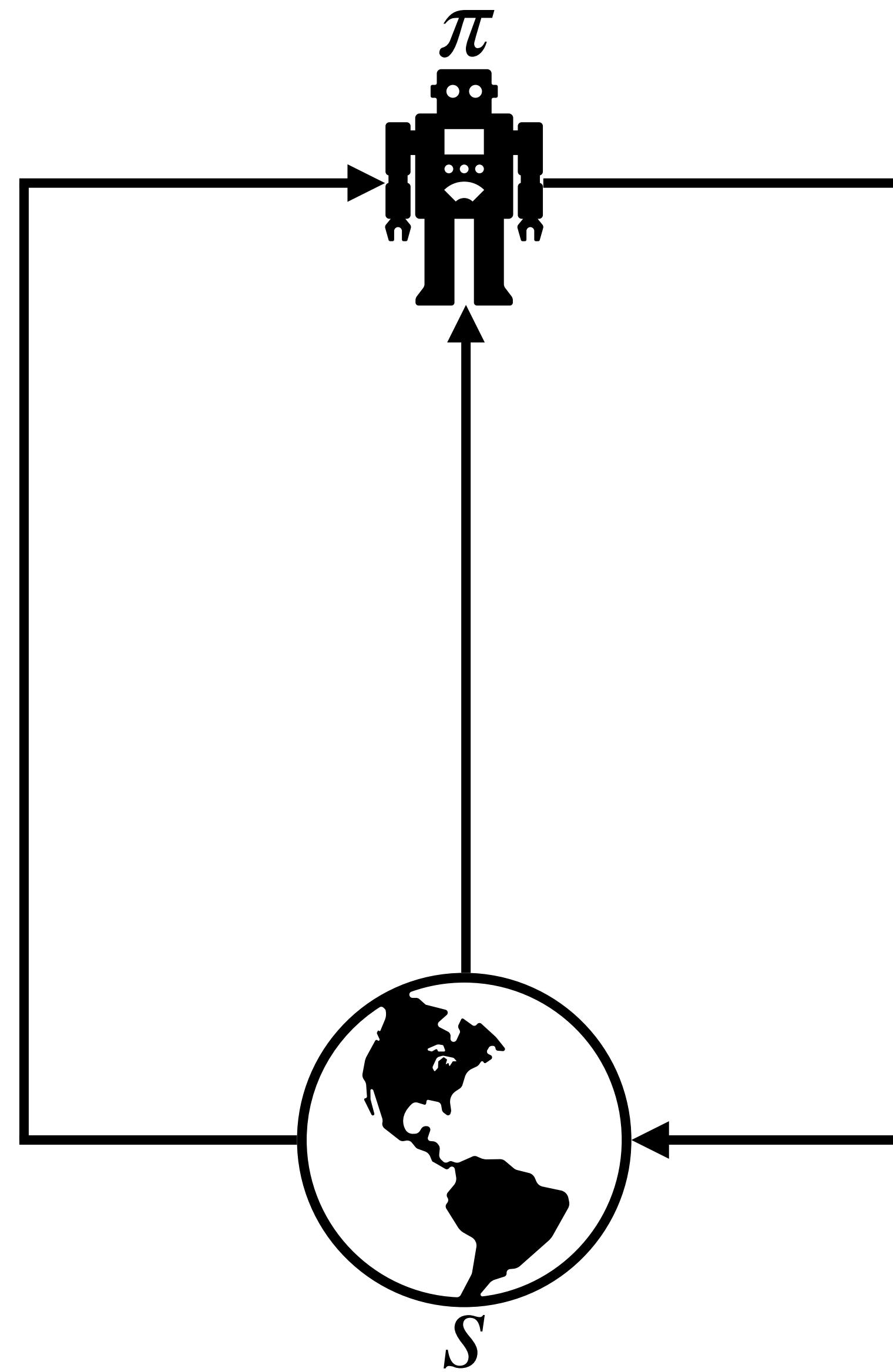
While the panda turned gibbon in the eyes of a machine is a harmless example of an adversarial attack, there are other forms of danger we must watch out for.

For instance, adversarial examples can also be used to [hijack the ML models behind autonomous vehicles](#), causing them to misclassify 'stop' signs as 'yield', as seen below.

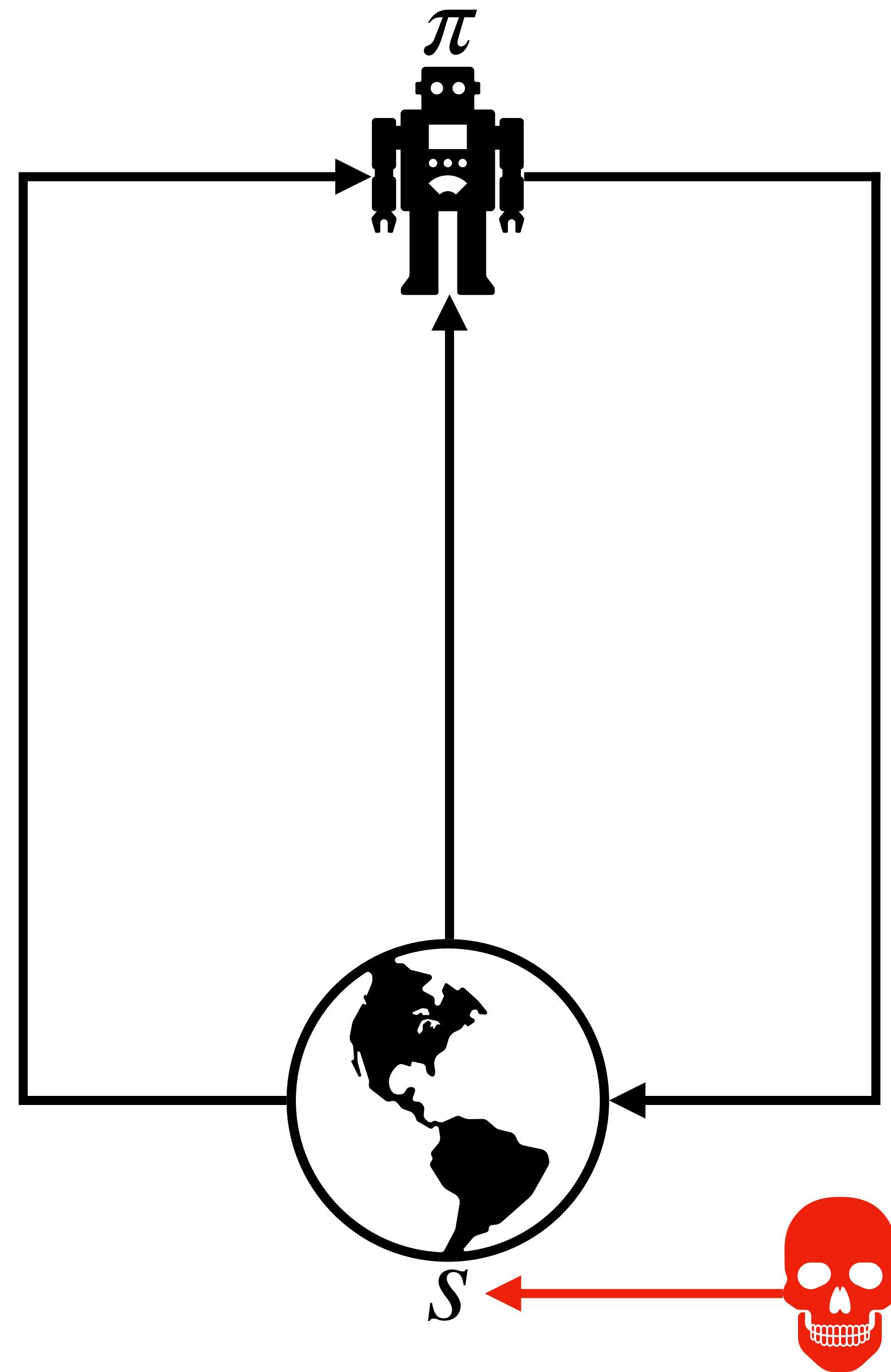


Source: [Kumar et al, 2021](#)

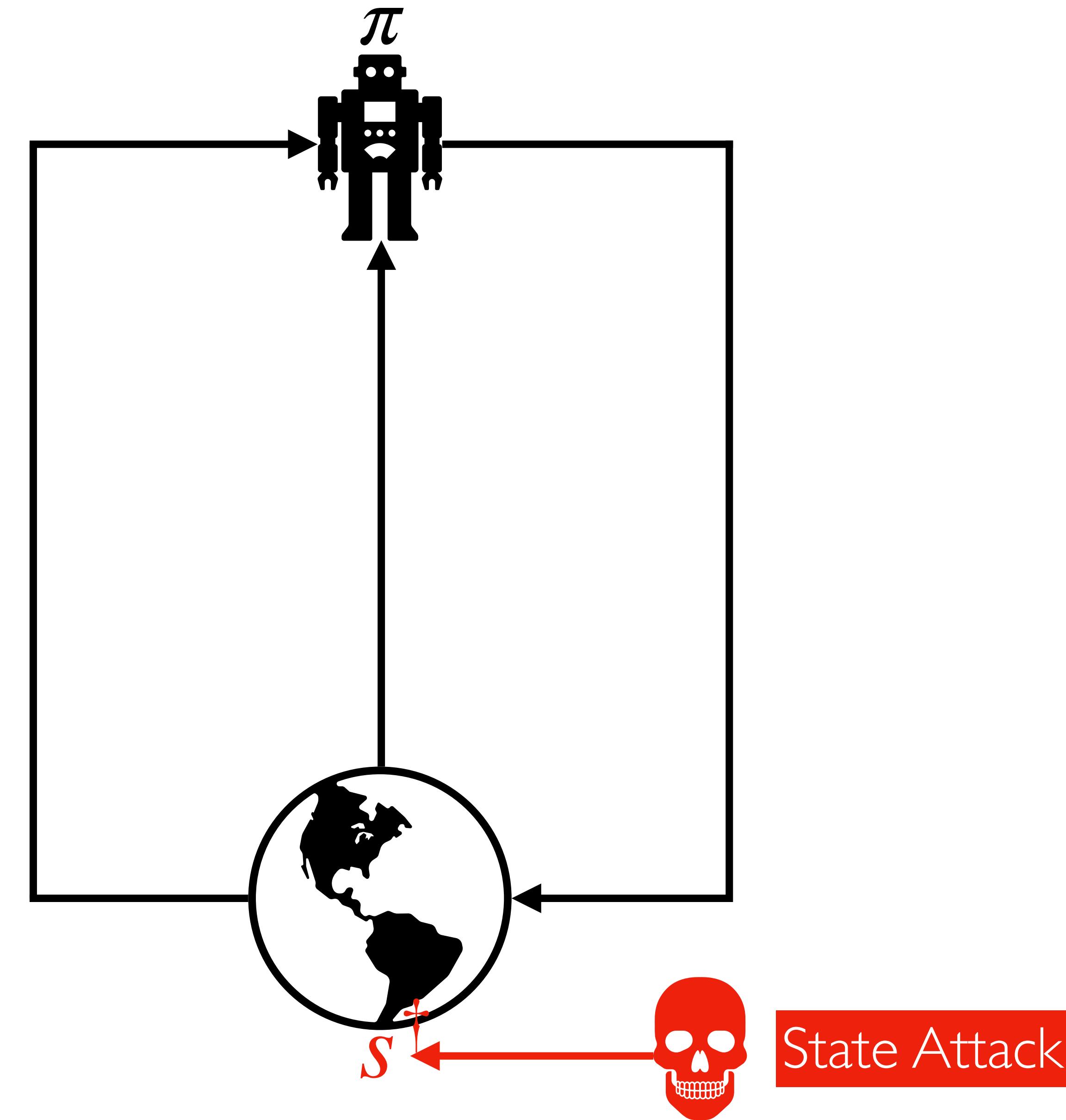
Attack Surfaces



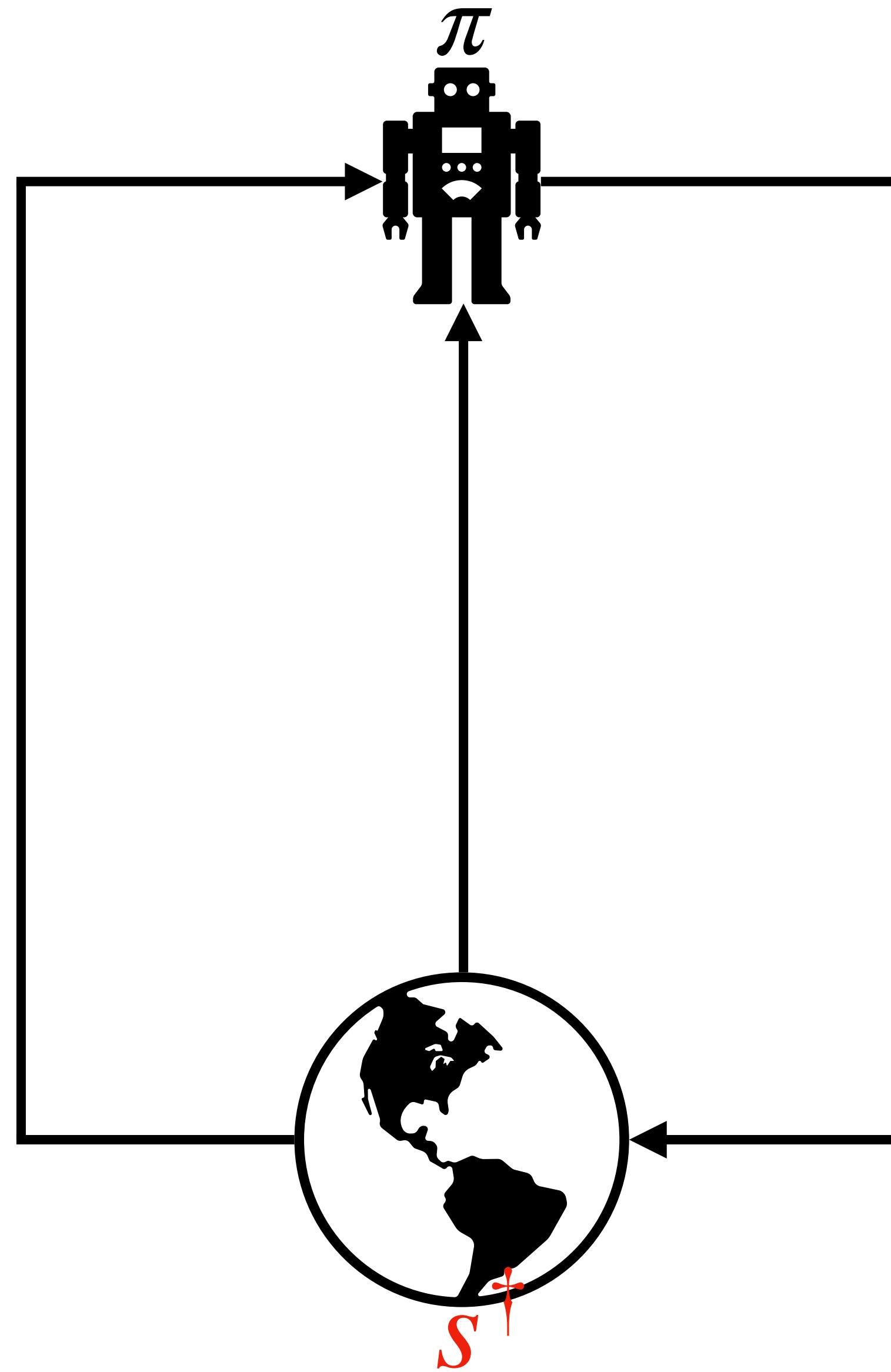
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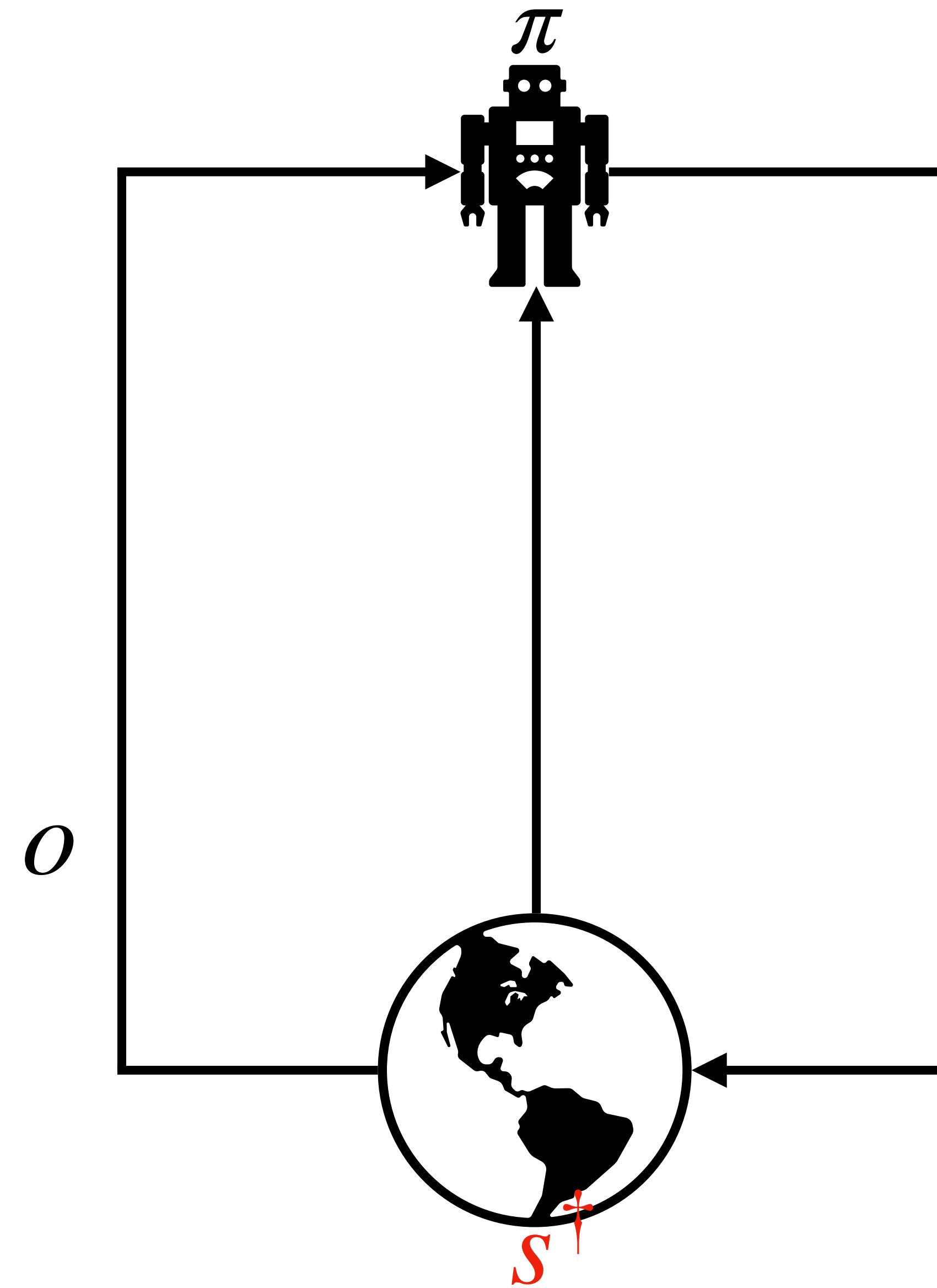
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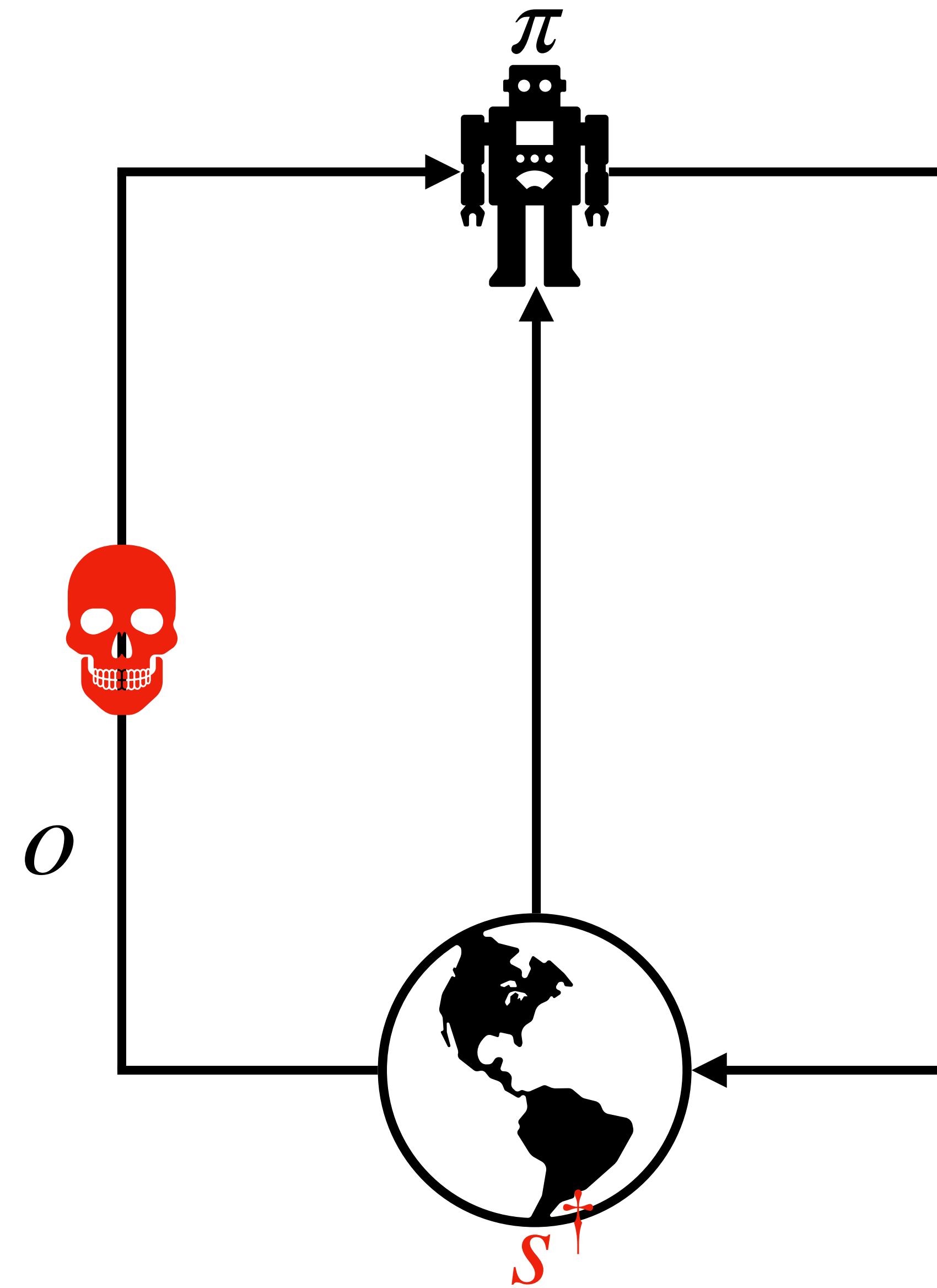
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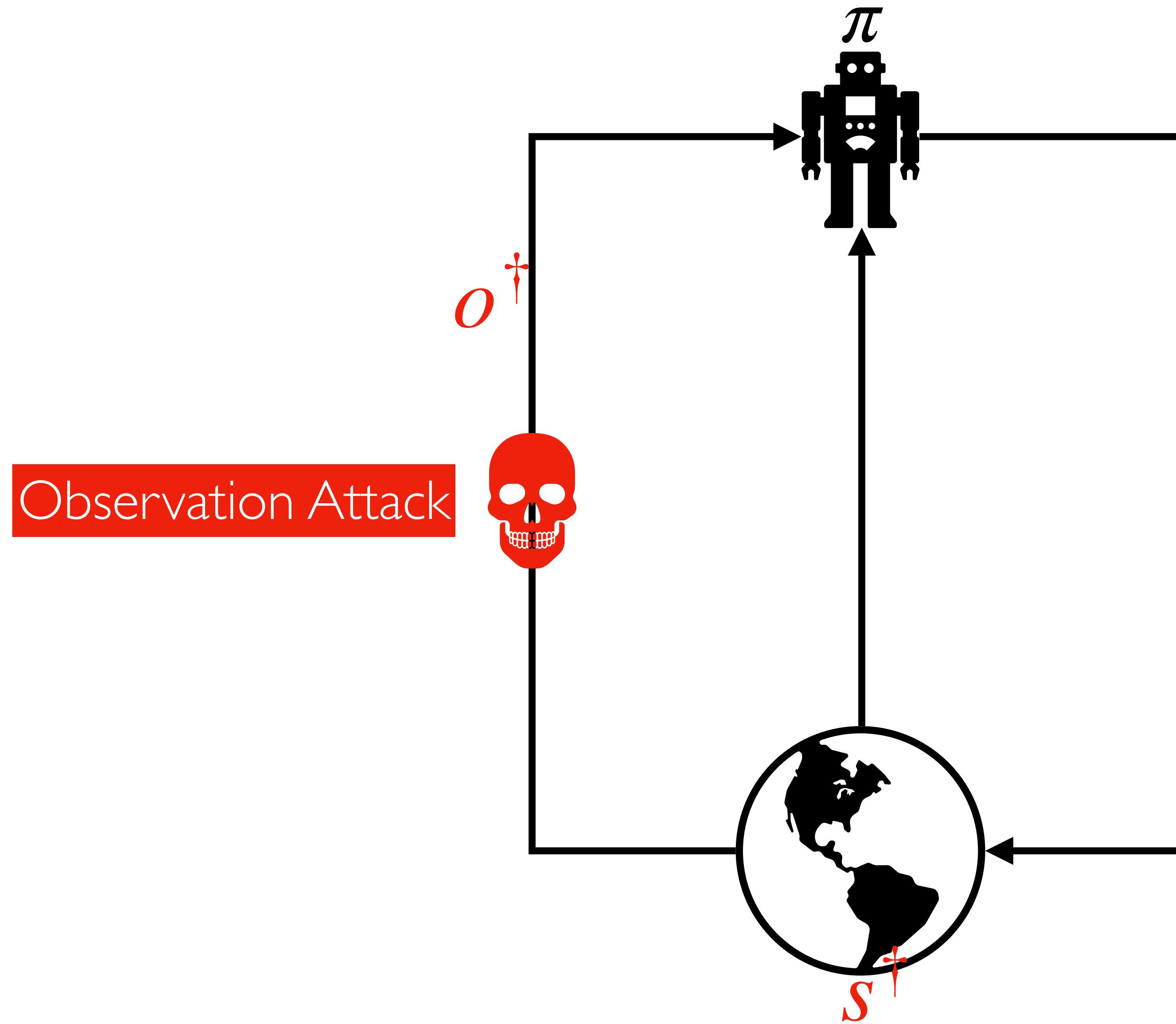
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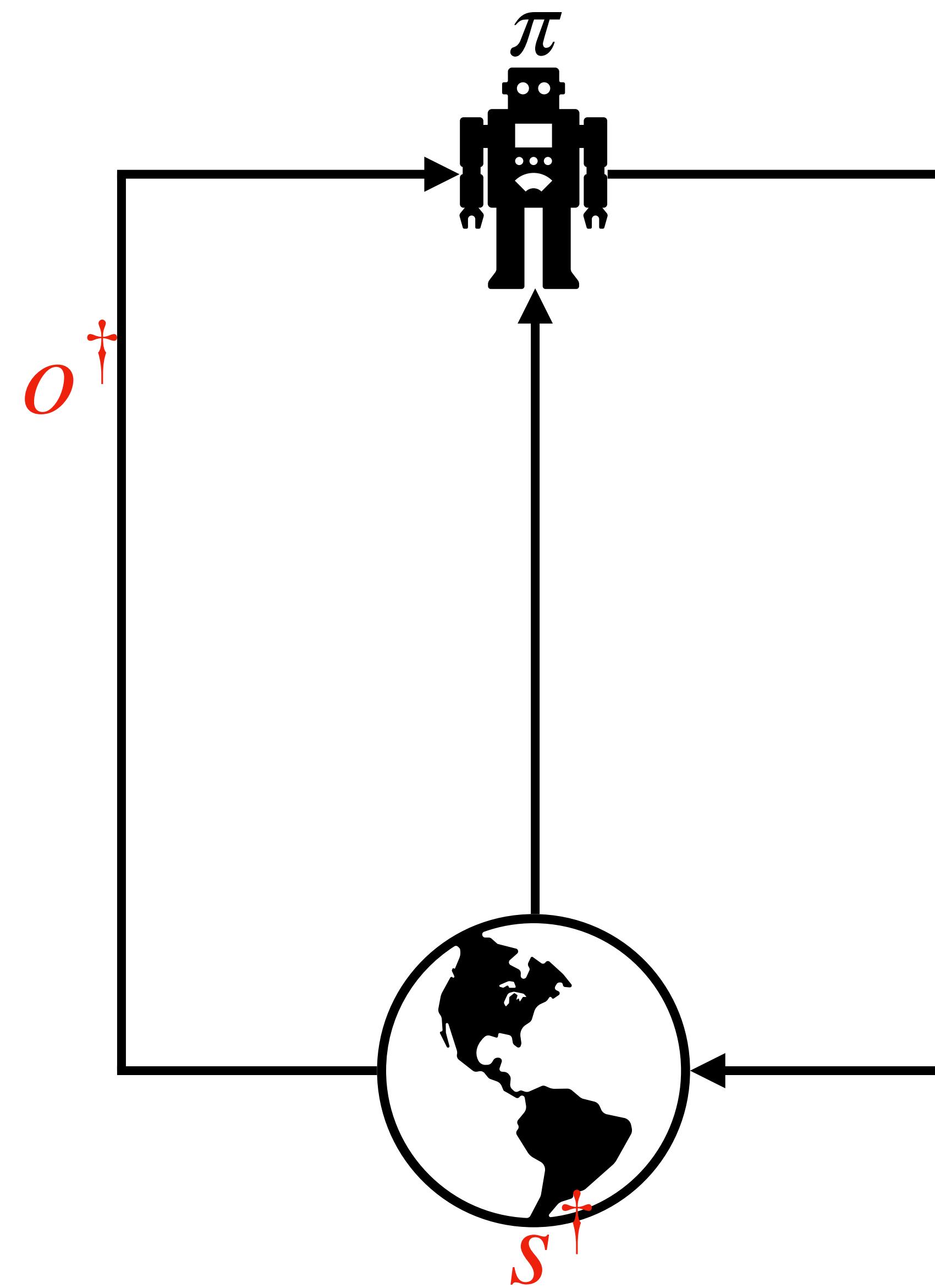
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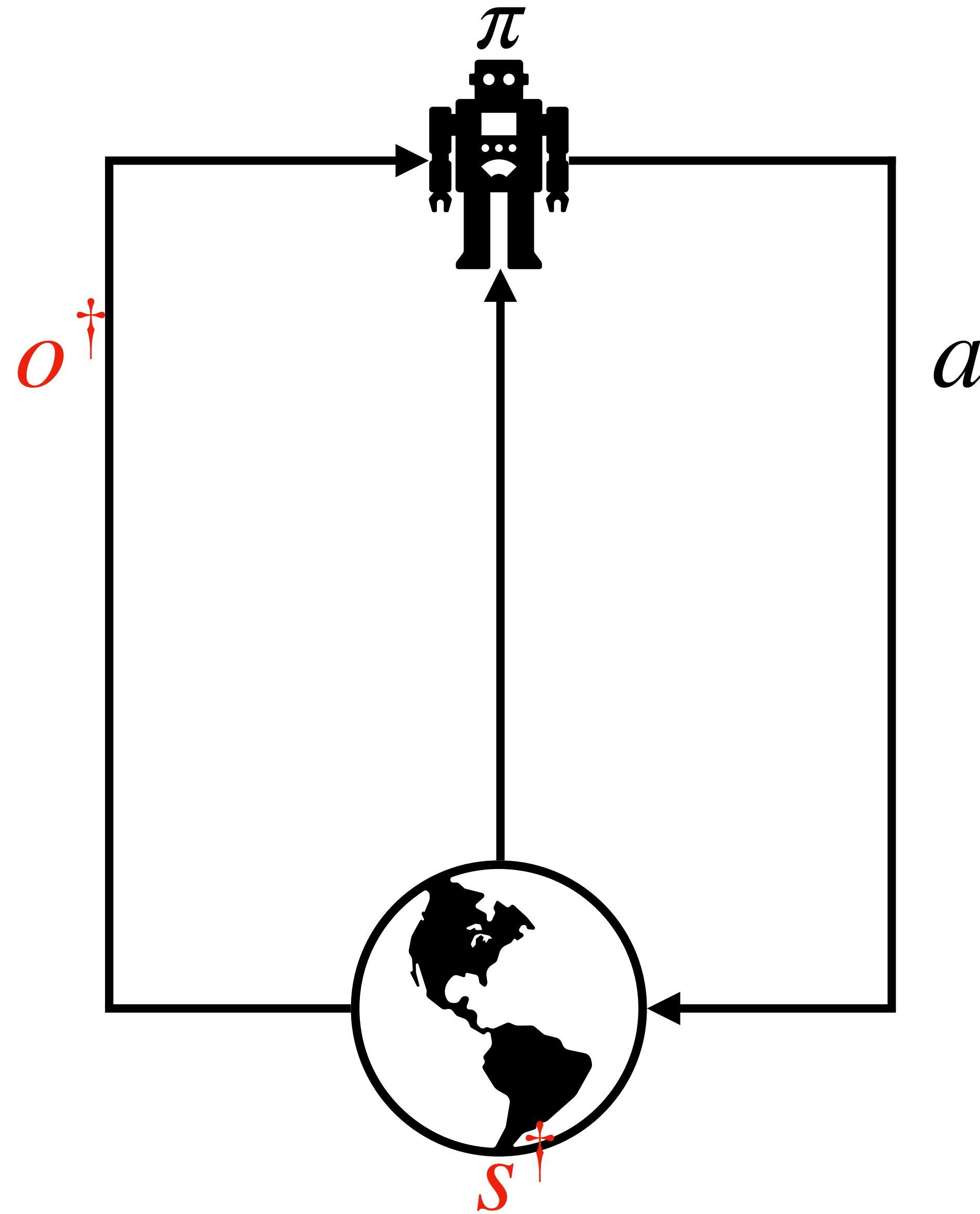
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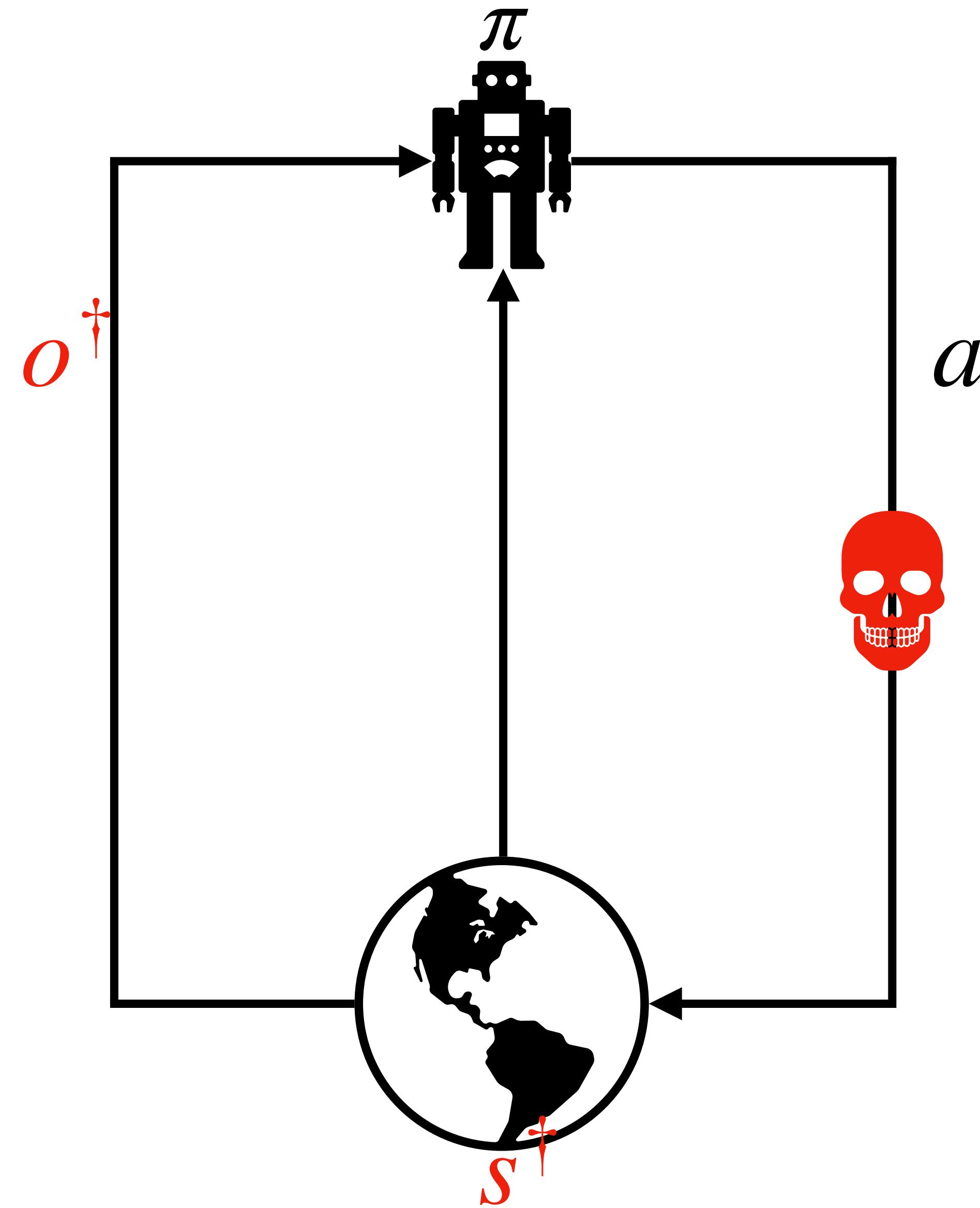
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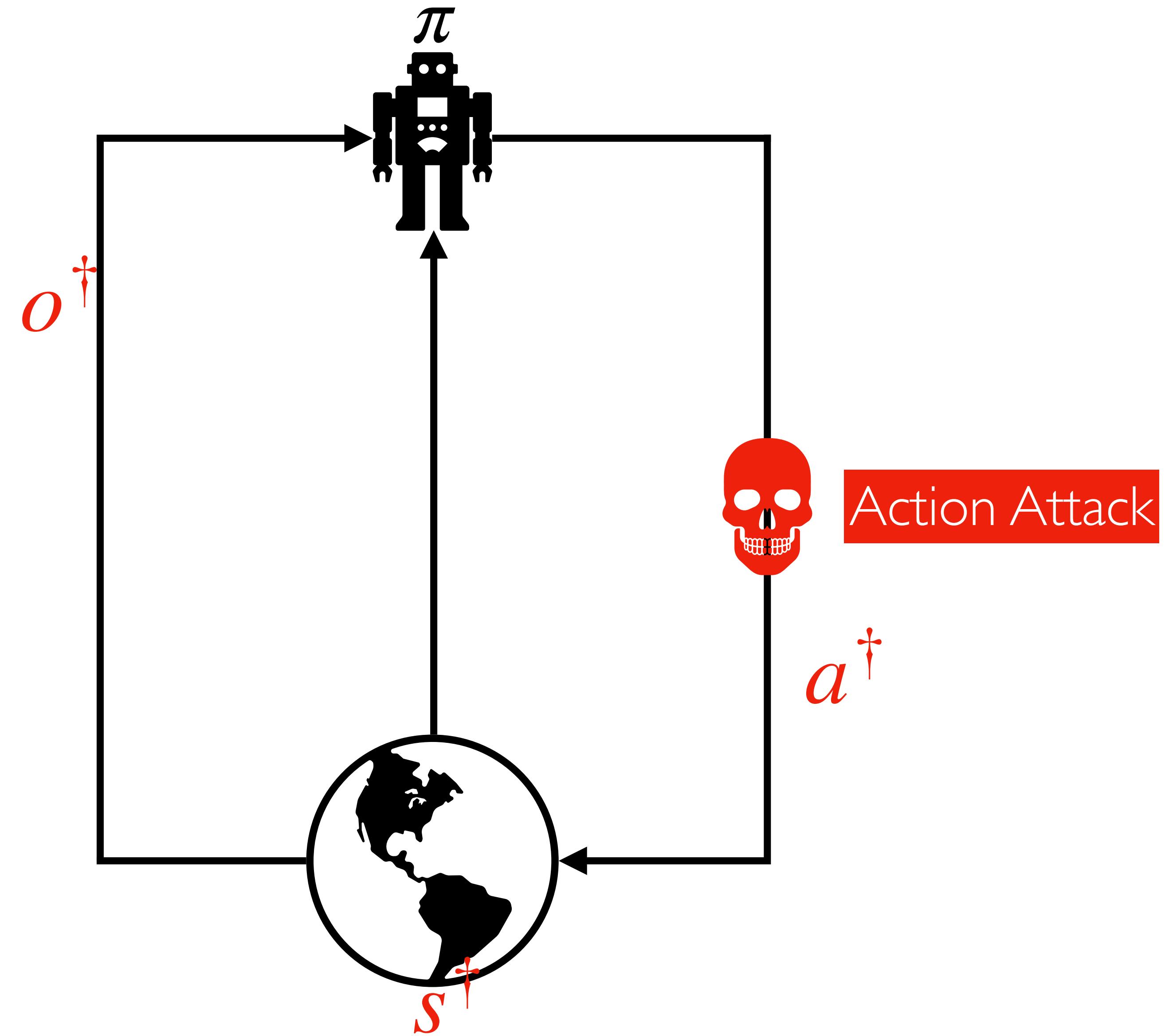
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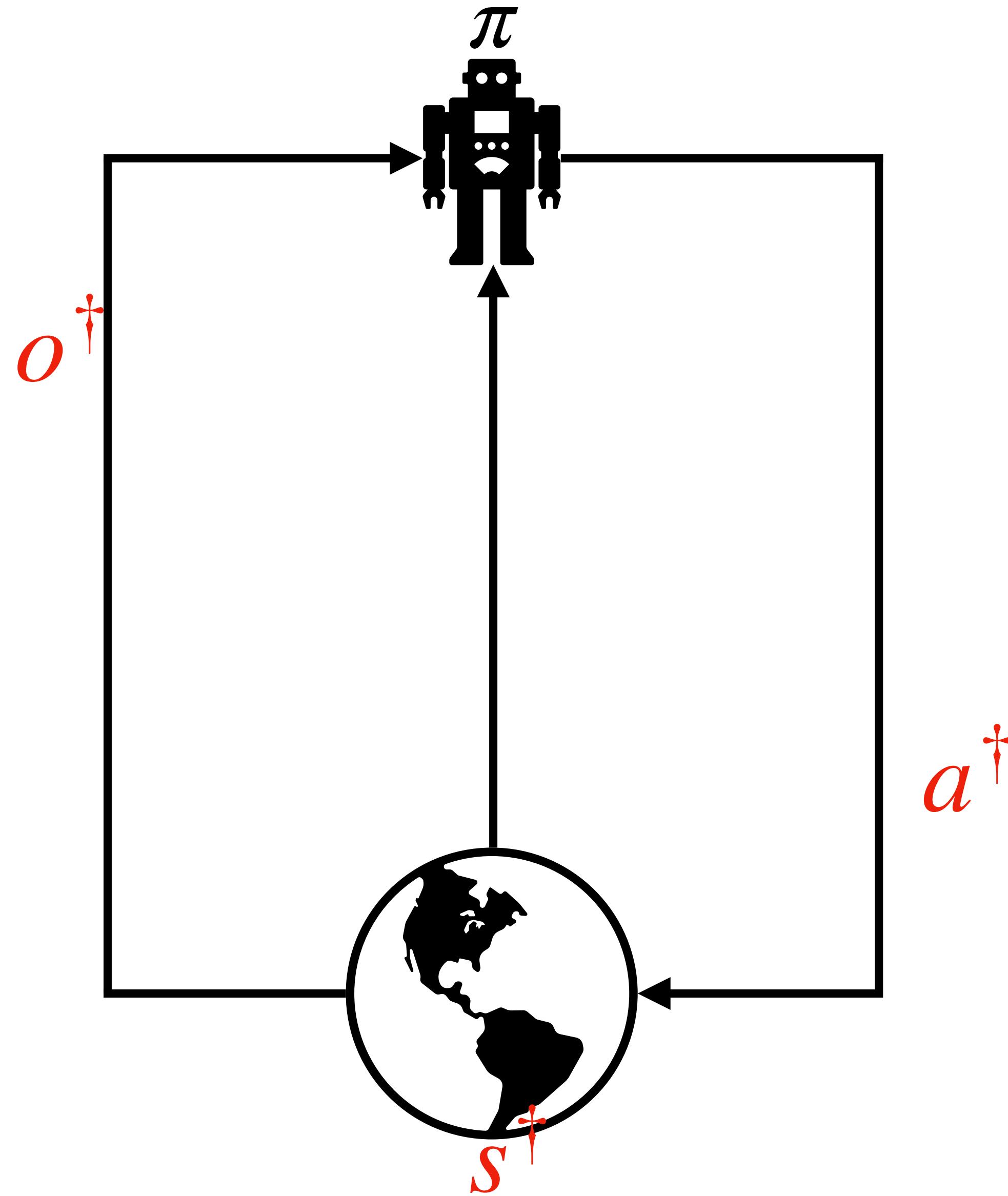
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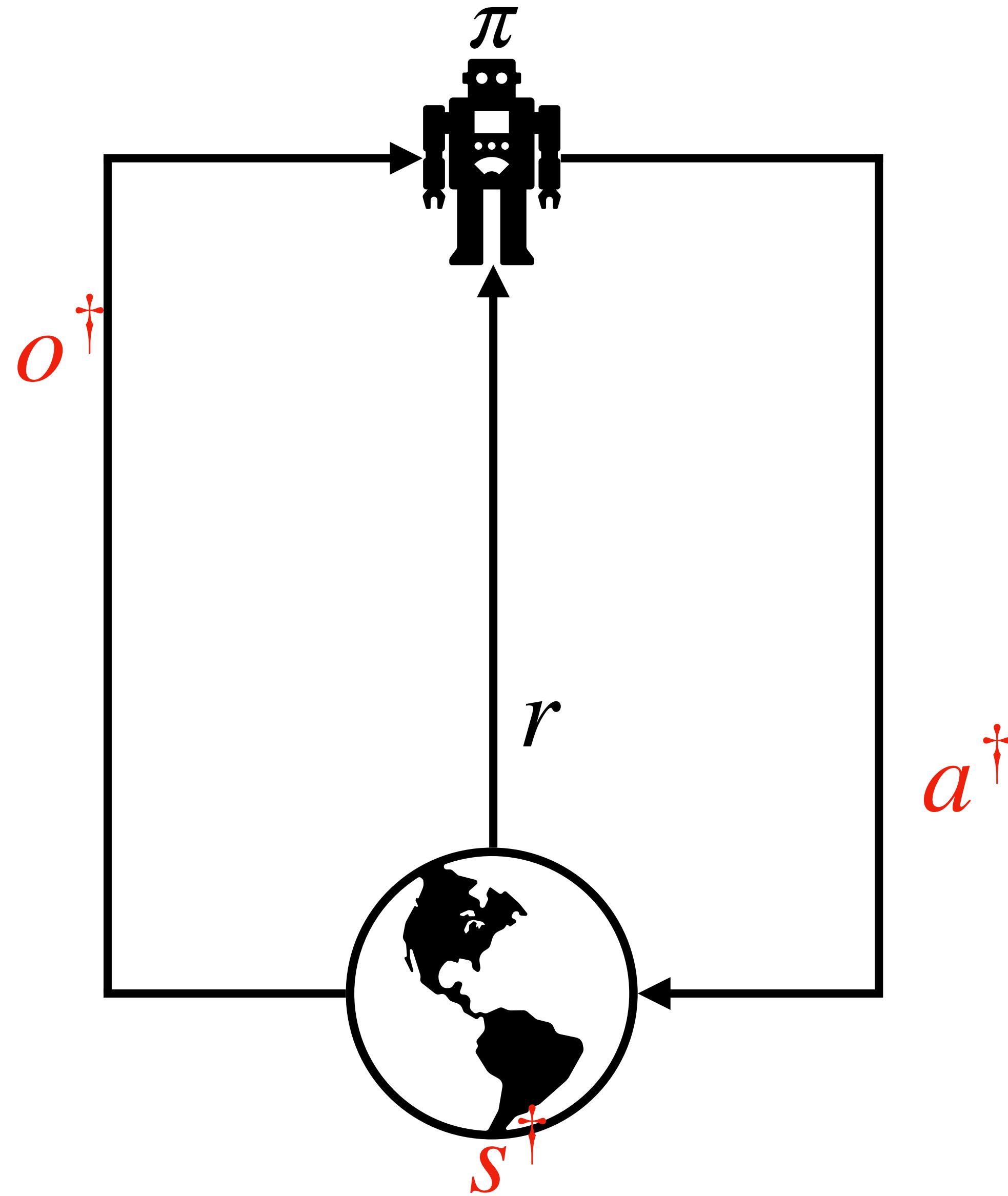
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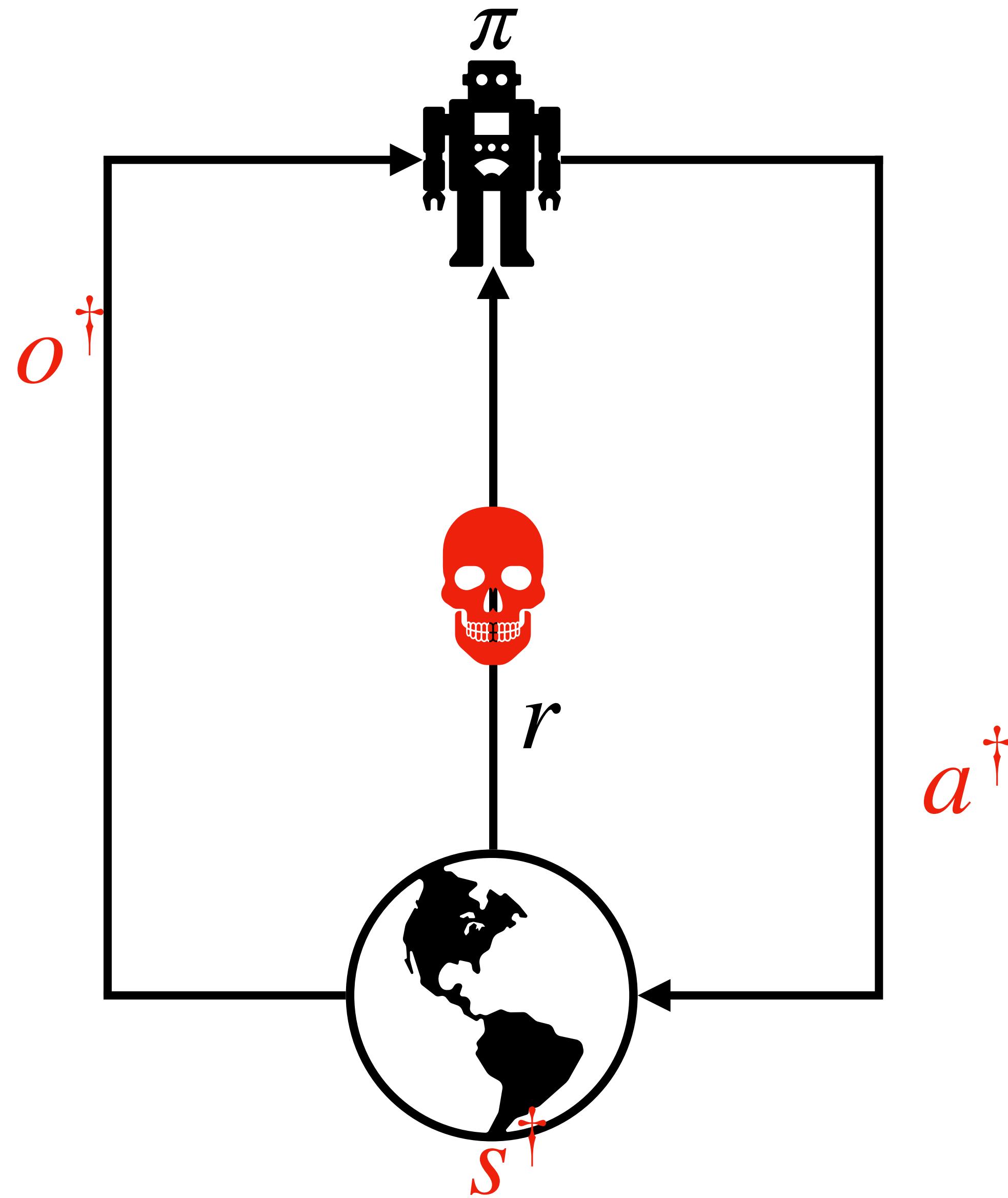
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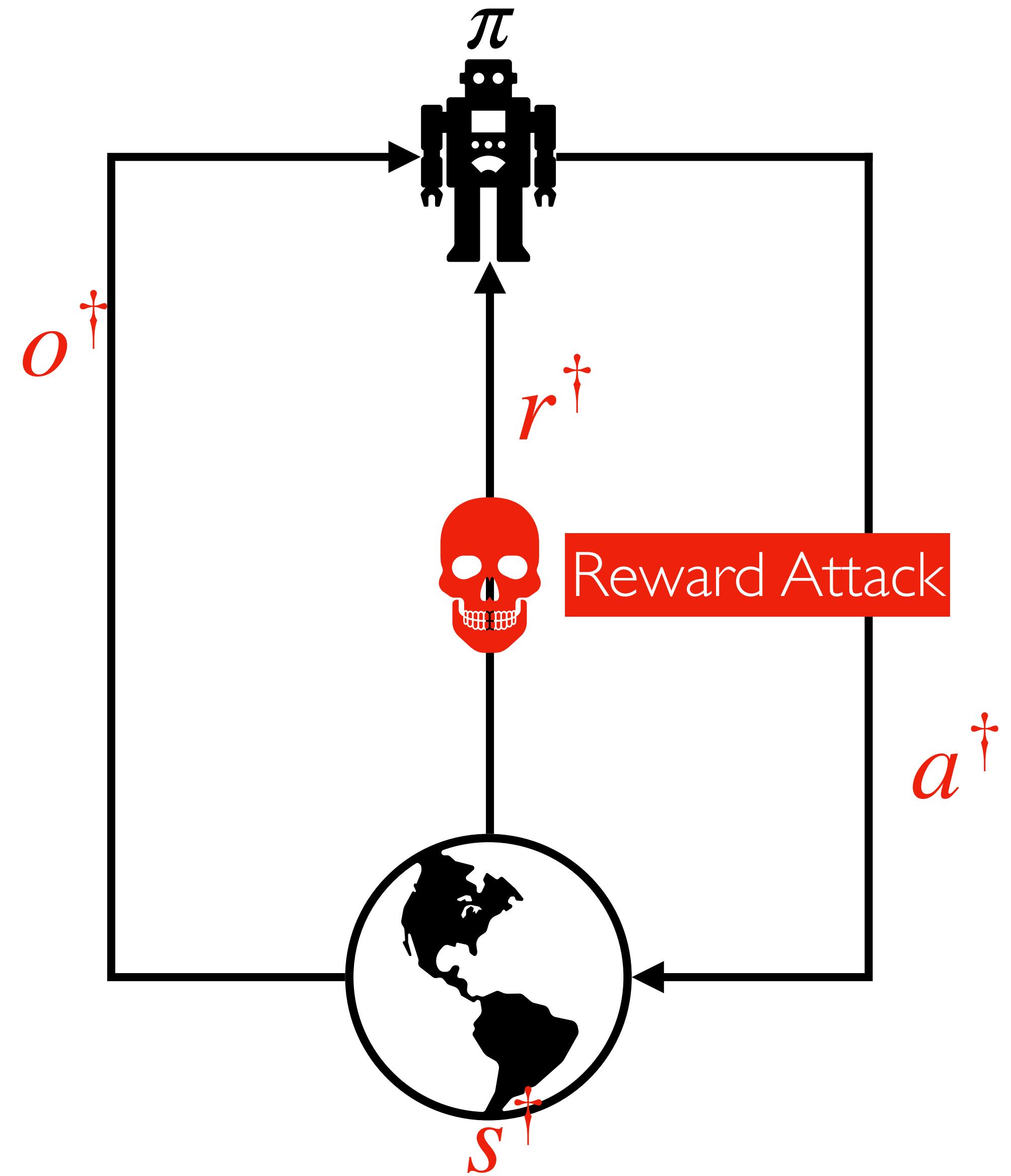
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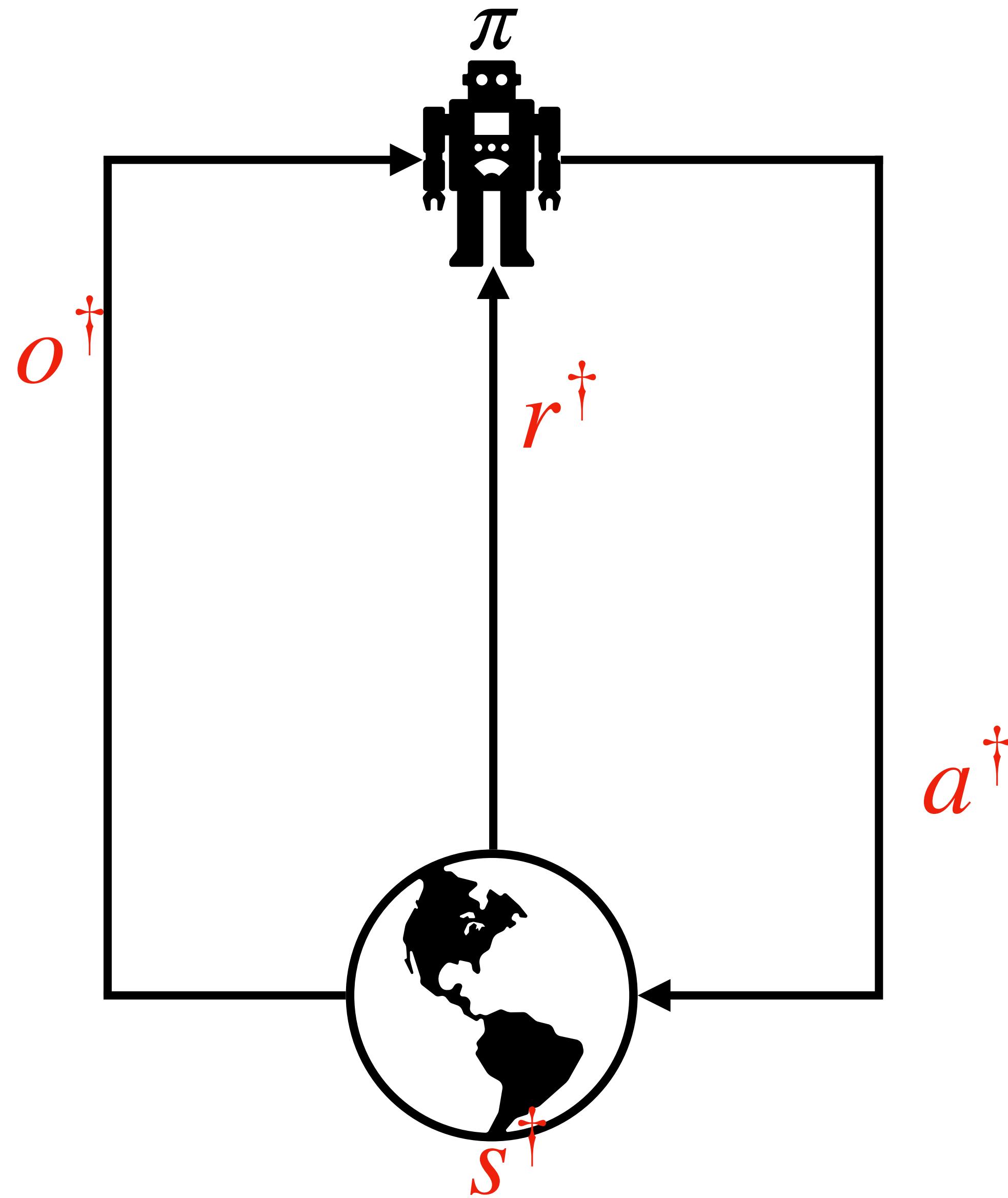
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- **Reward Attack:** changes the agent's reward from r_t to r_t^\dagger .

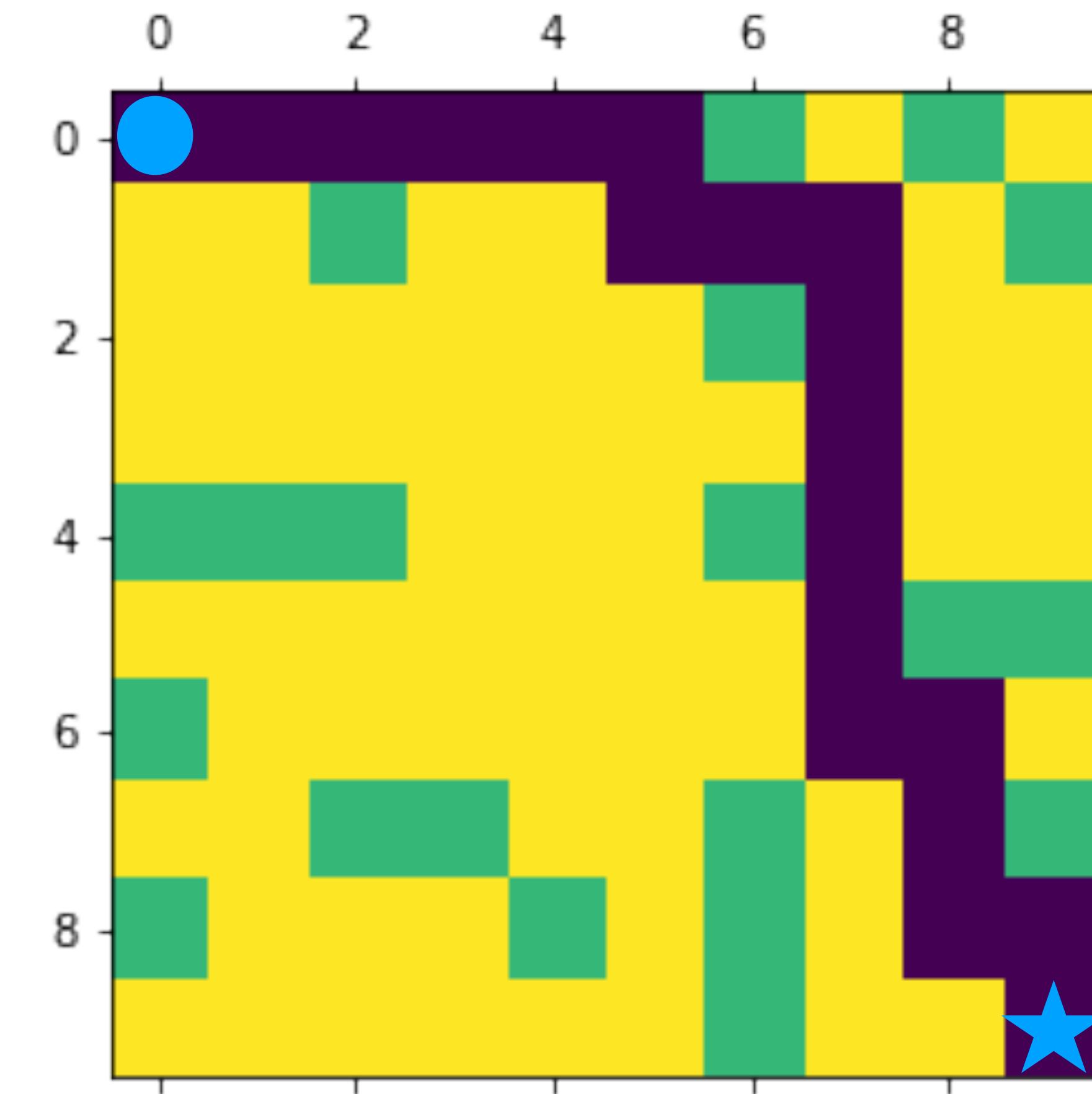
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The attacker can manipulate any element of the interaction tuple (s, a, r) .

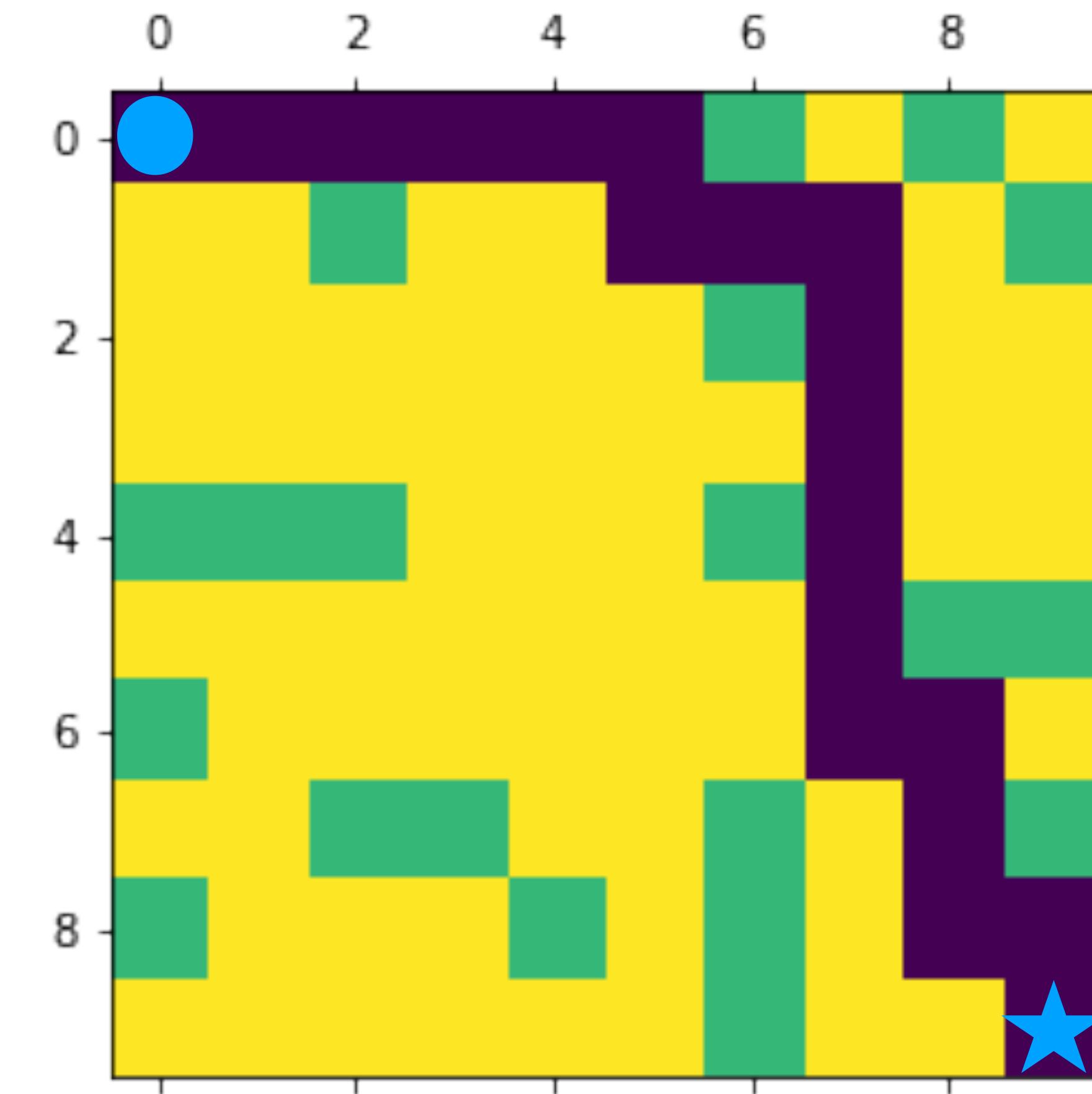
Maze Environment

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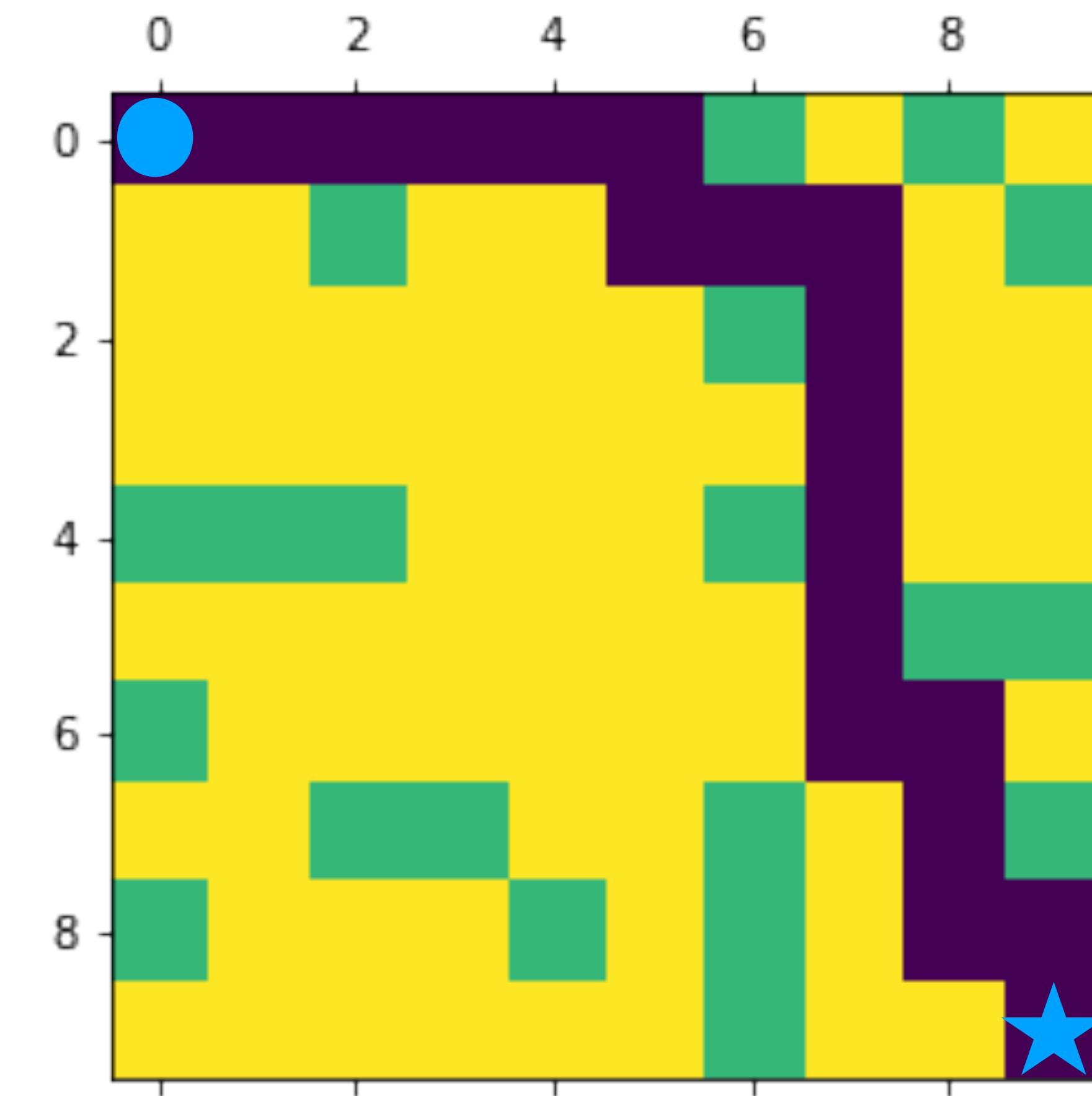
Maze Environment

- Green Squares are obstacles.



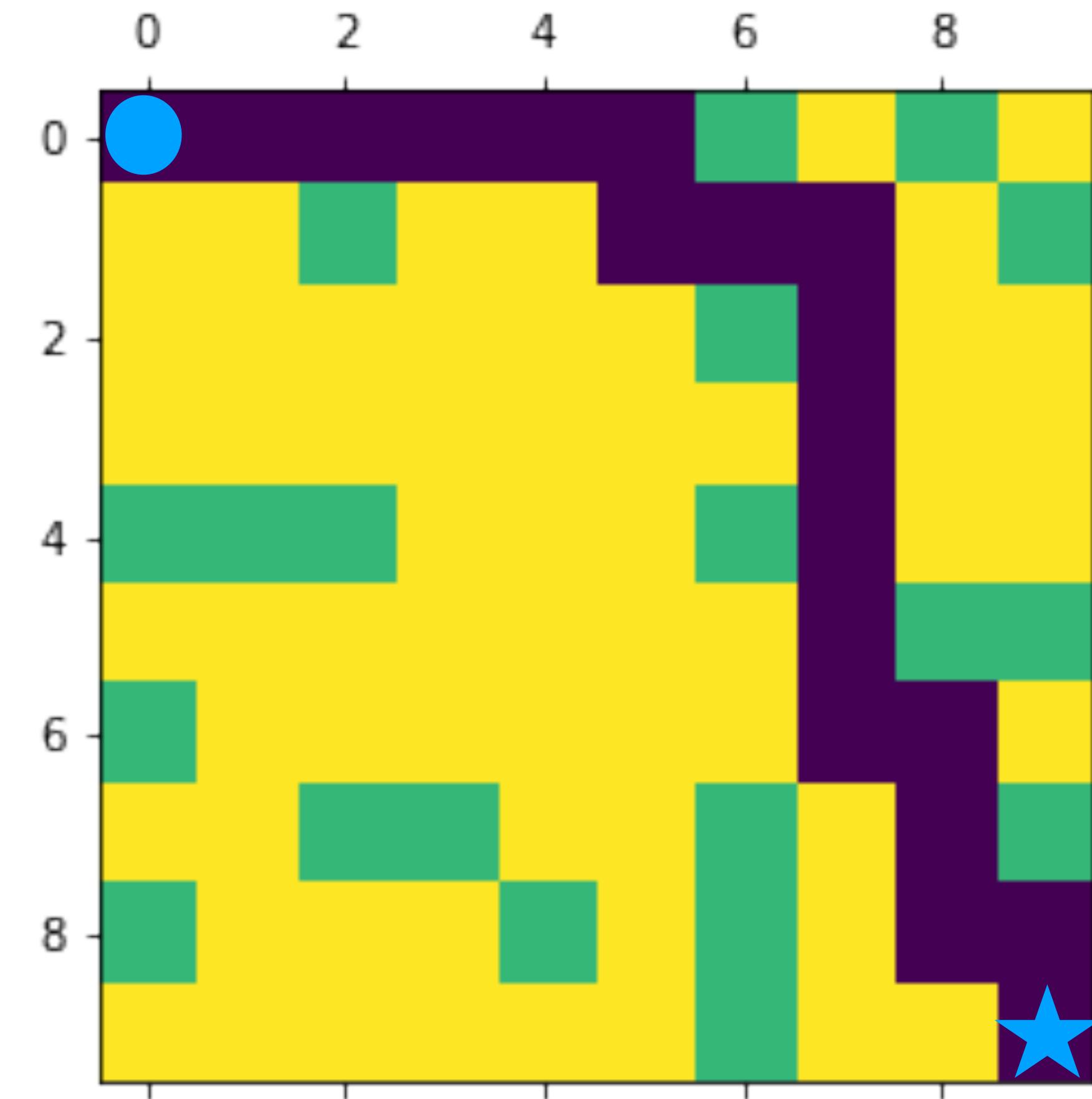
Maze Environment

- **Green** Squares are obstacles.
- **Yellow** Squares are traversable.



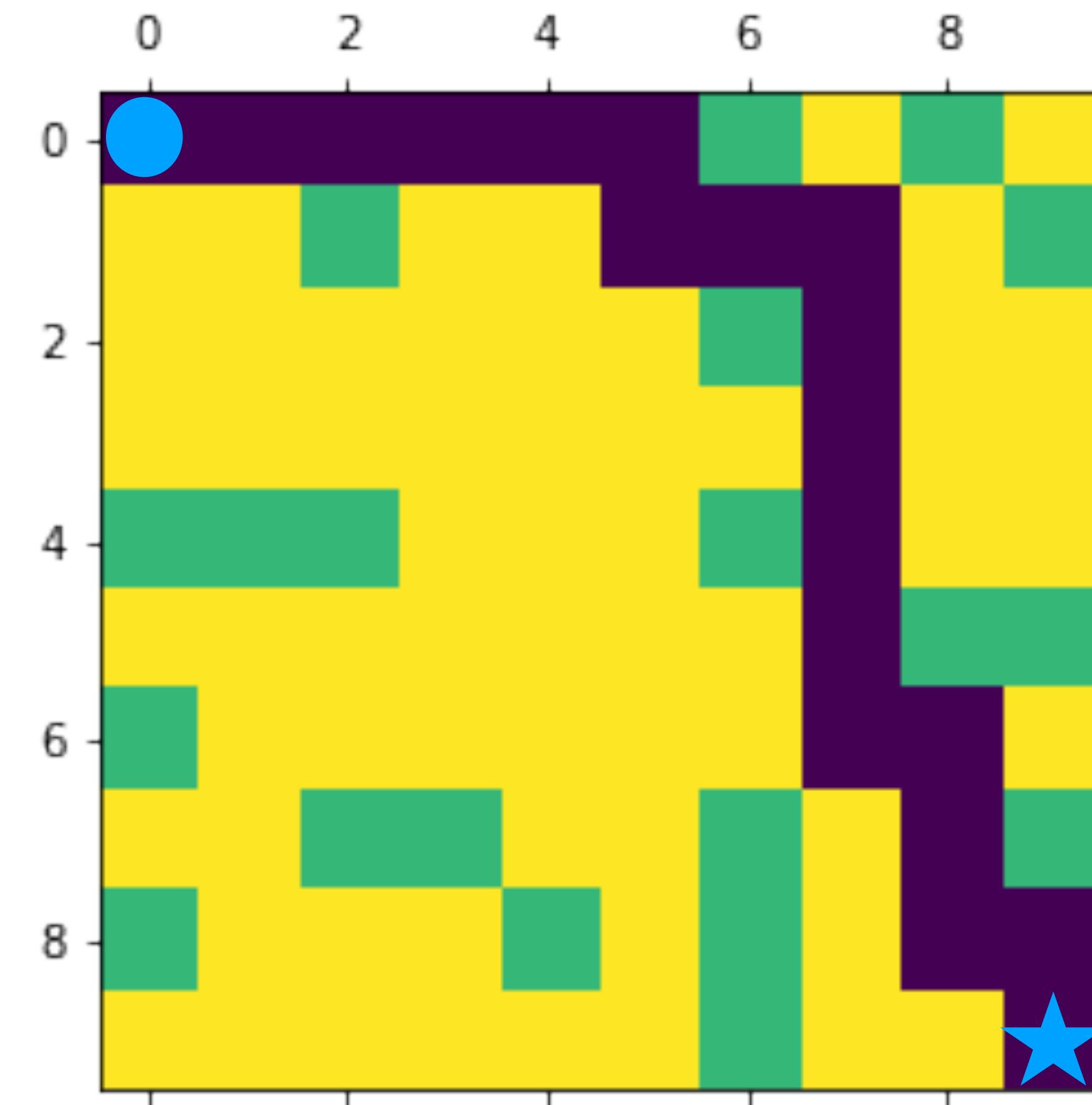
Maze Environment

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- The agent starts at top **left corner**.



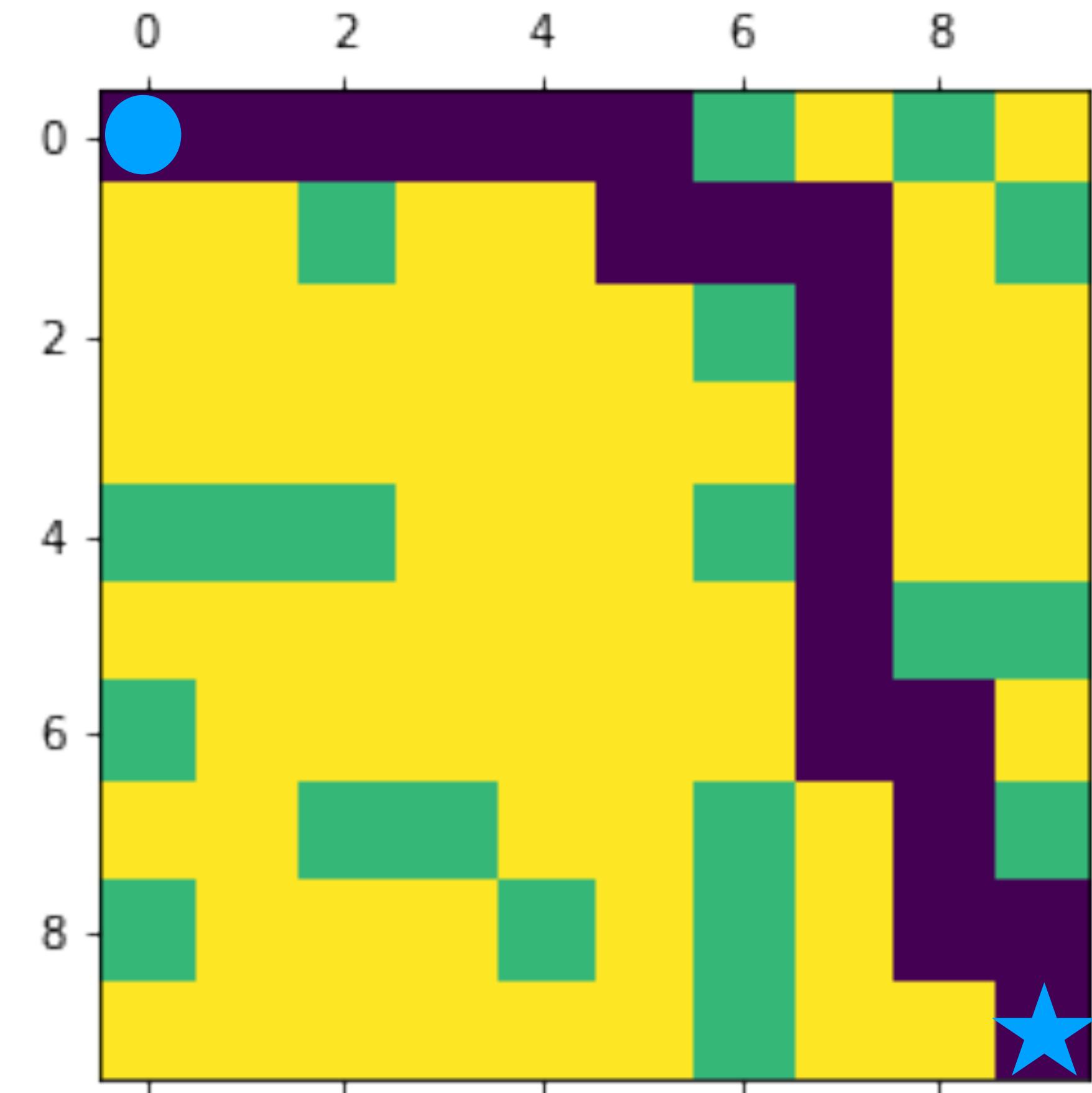
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Maze Environment

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- The agent starts at top **left corner**.
- The agent receives reward only at the **bottom right** corner.
- An optimal (shortest path) policy for the agent is in **purple**.



Perceived-State Attack

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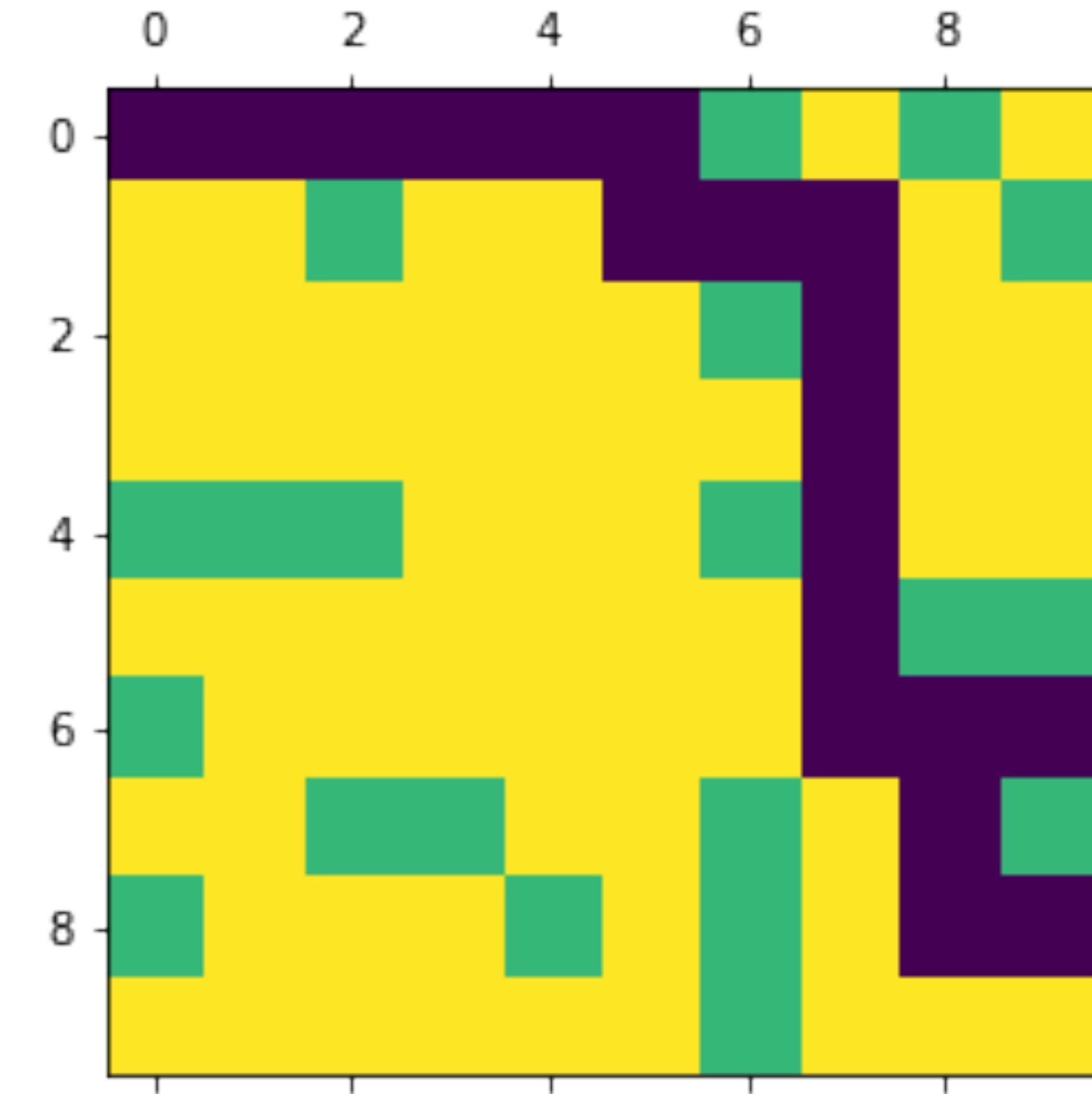
- Attacker shows agent s^\dagger .

Perceived-State Attack

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- Agent chooses action $\pi(s^\dagger)$ instead of $\pi(s)$

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Perceived-State Attack



Perceived-State Attack



Perceived-State Attack



Check out Shubham's full paper in
Neurips22!

**Provable Defense against Backdoor
Policies in Reinforcement Learning**

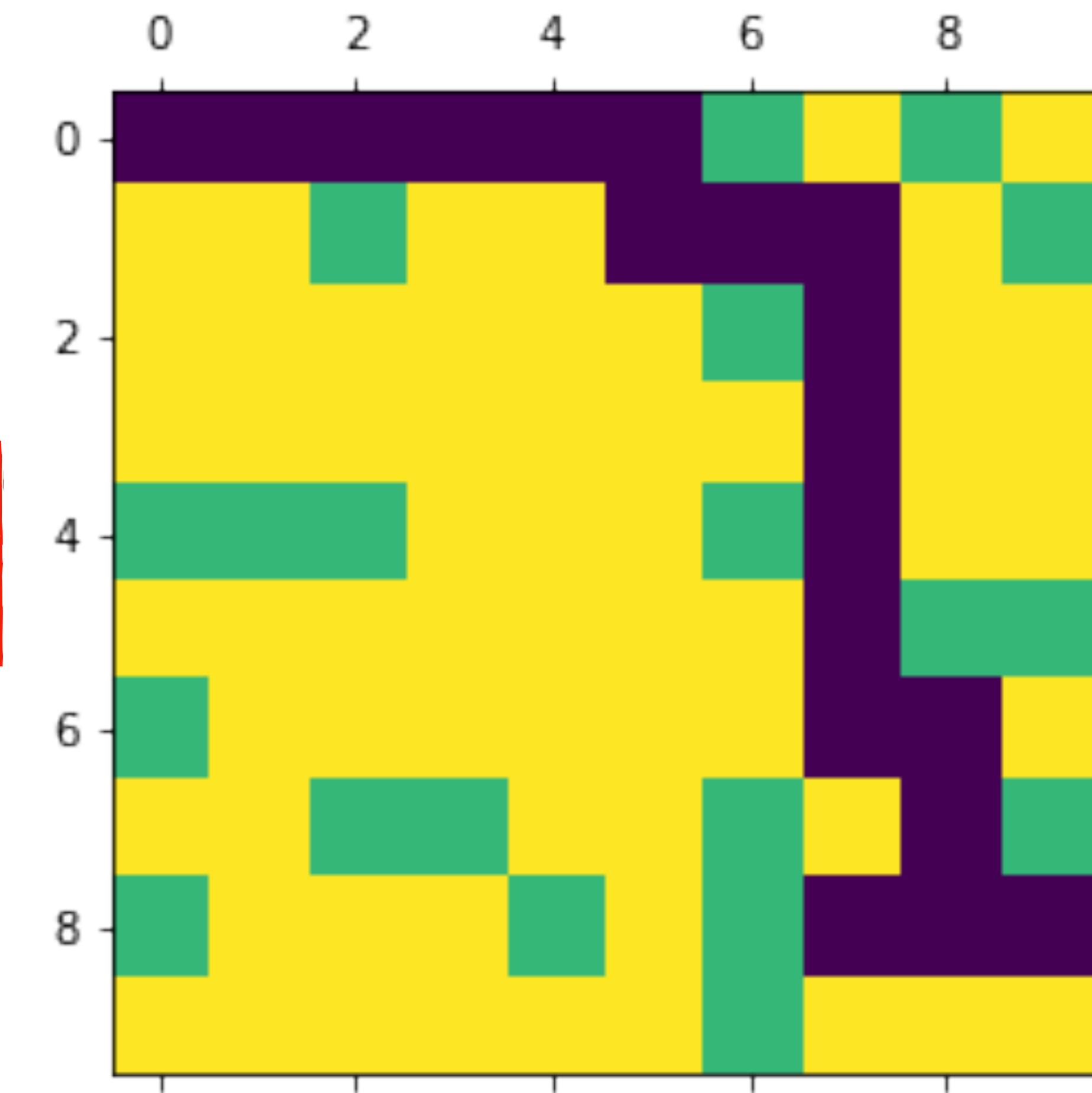
Action Attack

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Attacker intercepts $a = \pi(s)$ and sends a^\dagger to the environment instead.

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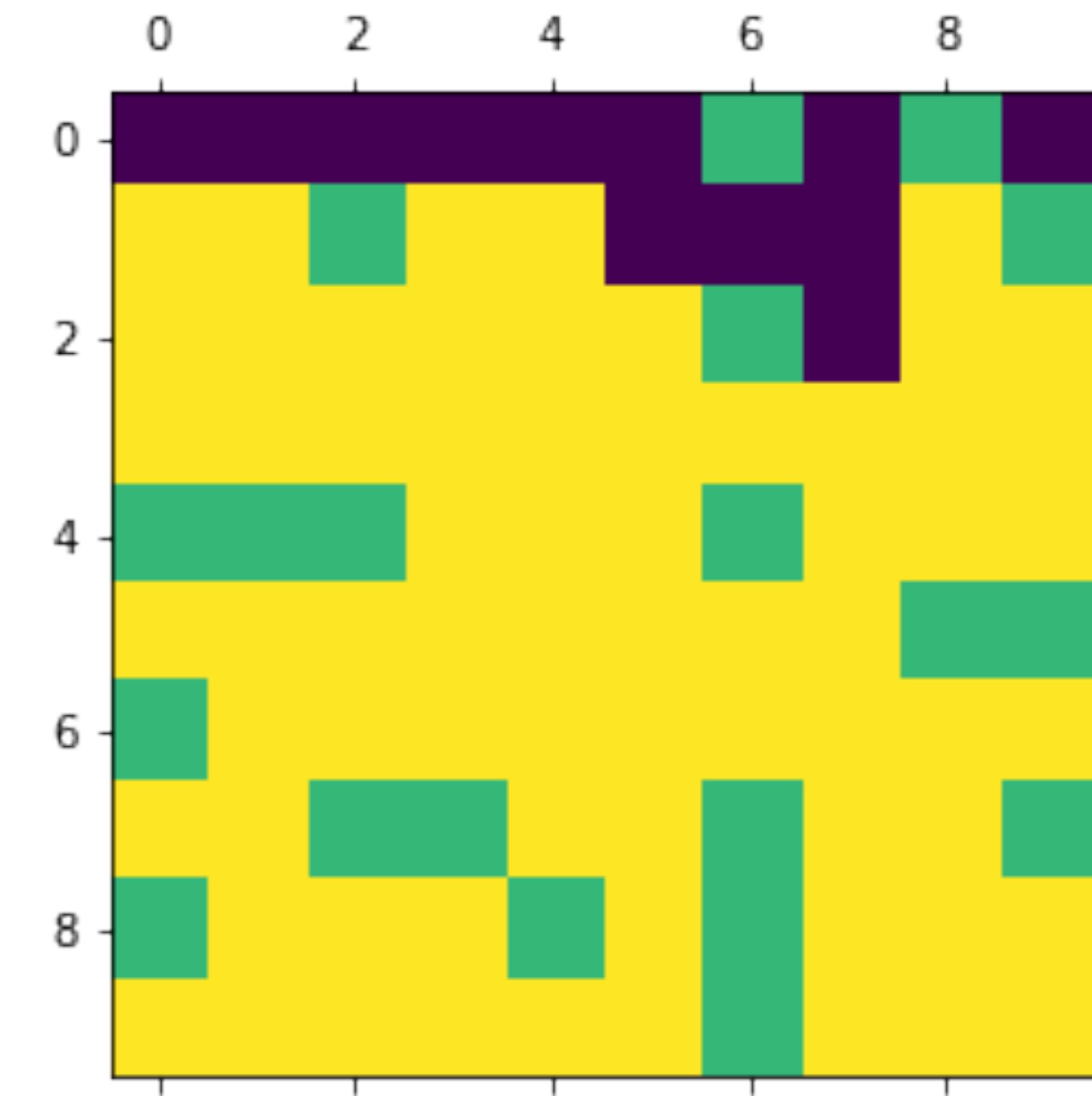
Attacker changes the environment's state to

$$s^\dagger$$

True-State Attack

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S 



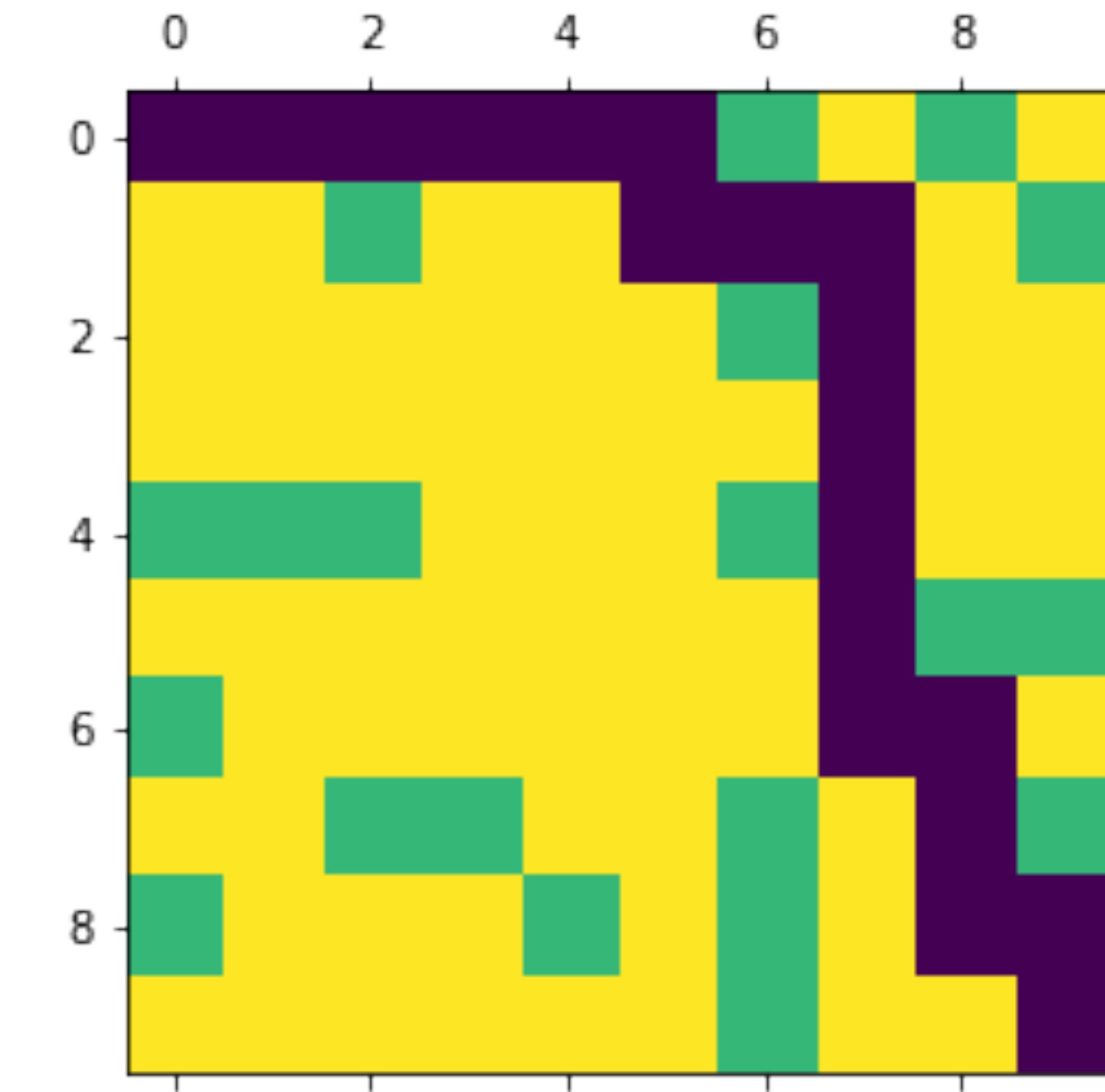
Reward Attack

Reward Attack

Attacker changes the reward the agent receives to r^\dagger

Reward Attack

Attacker changes the reward the agent receives to r^\dagger



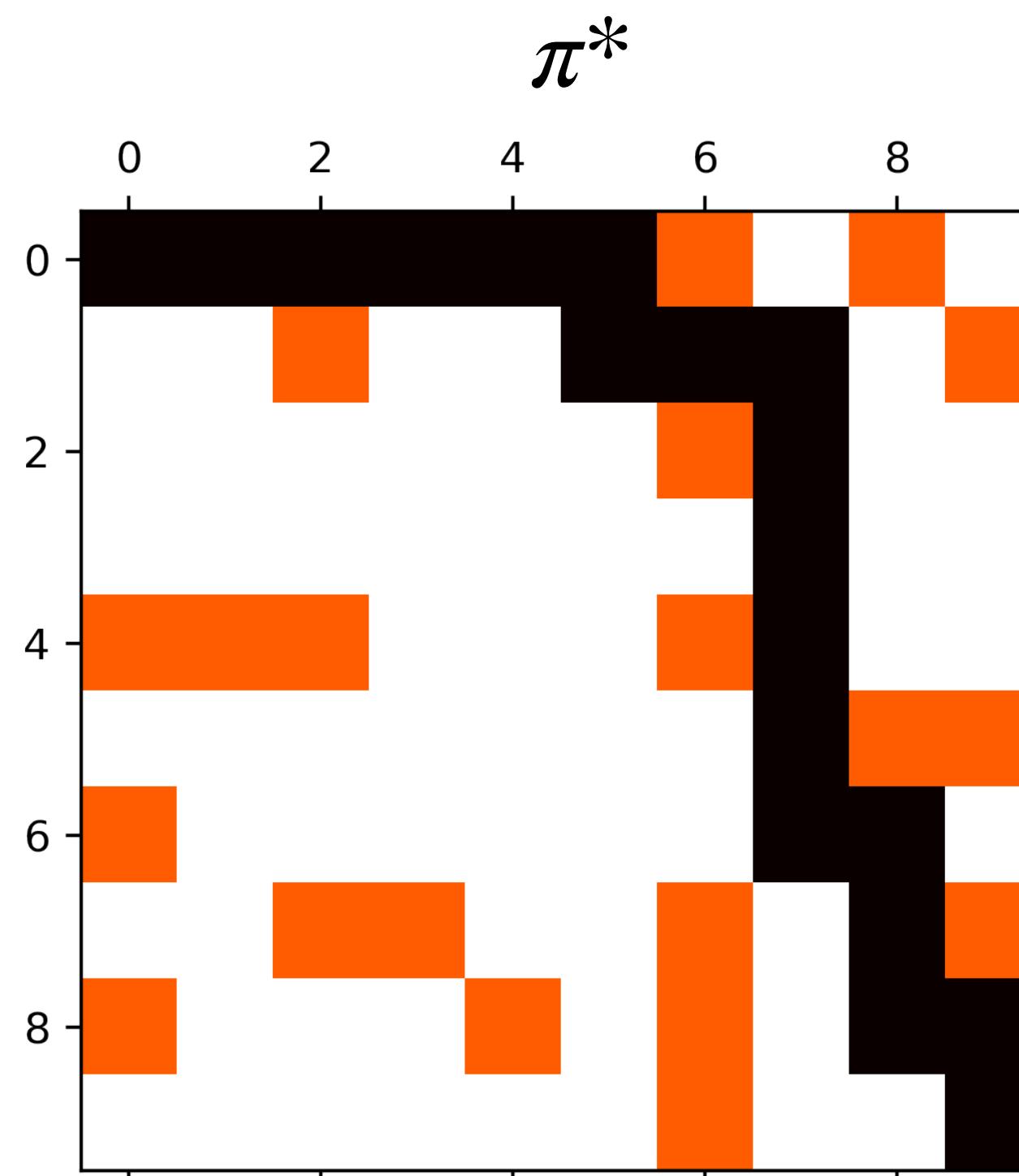
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Optimal policies may be sensitive to noise or attacks.

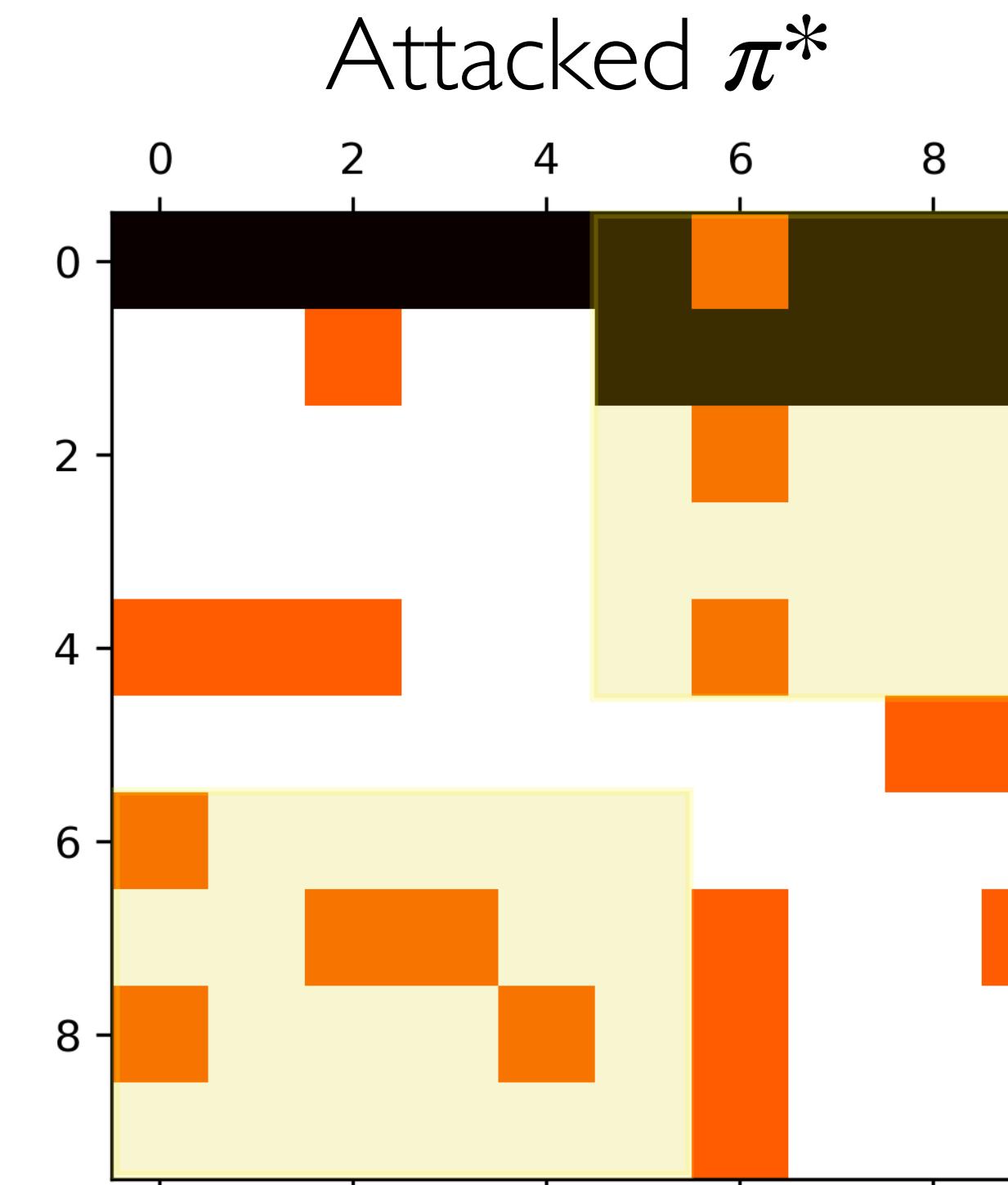
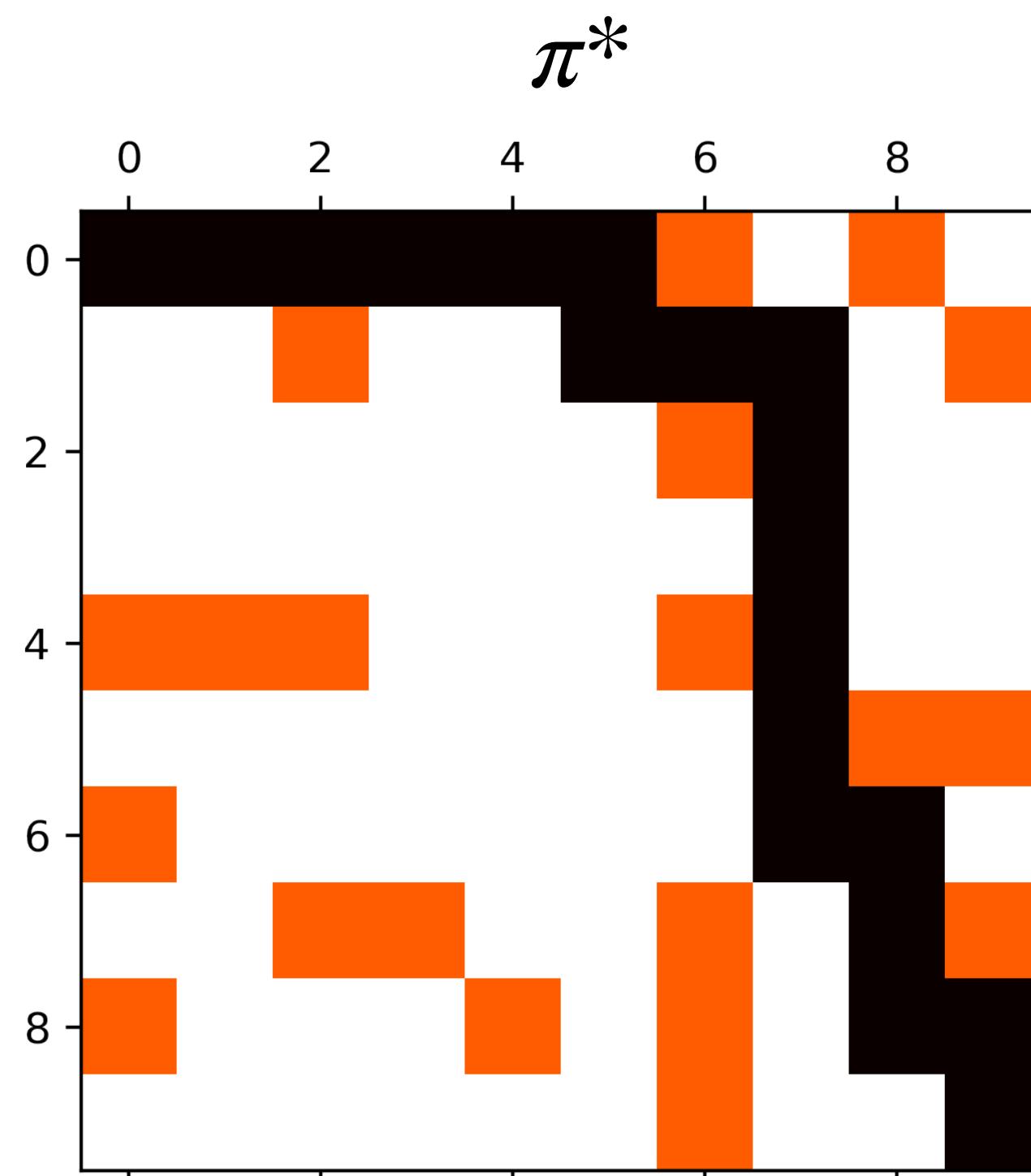
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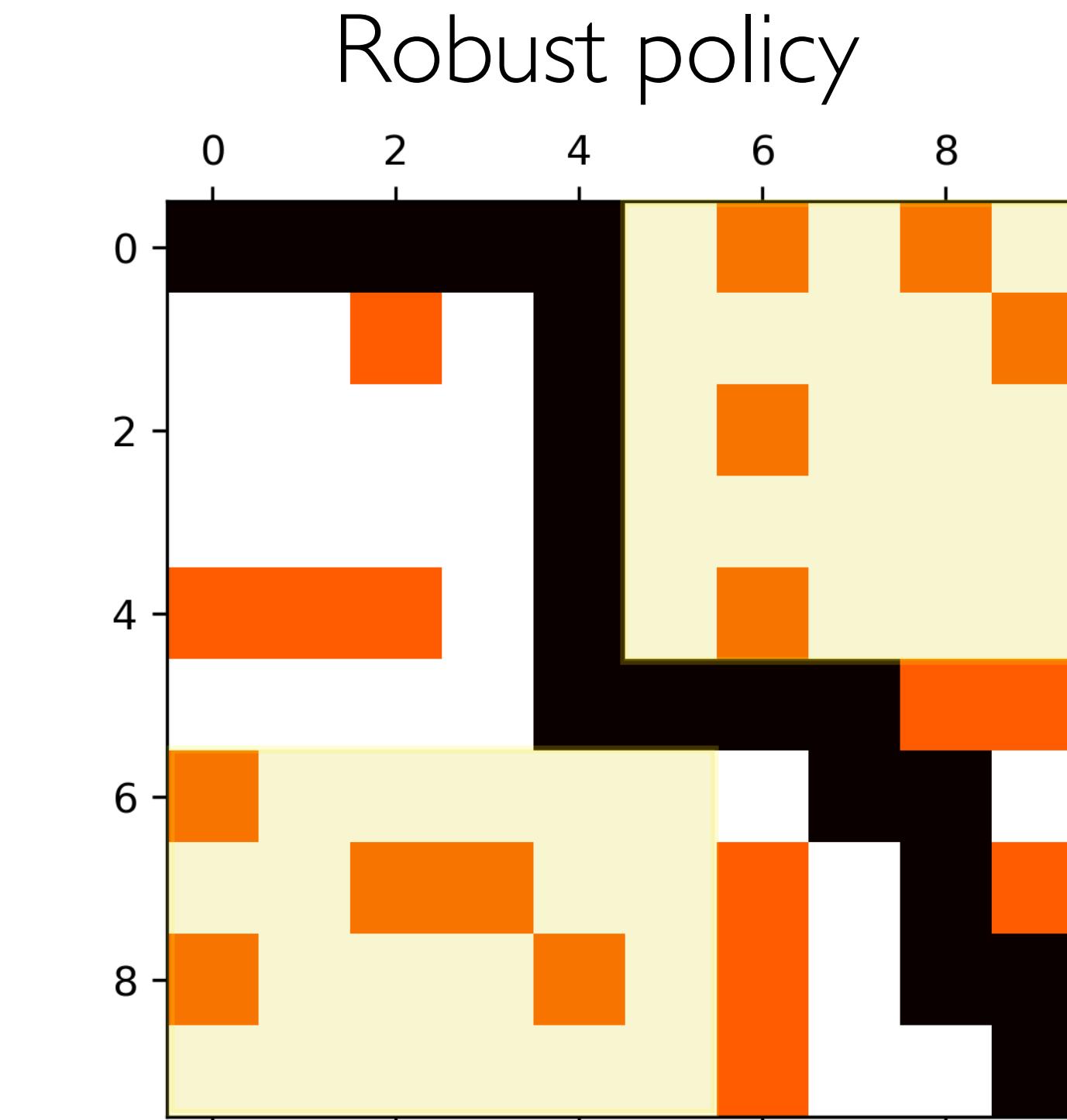
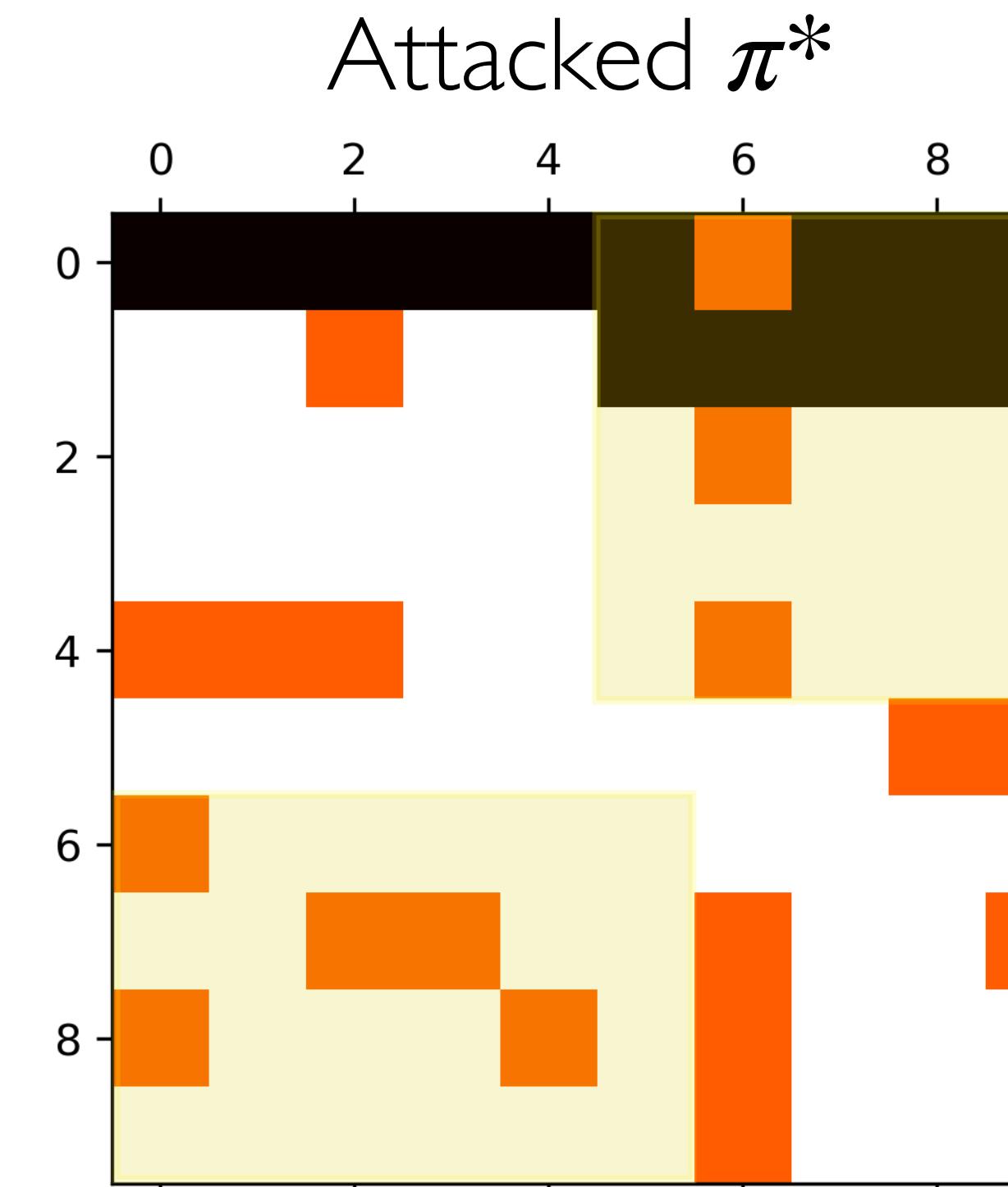
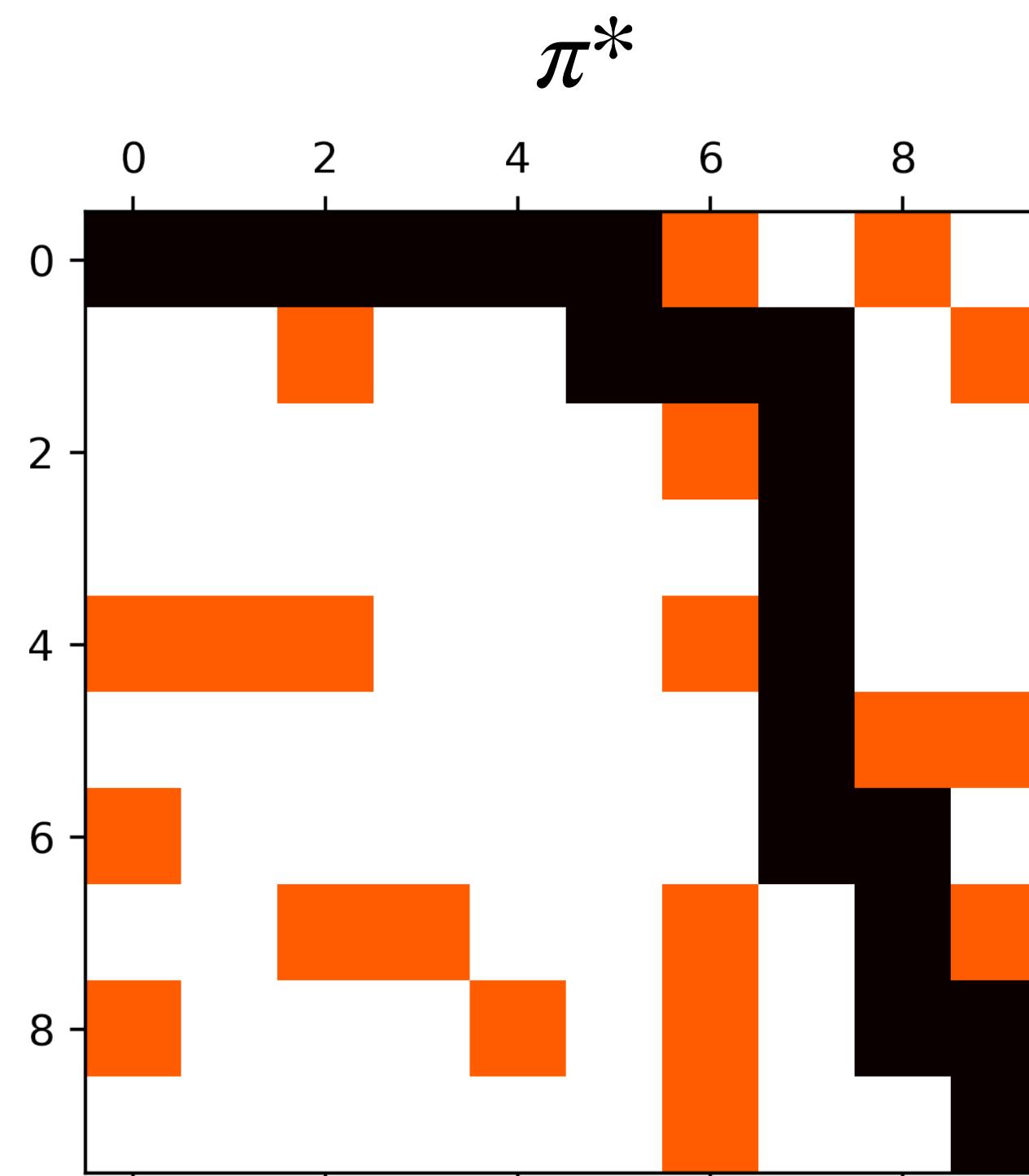
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Optimal Observation Attacks

Robust Deep Reinforcement Learning against Adversarial Perturbations on State Observations

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Bo Li⁴ Mingyan Liu⁵ Duane Boning² Cho-Jui Hsieh¹

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[Training-time] Action and Reward Attacks

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Defense against a specific reward attack algorithm

Defense Against Reward Poisoning Attacks in Reinforcement Learning

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Not robust! Attacker can change its algorithm later.

The Attack Problem

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Adversarial Decomposition

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We decompose the attacked π -M interaction based on the *flow of information*.

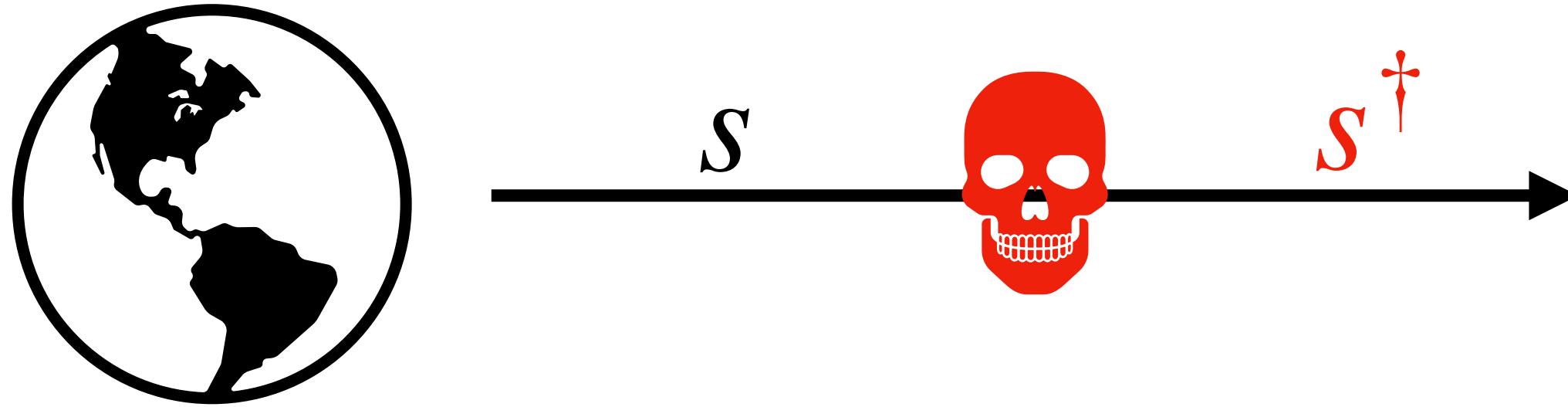
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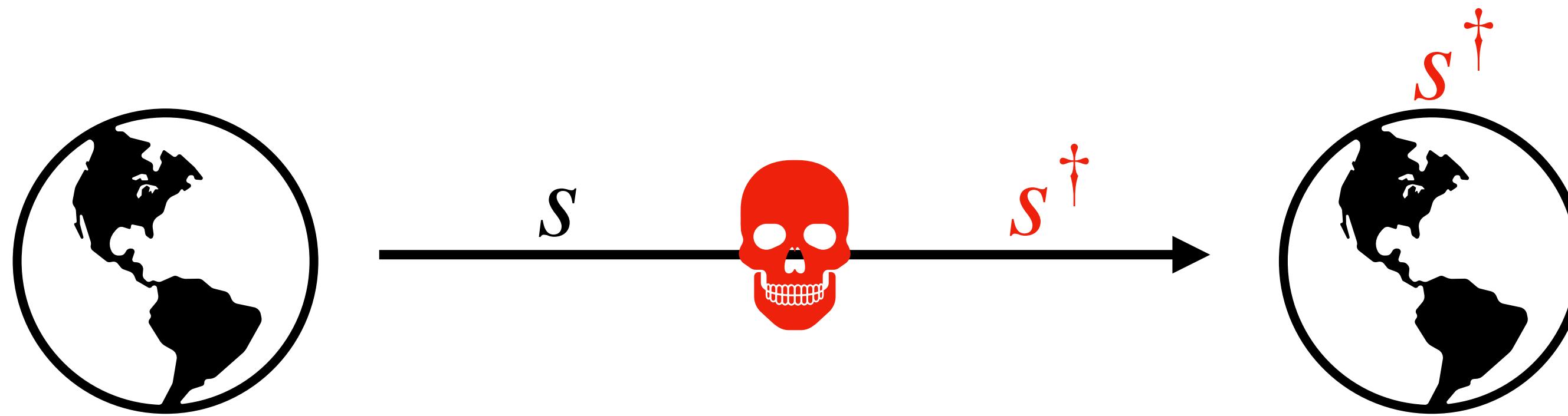
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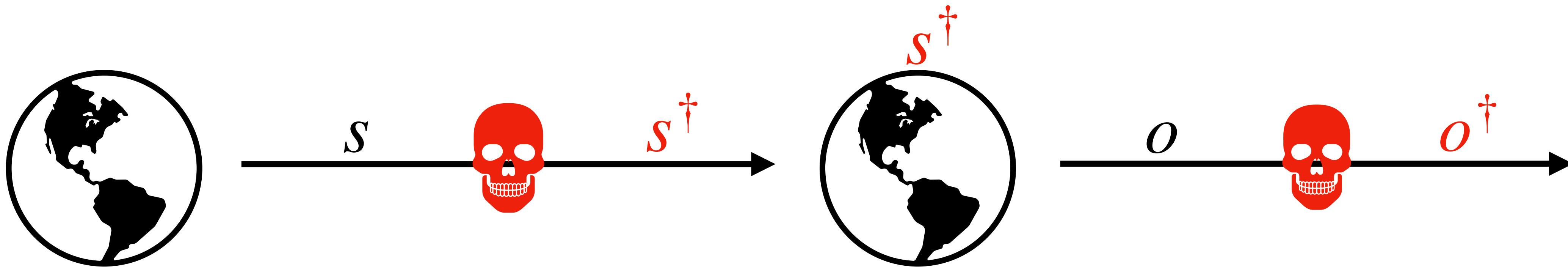
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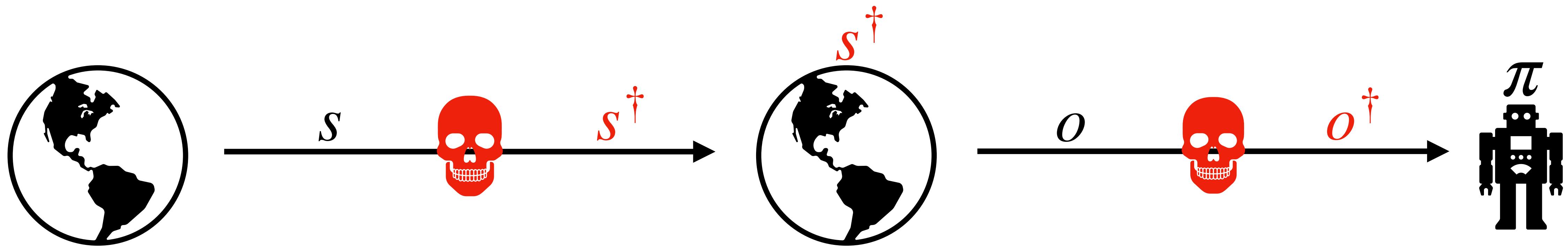
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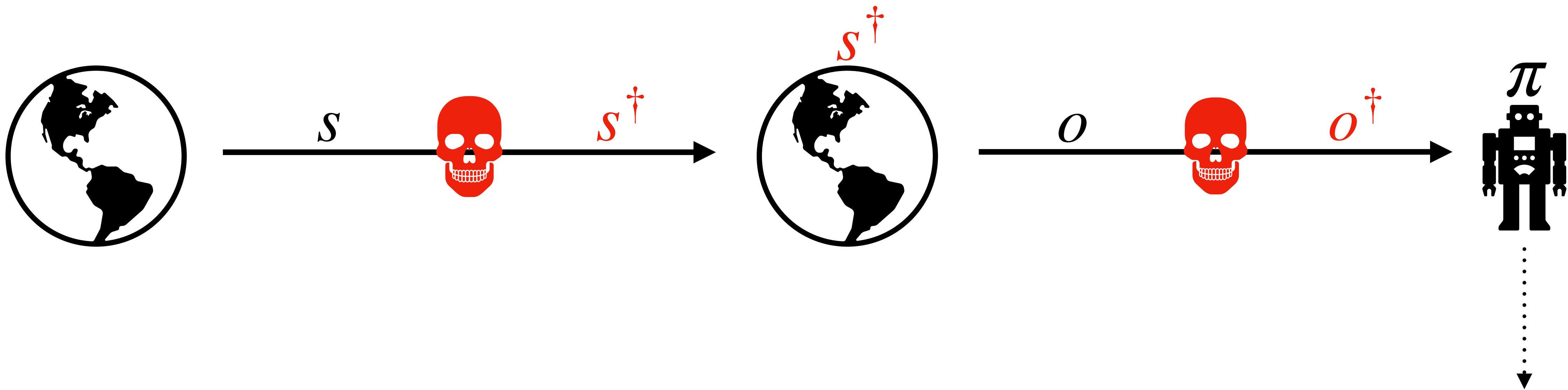
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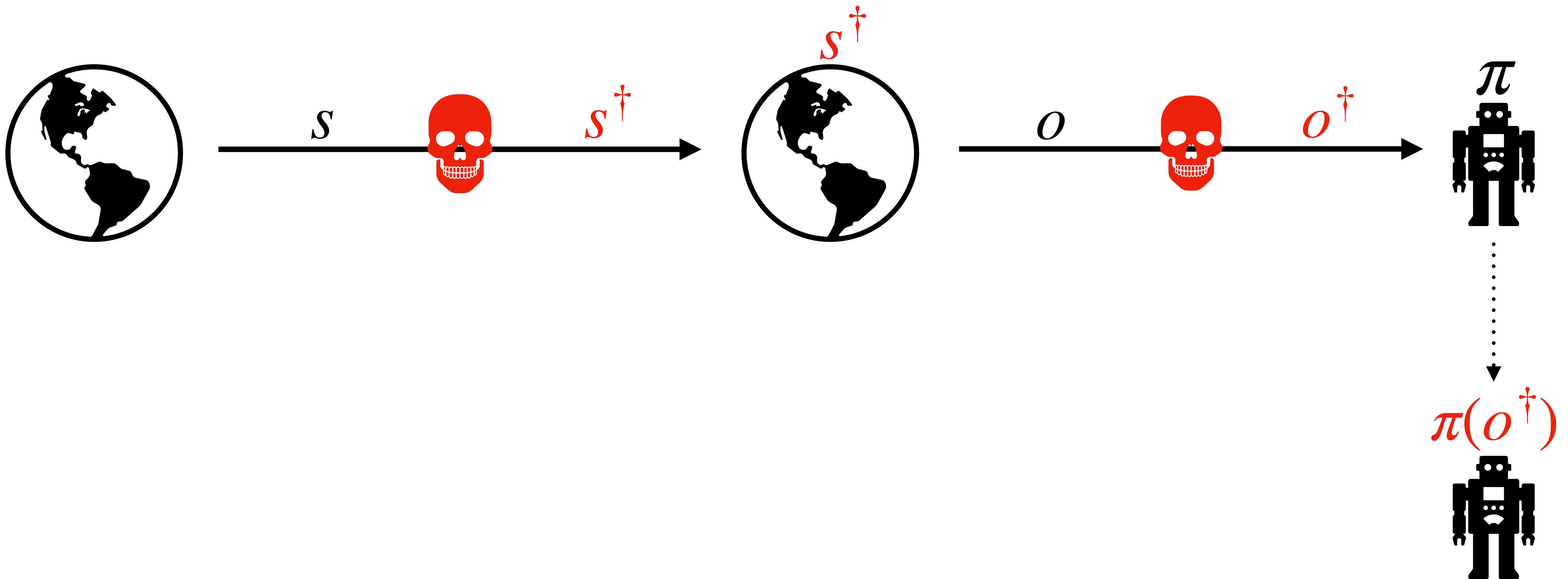
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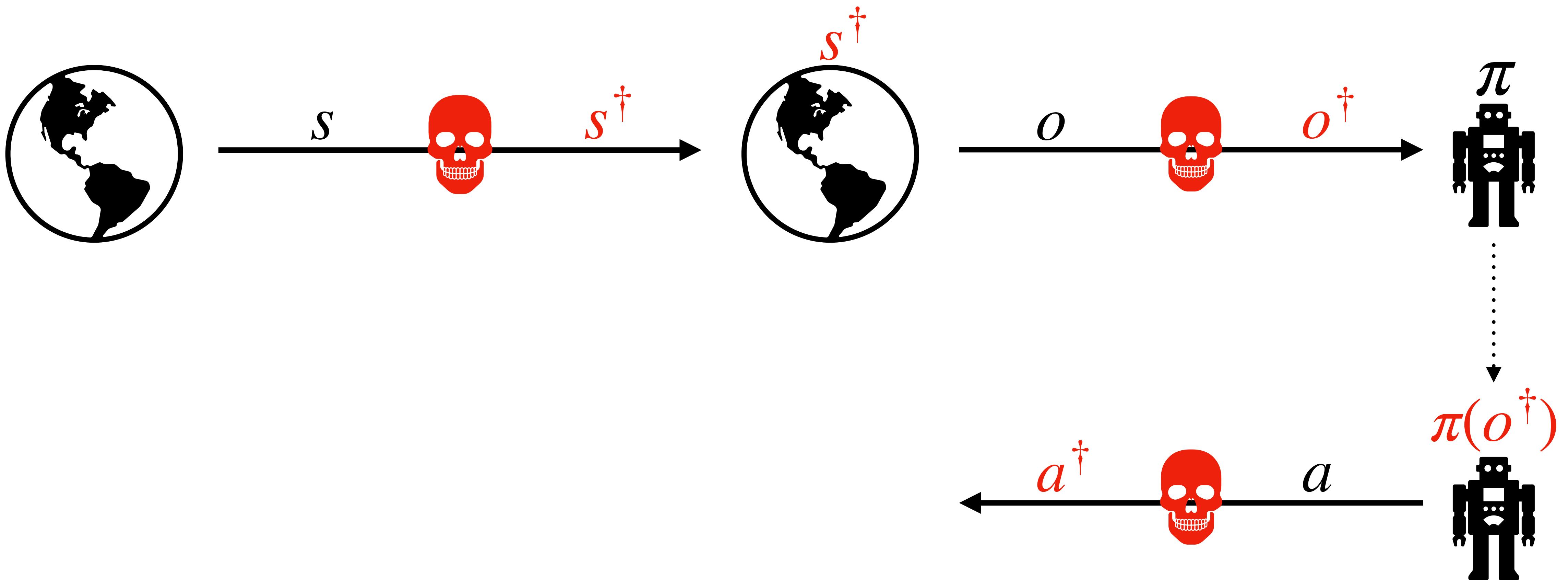
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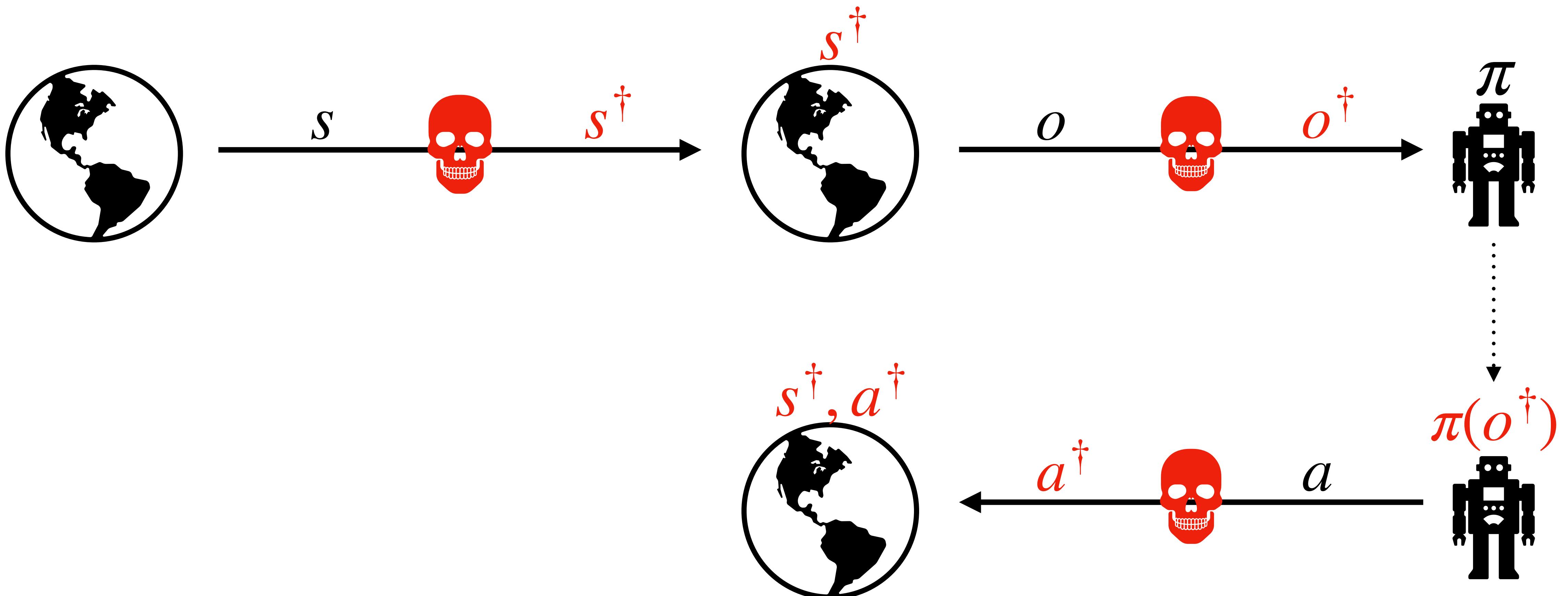
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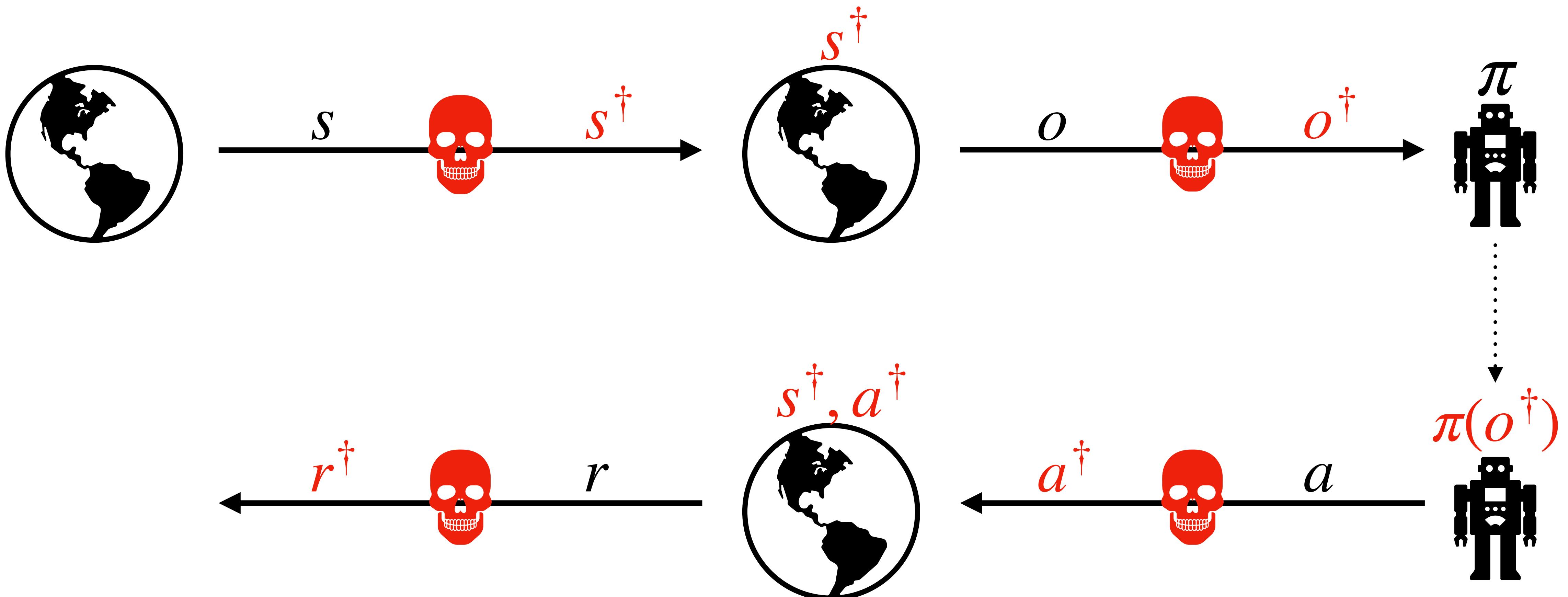
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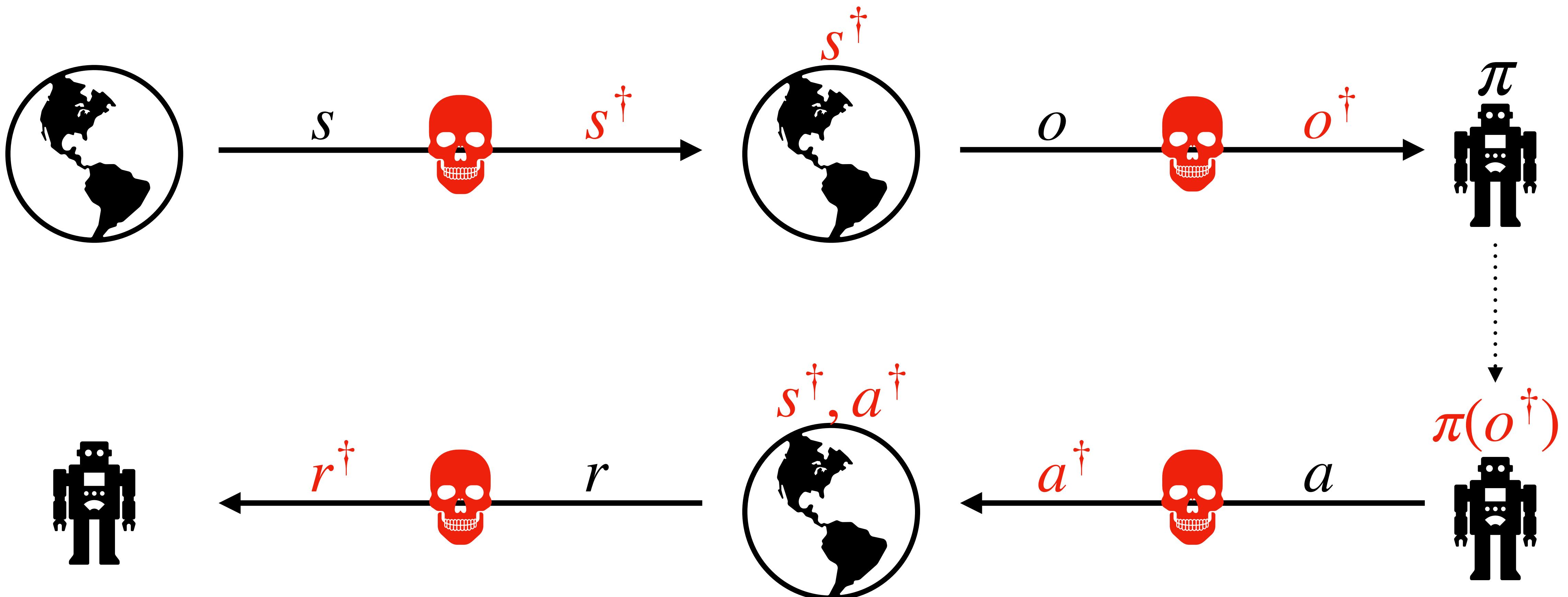
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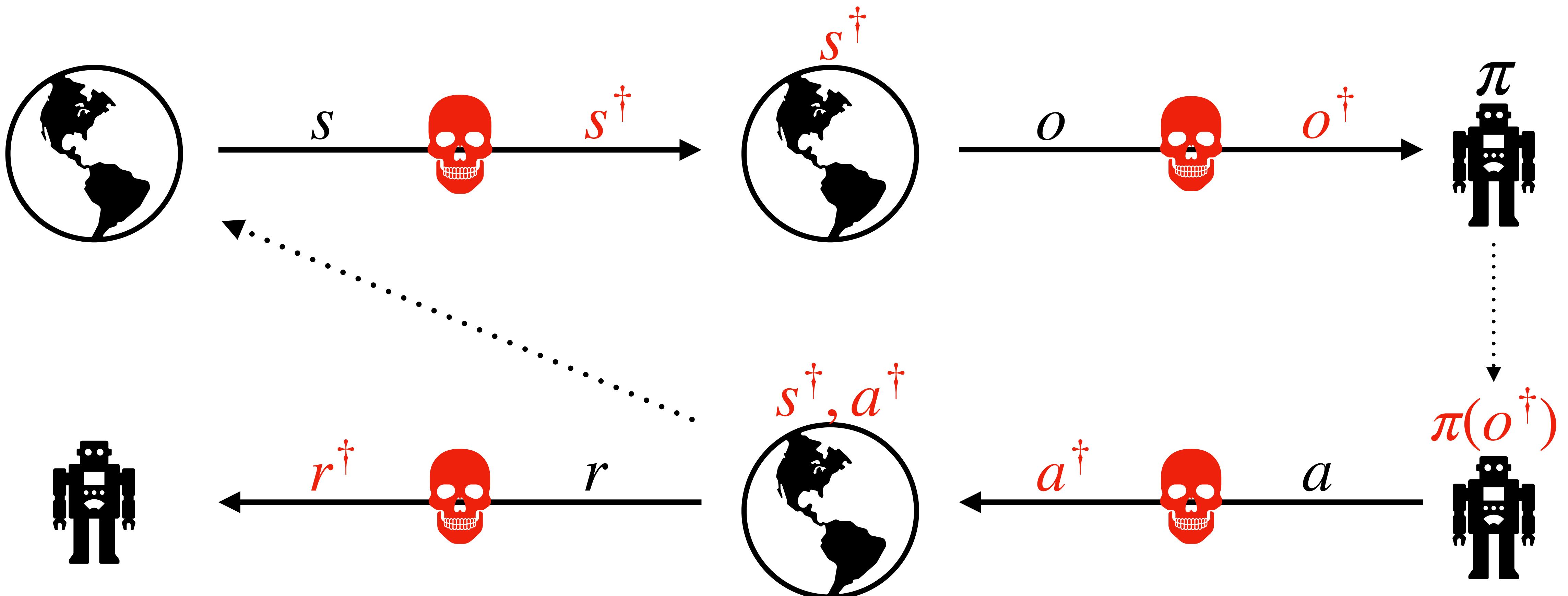
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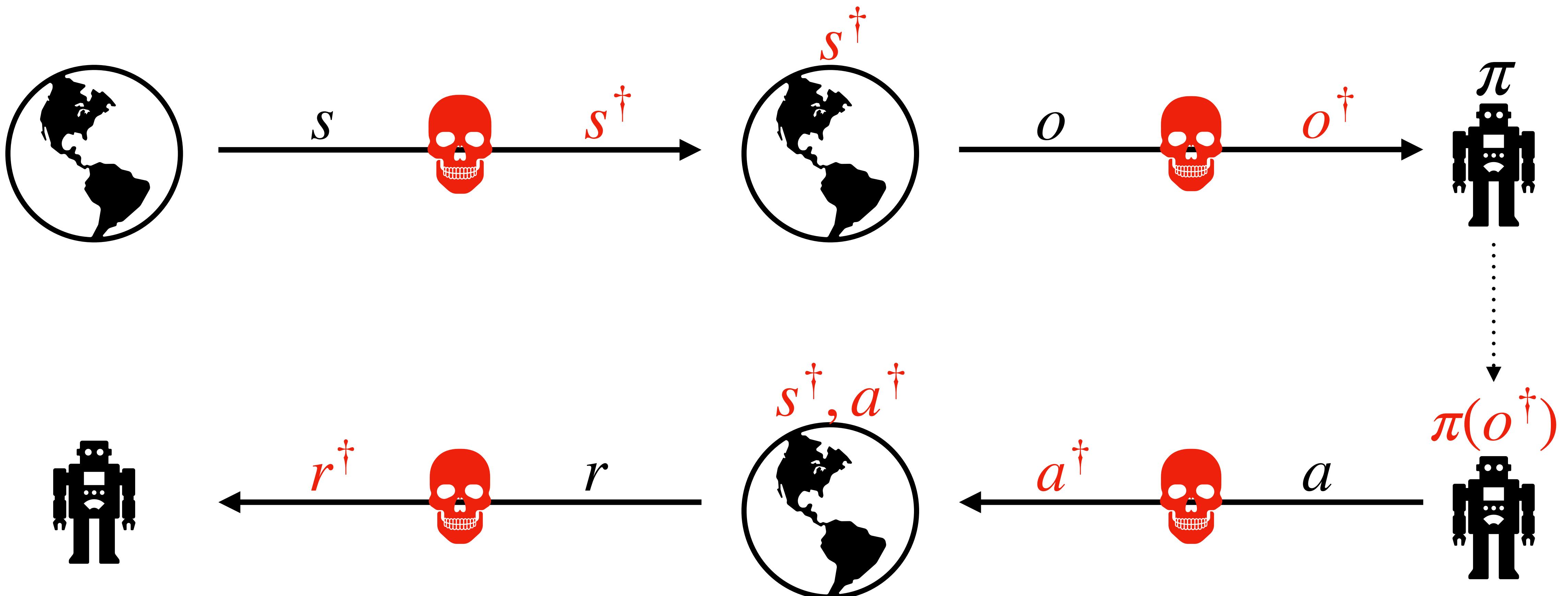
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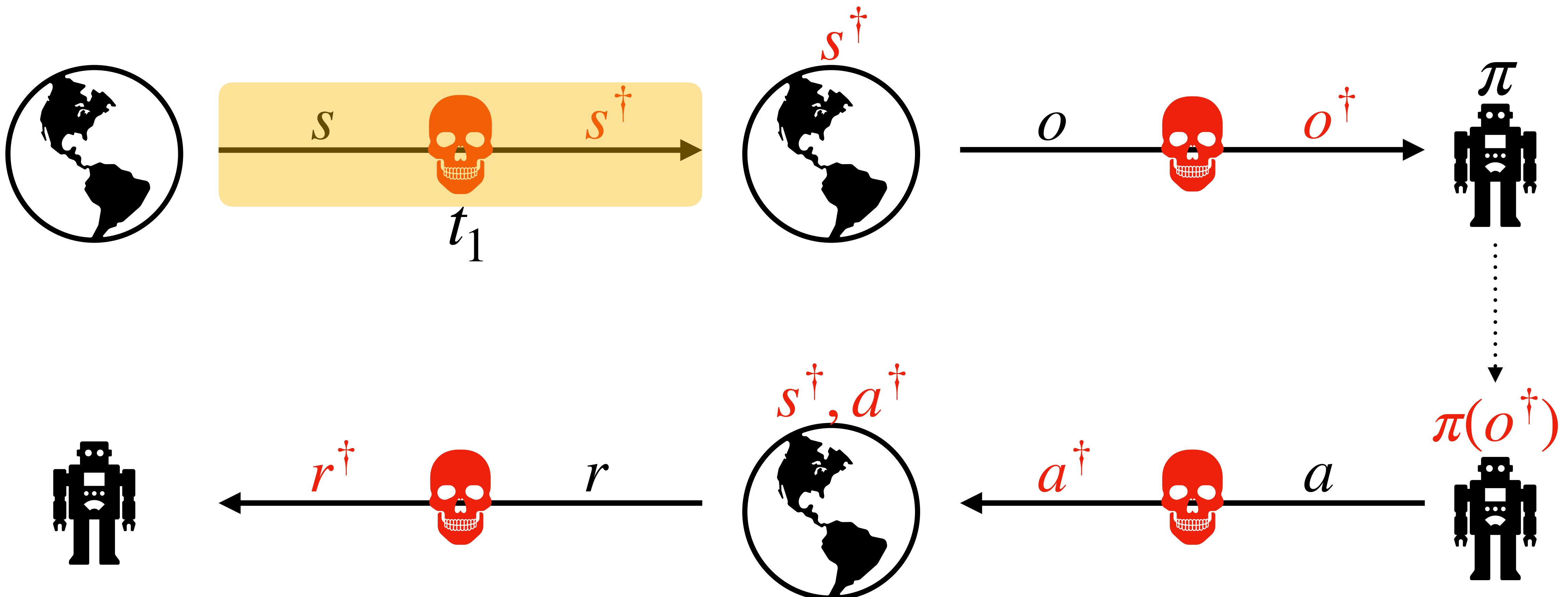
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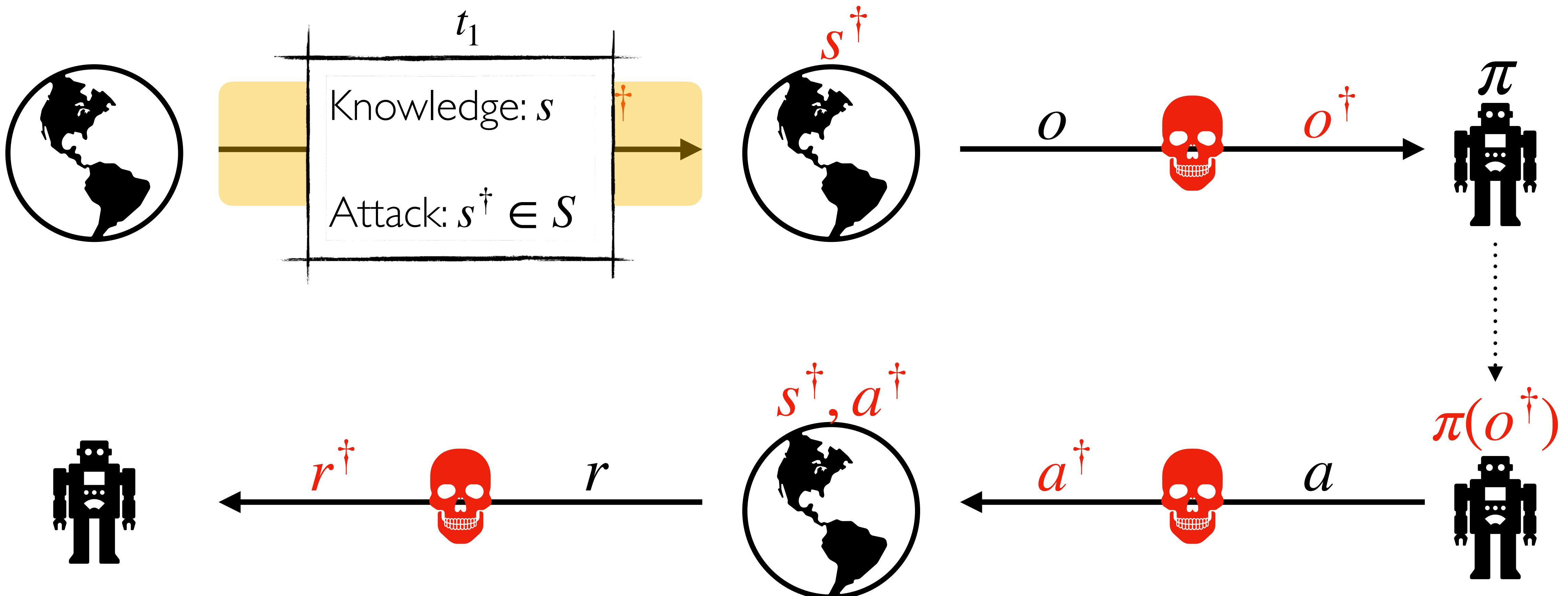
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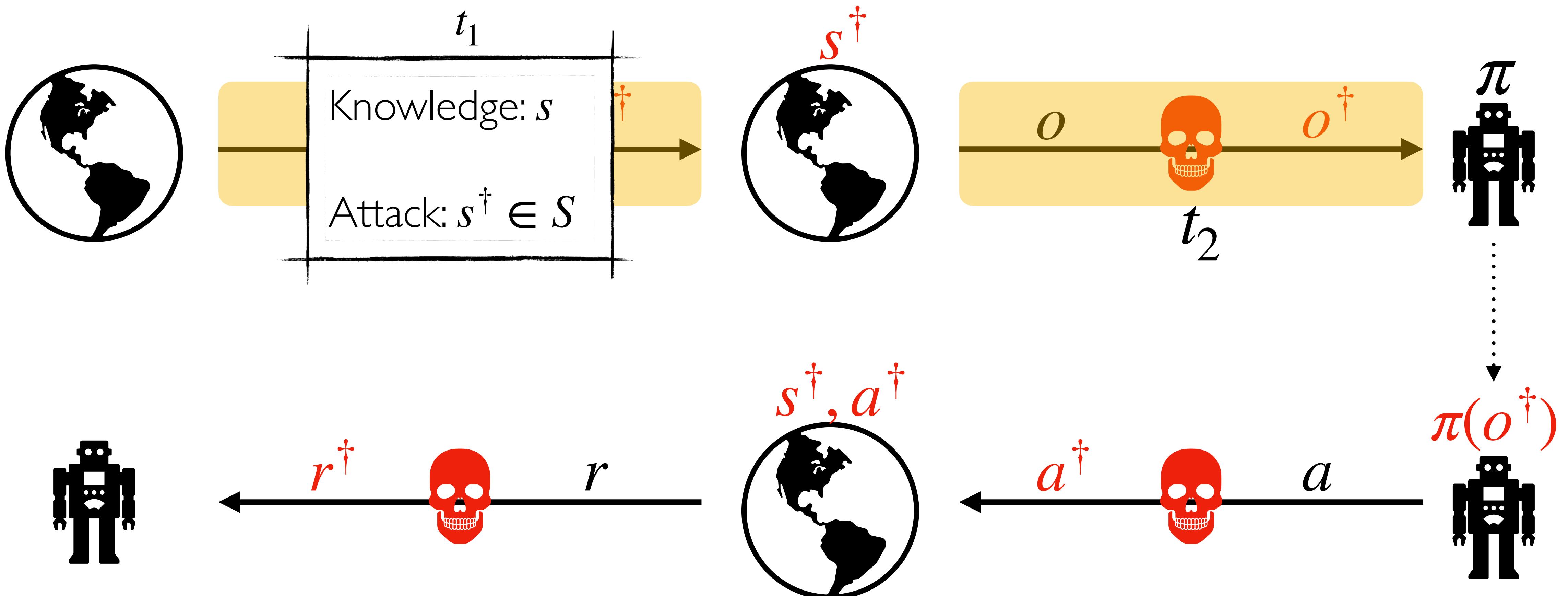
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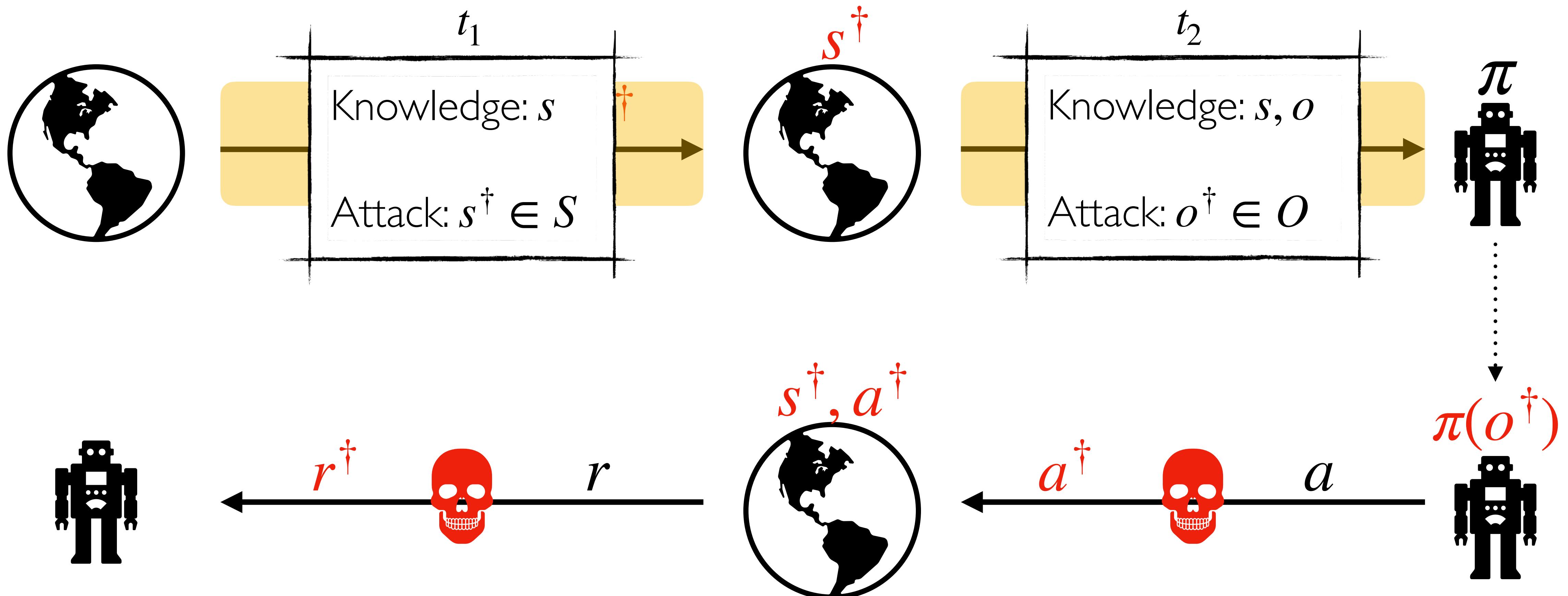
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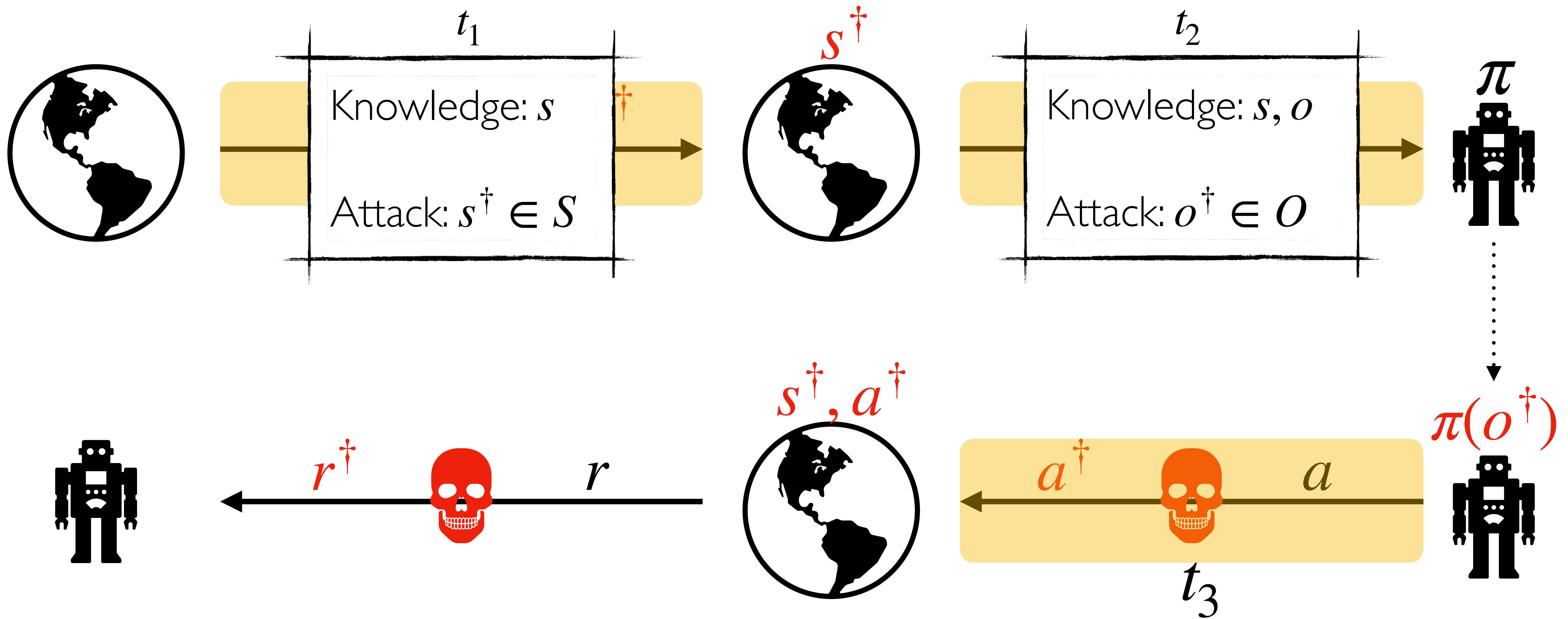
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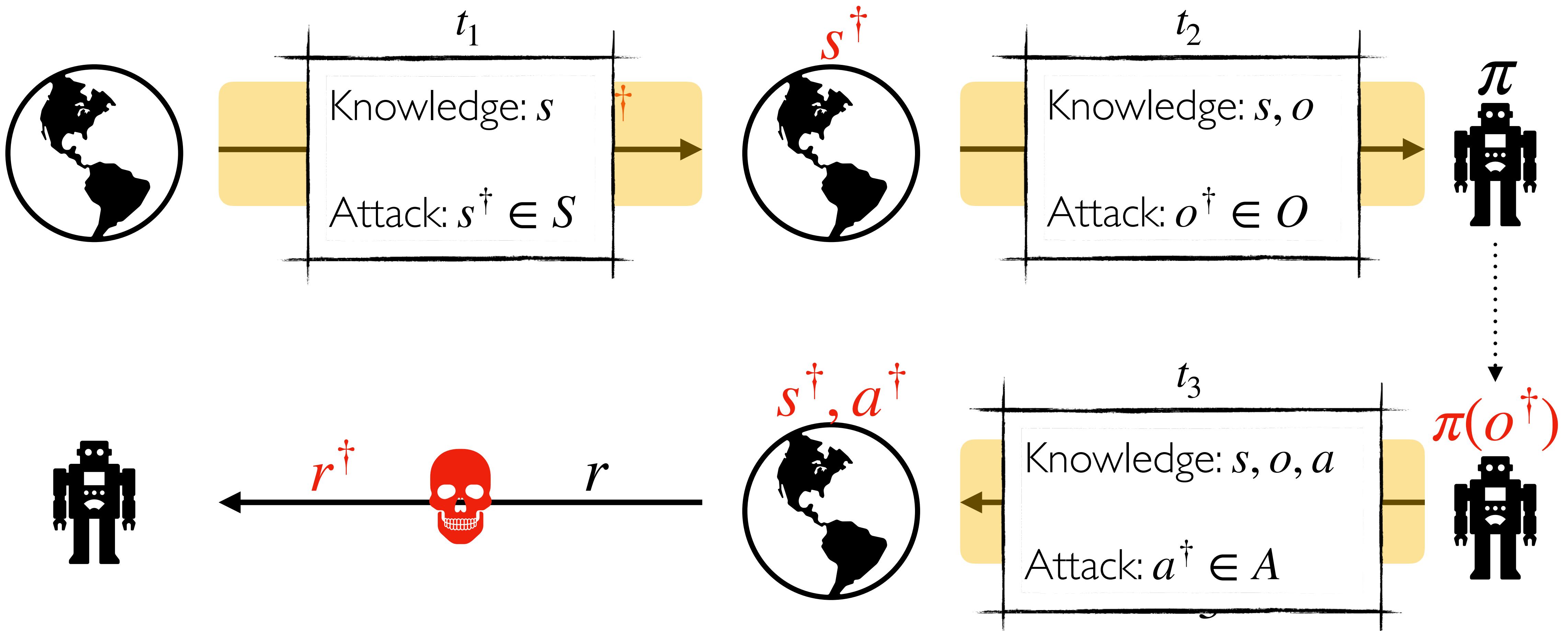
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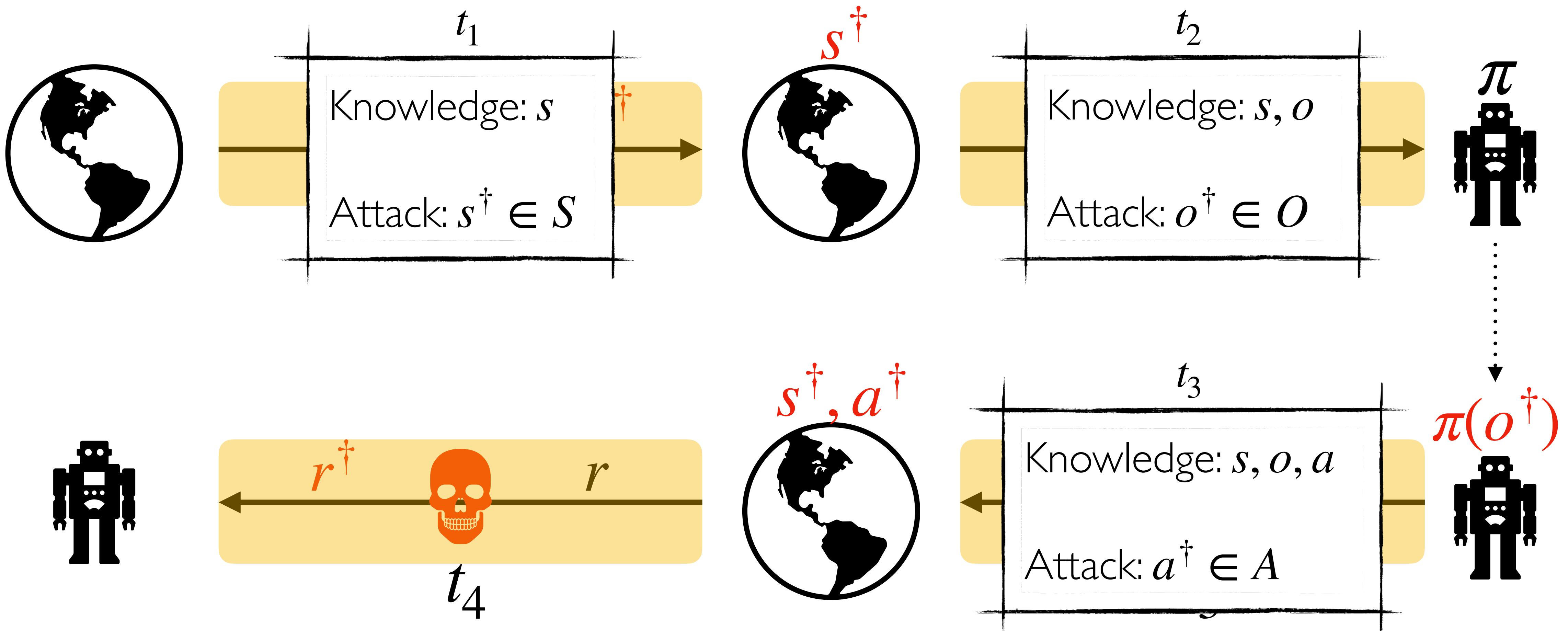
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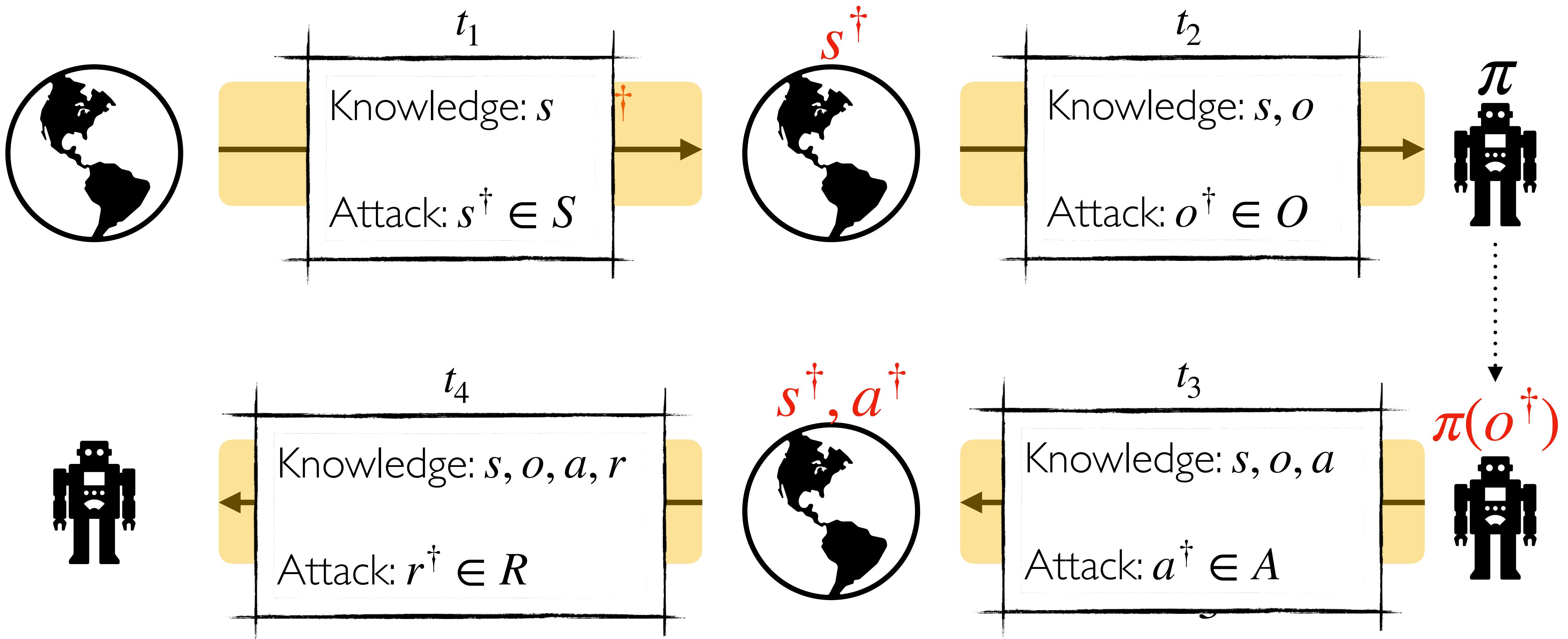
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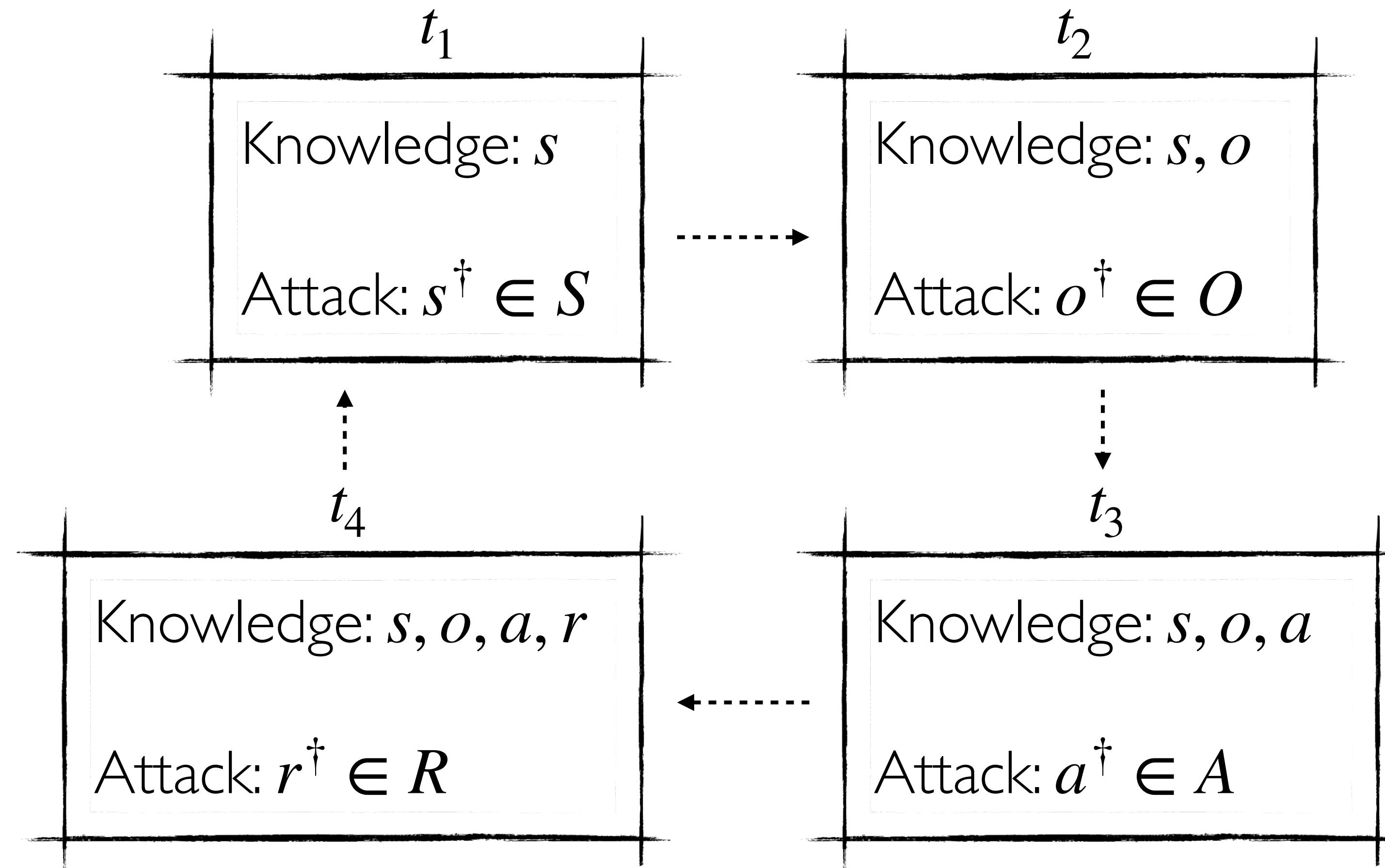


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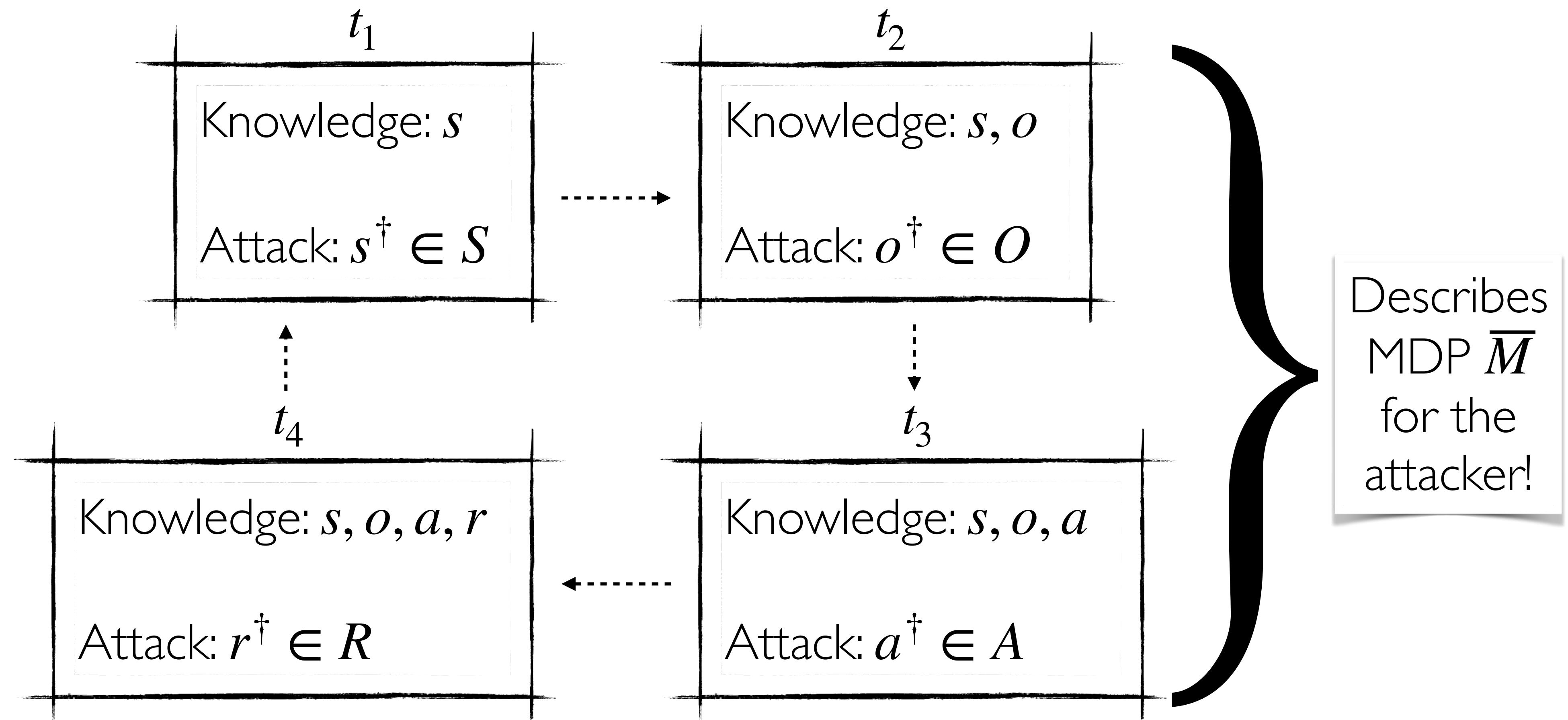
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Attacker's Perspective



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Meta MDP

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Attacker's interaction with π and M evolves according to MDP \bar{M} .

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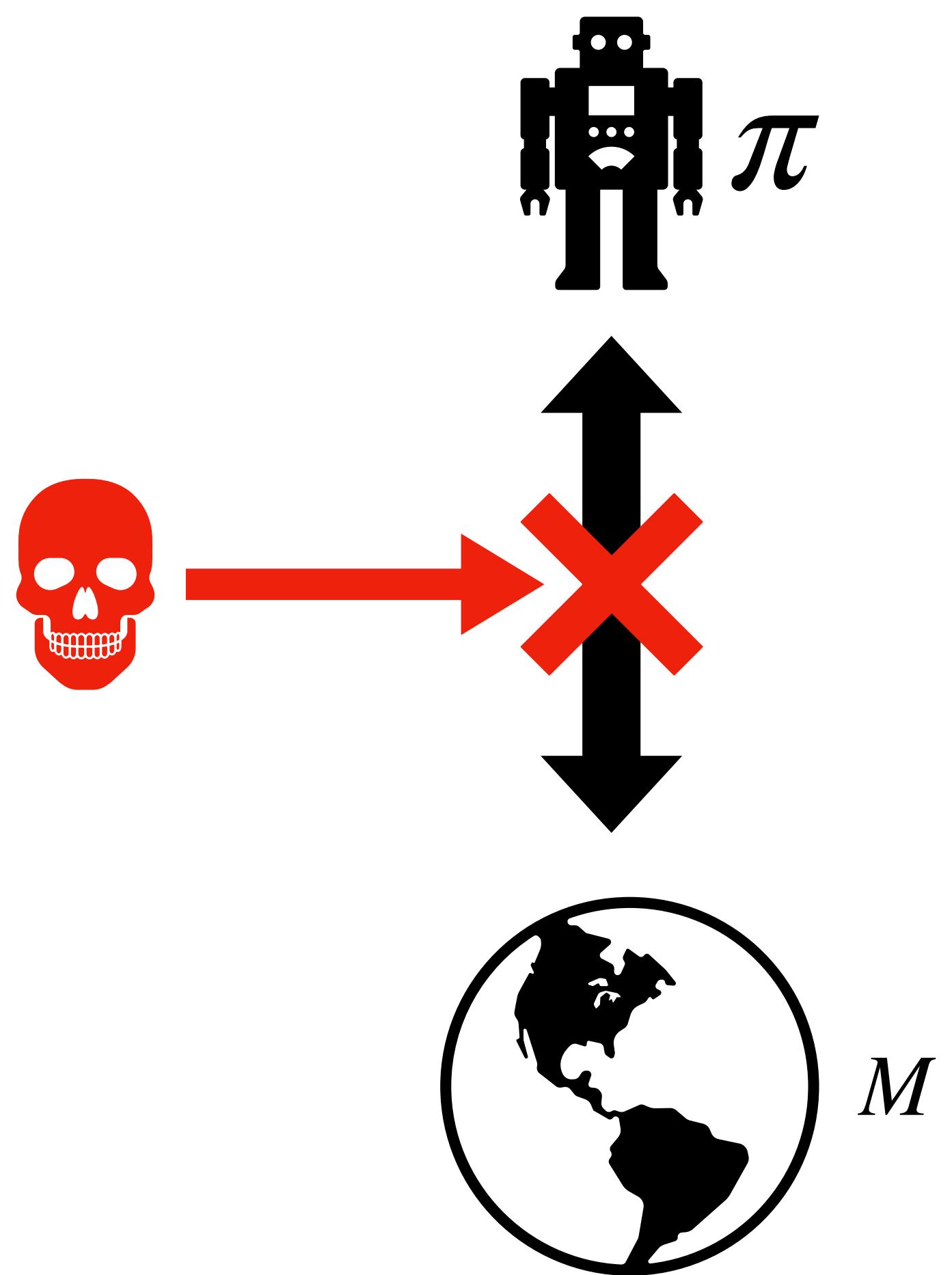
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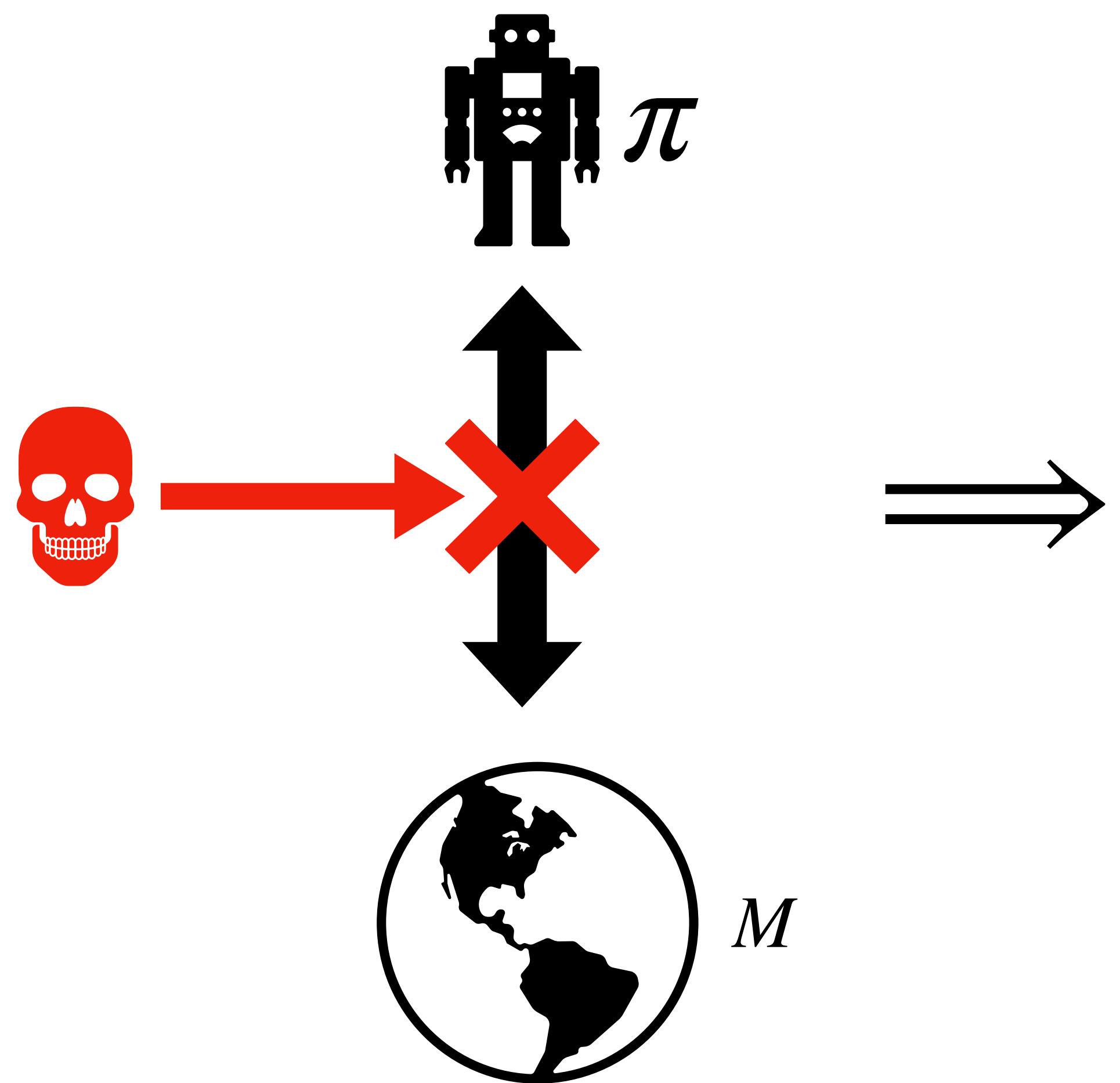
Proposition: Any optimal policy for \bar{M} is an optimal attack policy.

Reduction to RL

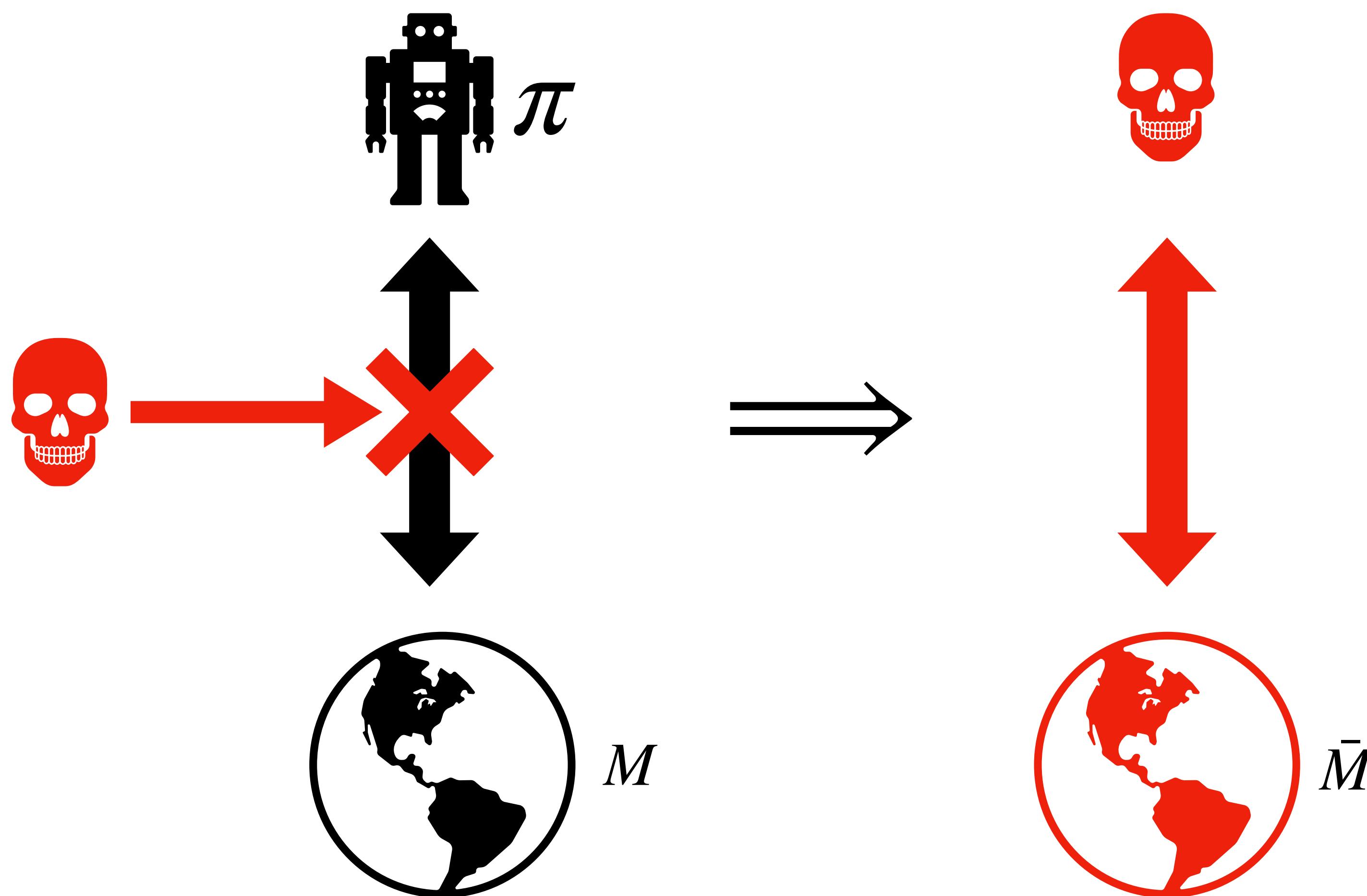
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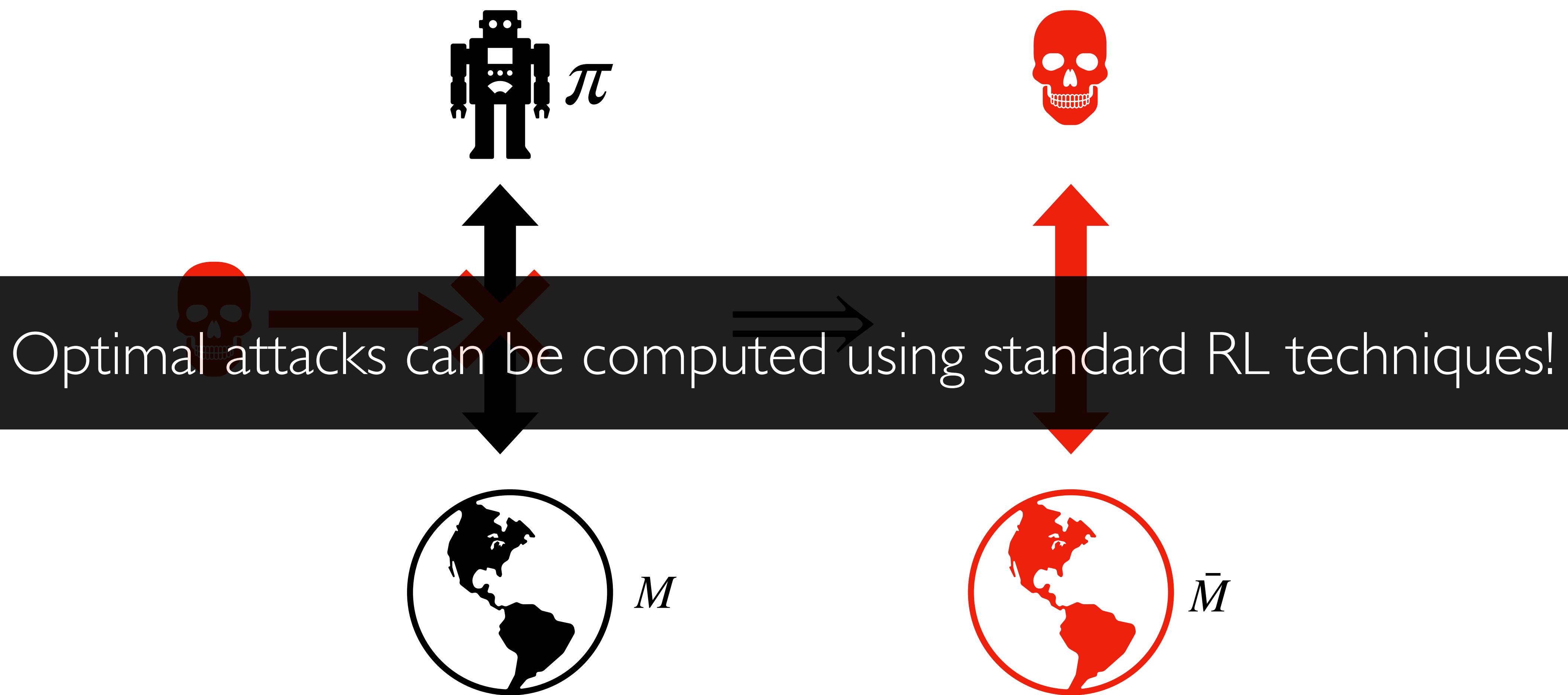
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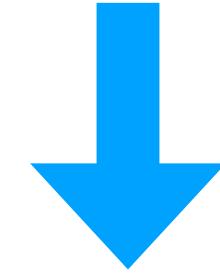
Computational Efficiency

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\bar{M} has only polynomially larger state and action space than M .

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Attacking RL *efficiently* reduces to RL!

\bar{M} has only polynomially larger state and action space than M .

Can we defend against attacks?

Defense

The Defense Problem

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Let $(V_1^{\pi,\nu}, V_2^{\pi,\nu})$ denote the victim's and attacker's value, respectively.

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Definition 2 (Defense Problem). The victim seeks a policy π^* that maximizes its expected reward from the victim-attacker- M interaction under the worst-case attack:

$$\pi^* \in \arg \max_{\pi \in \Pi} \min_{\nu \in BR(\pi)} V_1^{\pi,\nu}.$$

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$$BR(\pi) := \arg \max_{\nu \in N} V_2^{\pi,\nu}$$

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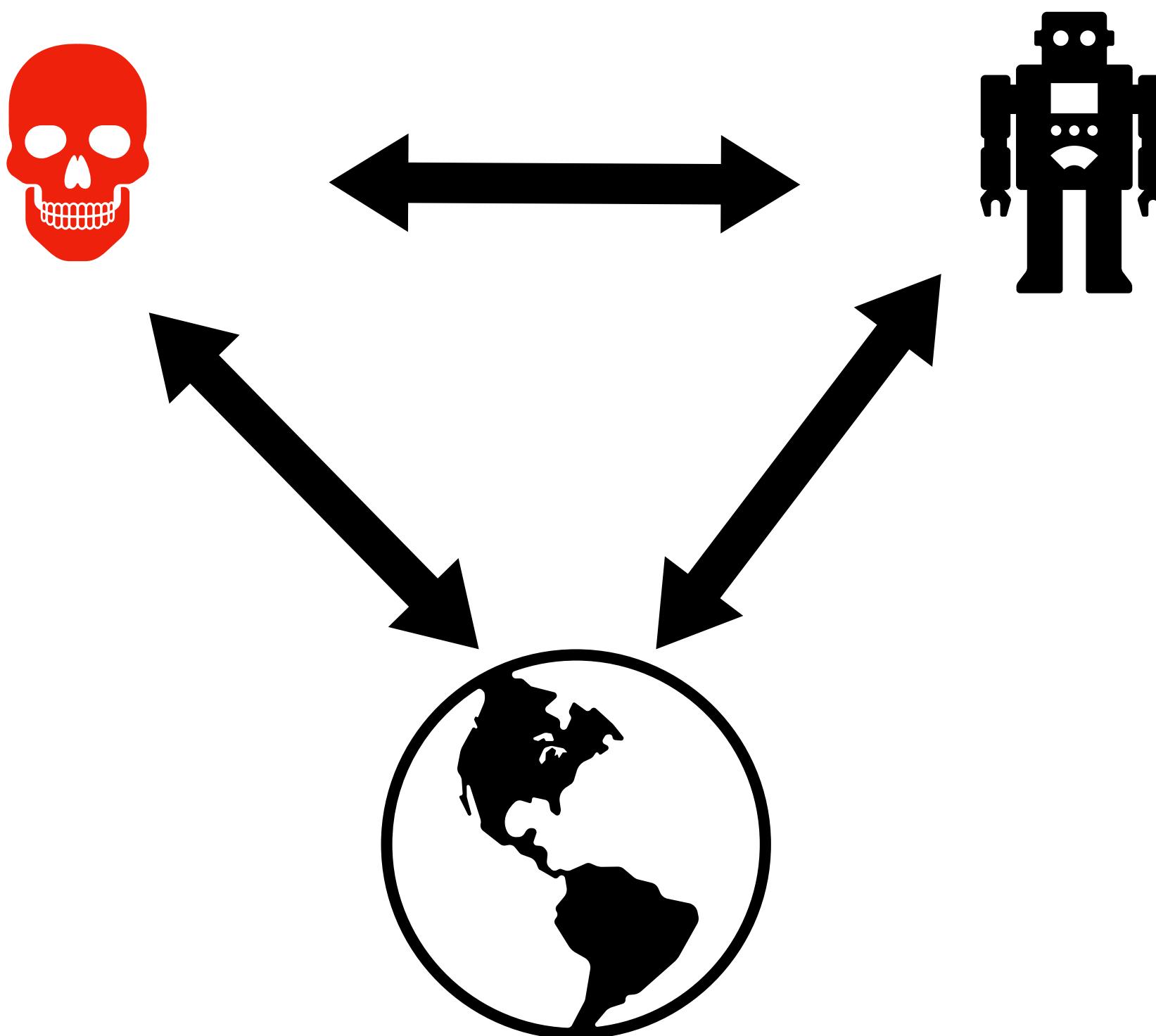
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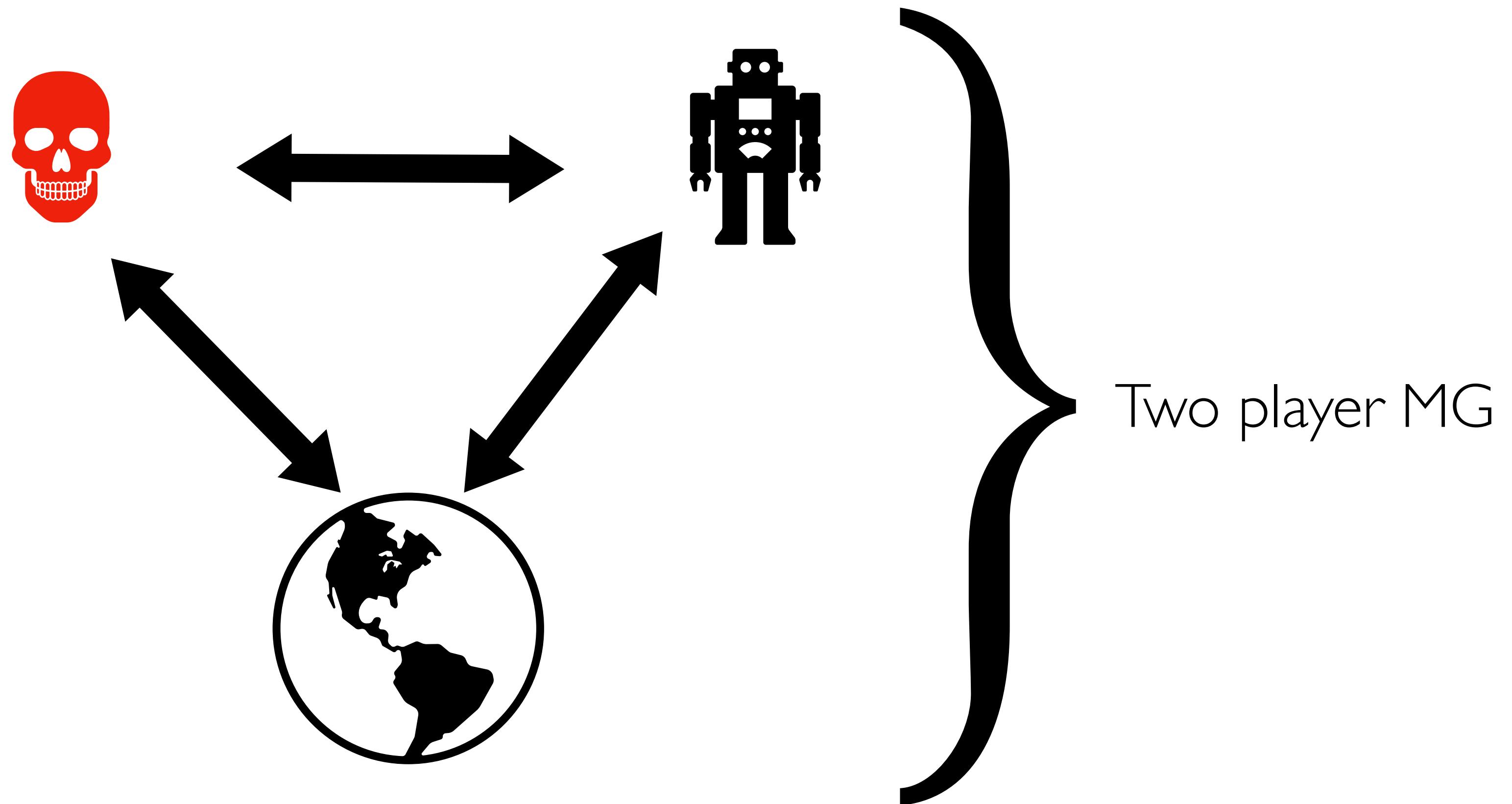
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Reduction to MARL

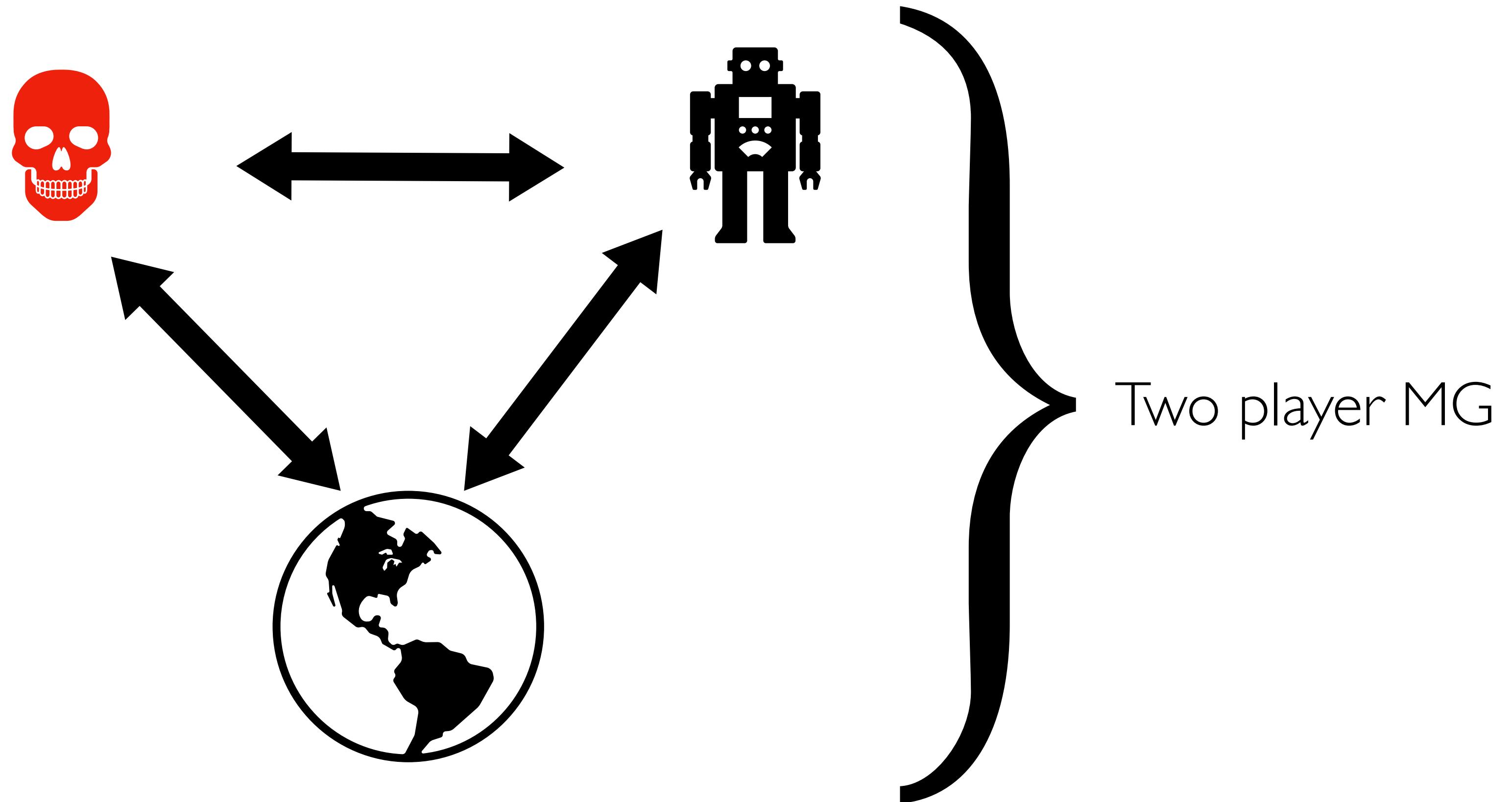
Reduction to MARL



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Defense corresponds to a Weak Stackelberg Equilibrium (WSE).

Challenges

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Proposition: The defense problem is as hard as solving POMDPs.

Thus, the defense problem is NP-hard to even approximate.

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Key: restrict observation attacks.

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P2

s

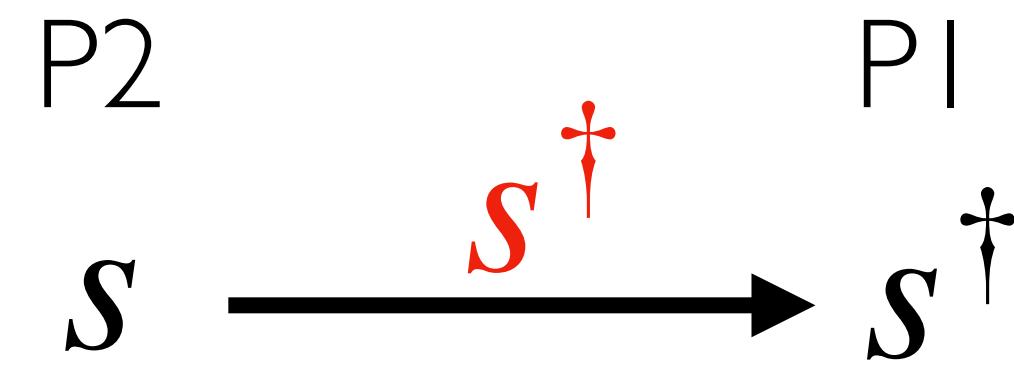
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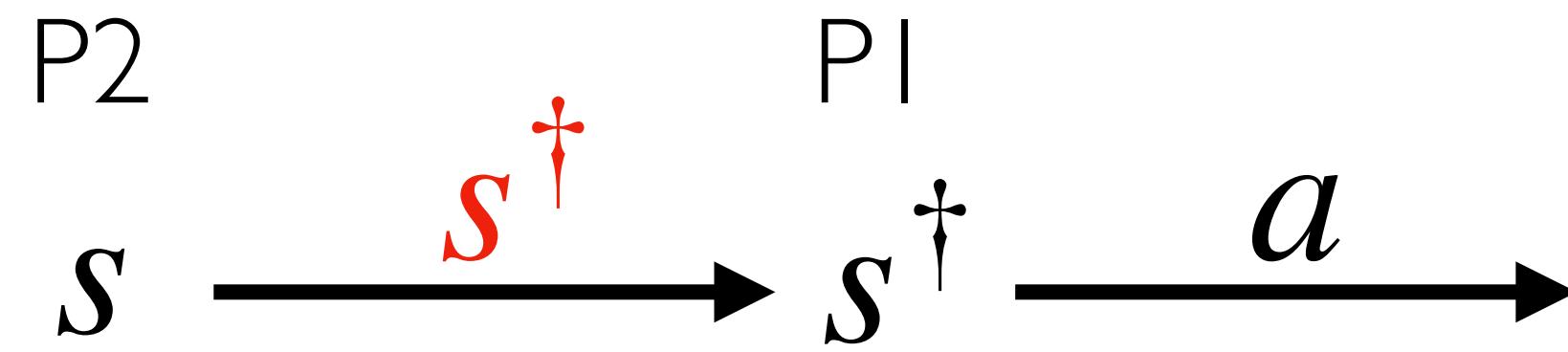
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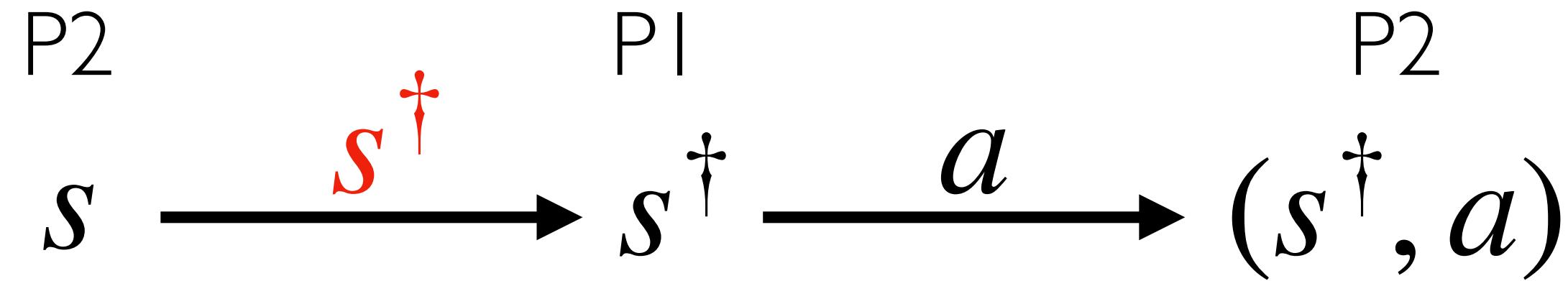
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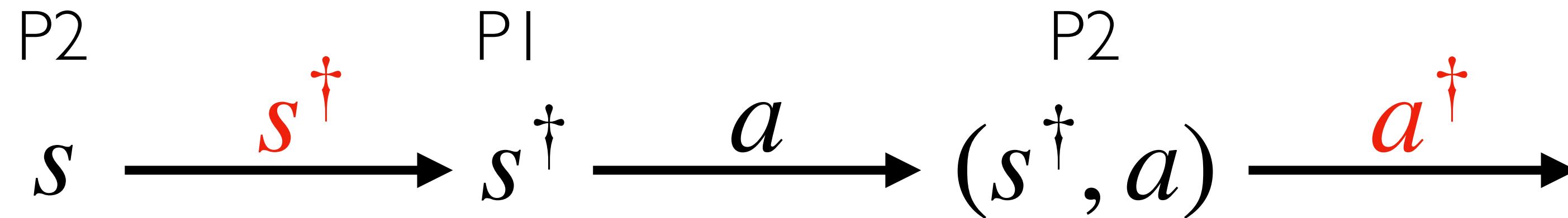
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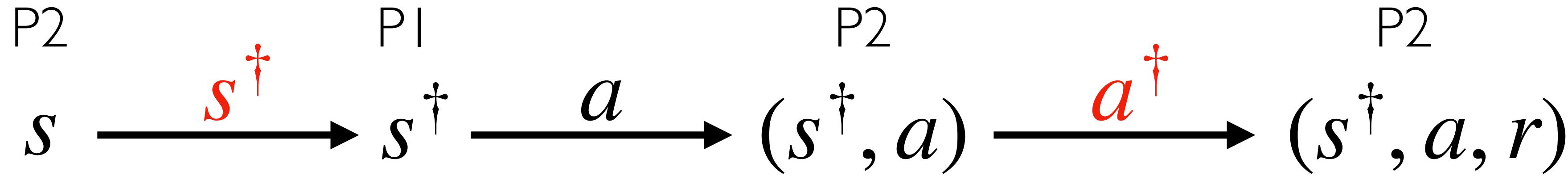
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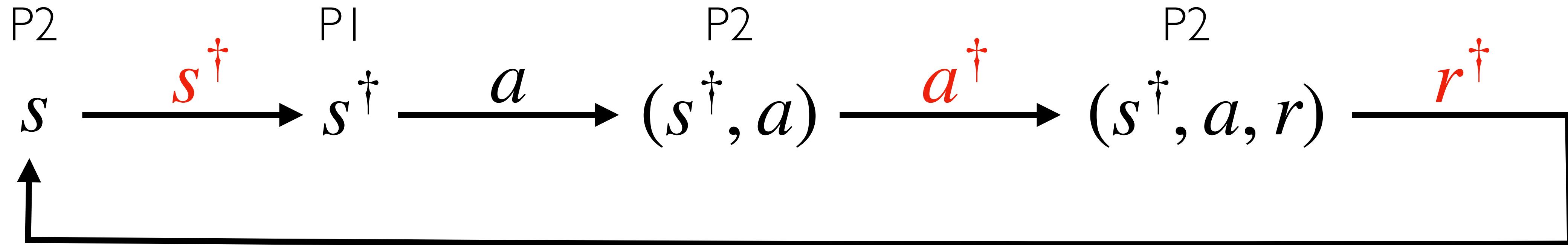
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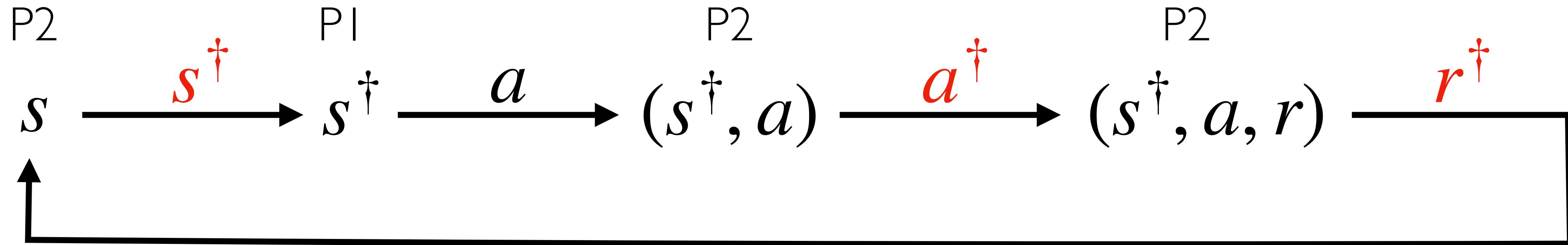
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Game evolves like a *turn-based* Markov game \overline{G} .

Meta Turn-based Markov Game

- I. \bar{S} records the player's information at any subperiod:

$$\bar{\mathcal{S}}_1 = \mathcal{S} \quad \text{and} \quad \bar{\mathcal{S}}_2 = \mathcal{S} \cup (\mathcal{S} \cup \mathcal{A}) \cup (\mathcal{S} \cup \mathcal{A} \cup \mathcal{R})$$

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$$\bar{\mathcal{A}}_1 = \mathcal{A} \quad \text{and} \quad \bar{\mathcal{A}}_2(s) \subseteq \mathcal{S}, \quad \bar{\mathcal{A}}_2(s, a) \subseteq \mathcal{A}, \quad \bar{\mathcal{A}}_2(s, a, r) \subseteq \mathbb{R}$$

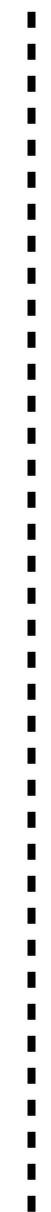
3. Transitions capture the evolution of information.

Proposition: Any WSE for \bar{G} is an optimal defense policy.

Efficient Reduction to MARL

Efficient Reduction to MARL

\overline{G}

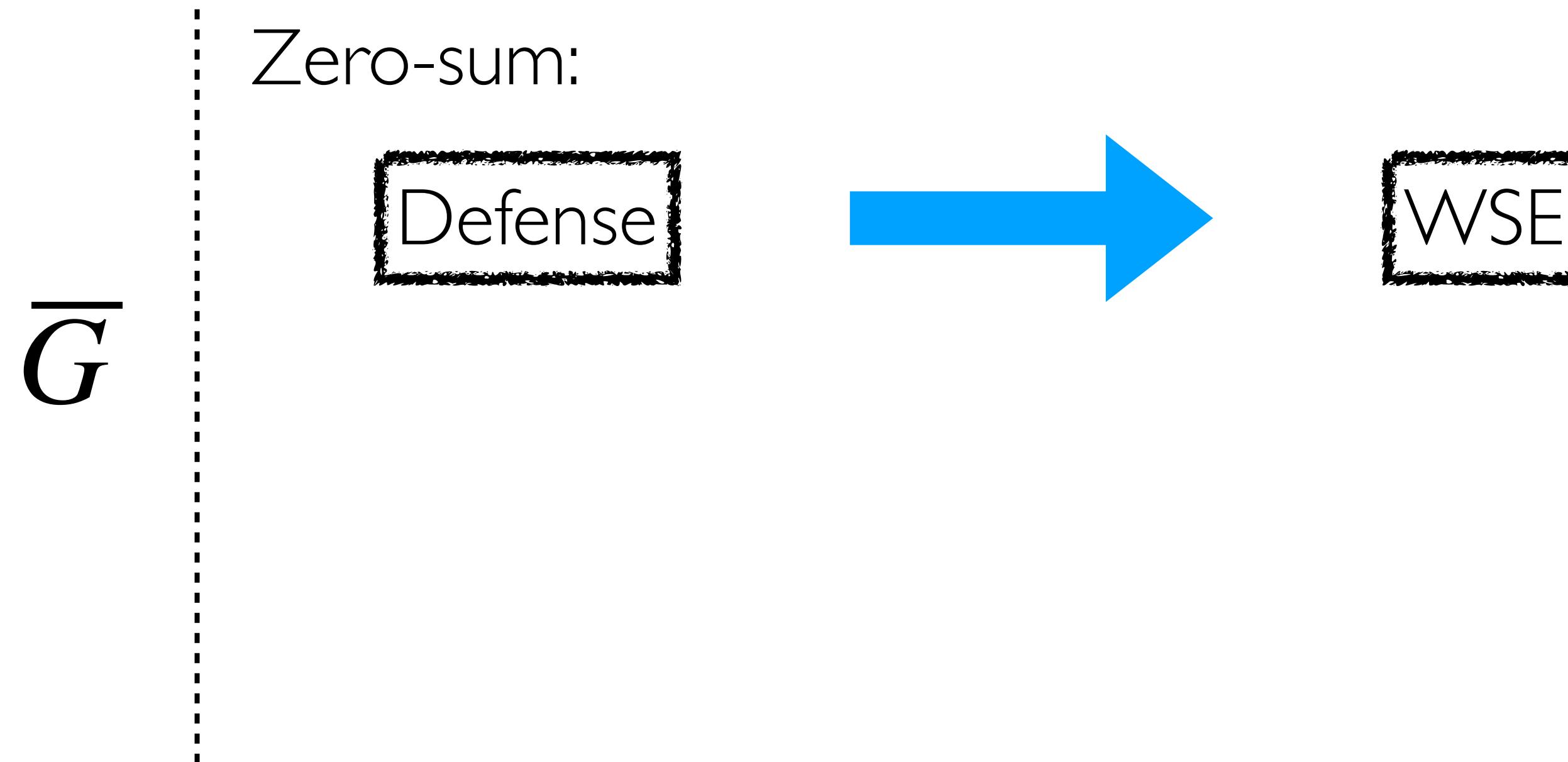


Efficient Reduction to MARL

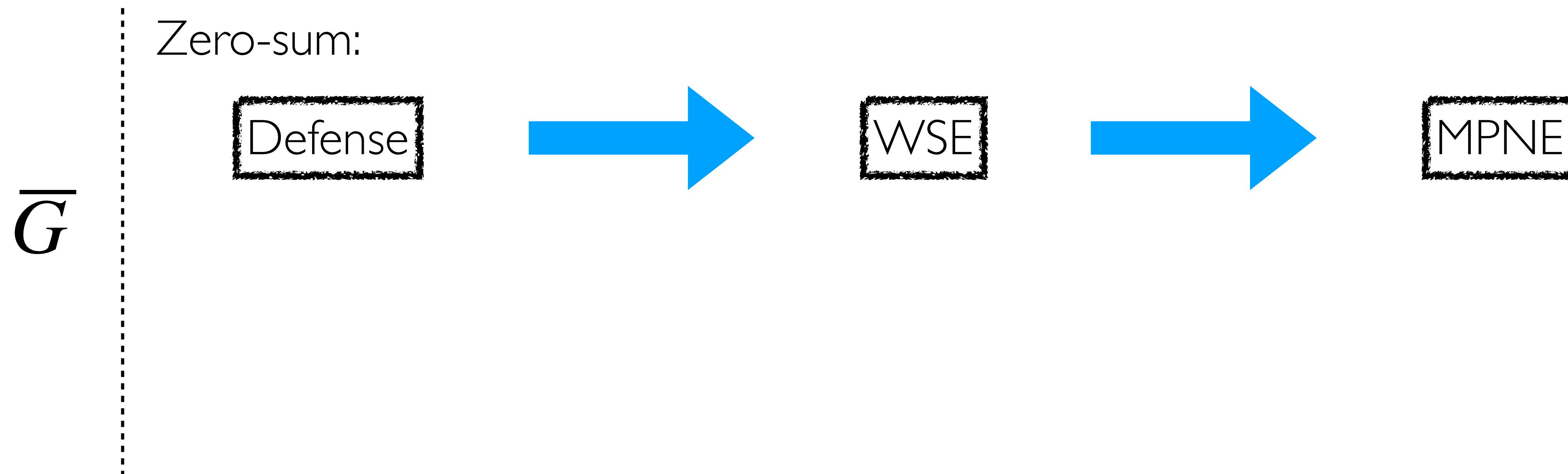
Zero-sum:

\overline{G}

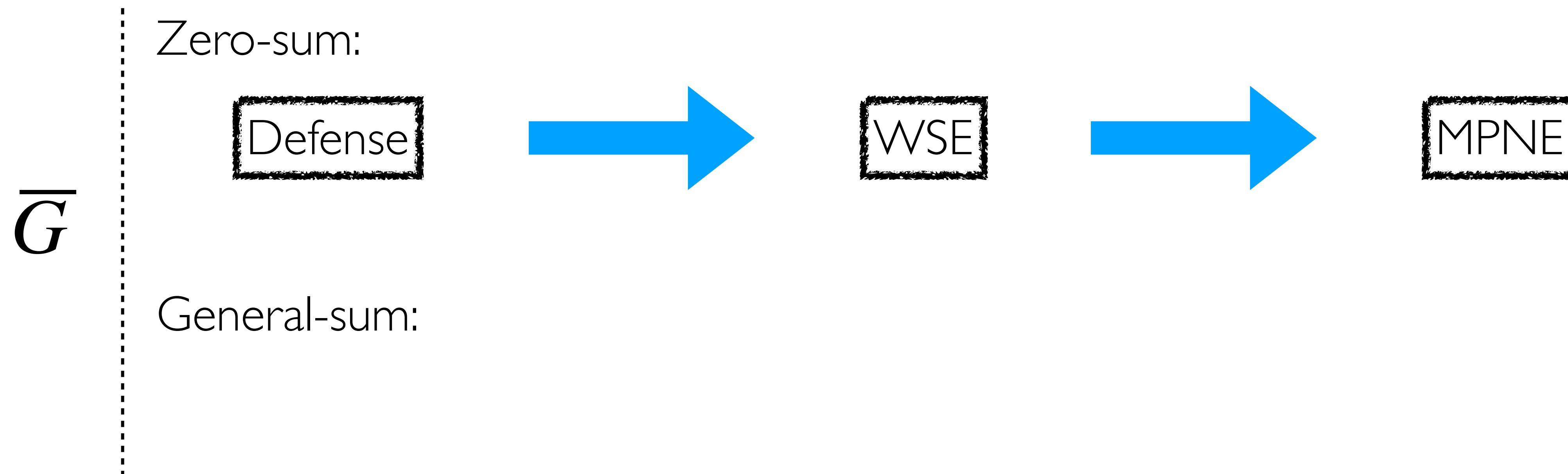
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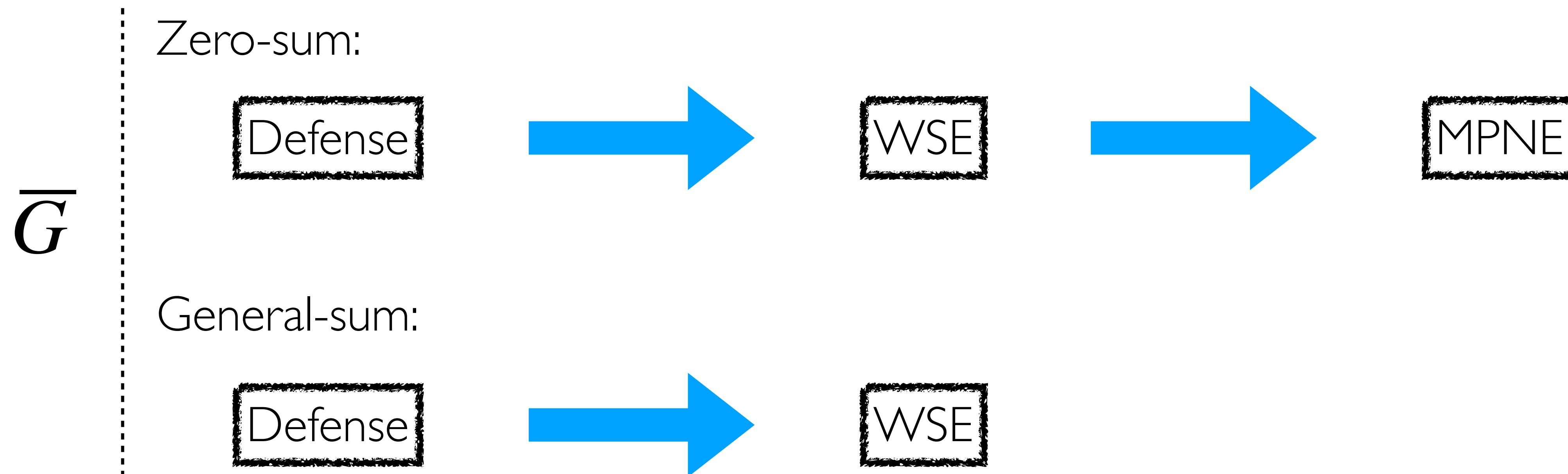
Efficient Reduction to MARL



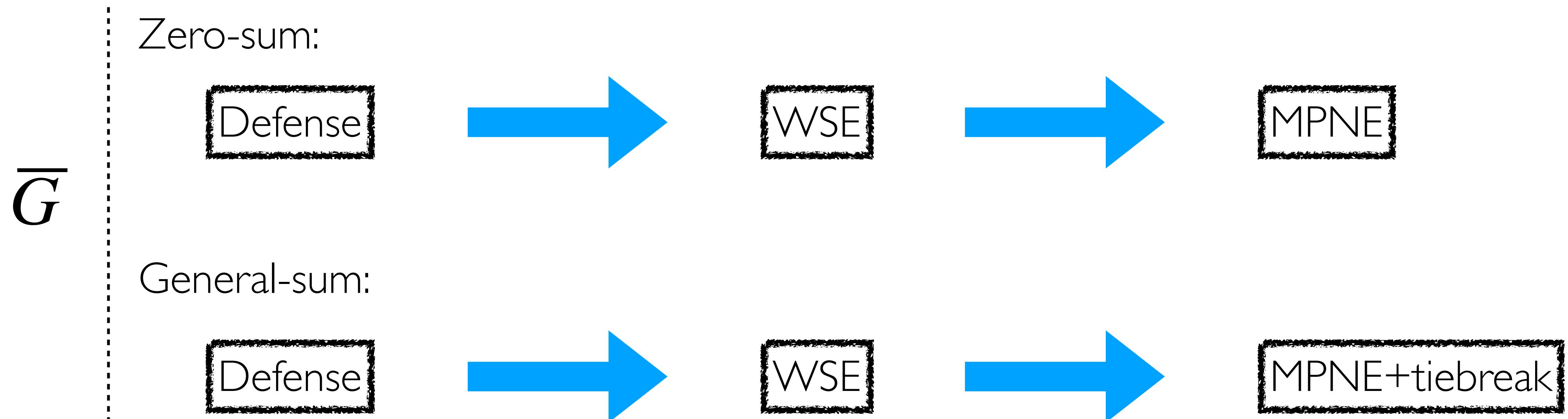
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Rollback Algorithm

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Special Case: Action Attacks

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I. Victim determines Attacker's best response to any action a :

$$BR_h(s, a) = \arg \max_{a^\dagger \in \overline{\mathcal{A}}(s, a)} [g_h(s, a, r_h(s, a)) + \mathbb{E}_{s' \sim P_h(s, a^\dagger)} [V_{h+1, 2}^*(s', \pi_{h+1}^*(s'))]]$$

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2. Victim picks a based on the worst-case best-response:

$$V_{h, 1}^*(s) = \max_{a \in \mathcal{A}} \min_{a^\dagger \in BR_h(s, a)} [r_h(s, a^\dagger) + \mathbb{E}_{s' \sim P_h(s, a^\dagger)} [V_{h+1, 1}^*(s')]]$$

Guarantees

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2. \bar{G} has finite-horizon.

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Complete characterization: hard \Leftrightarrow observation attacks!

2. \bar{G} has finite-horizon.

Conclusions

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Thank you!

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