



# Optimal Attack and Defense for Reinforcement Learning

*Jeremy McMahan, Young Wu, Xiaojin Zhu, Qiaomin Xie*

# RL Basics

# RL Interaction Protocol

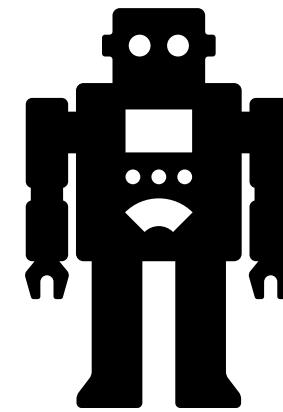
# RL Interaction Protocol

Models sequential decision making in uncertain environments



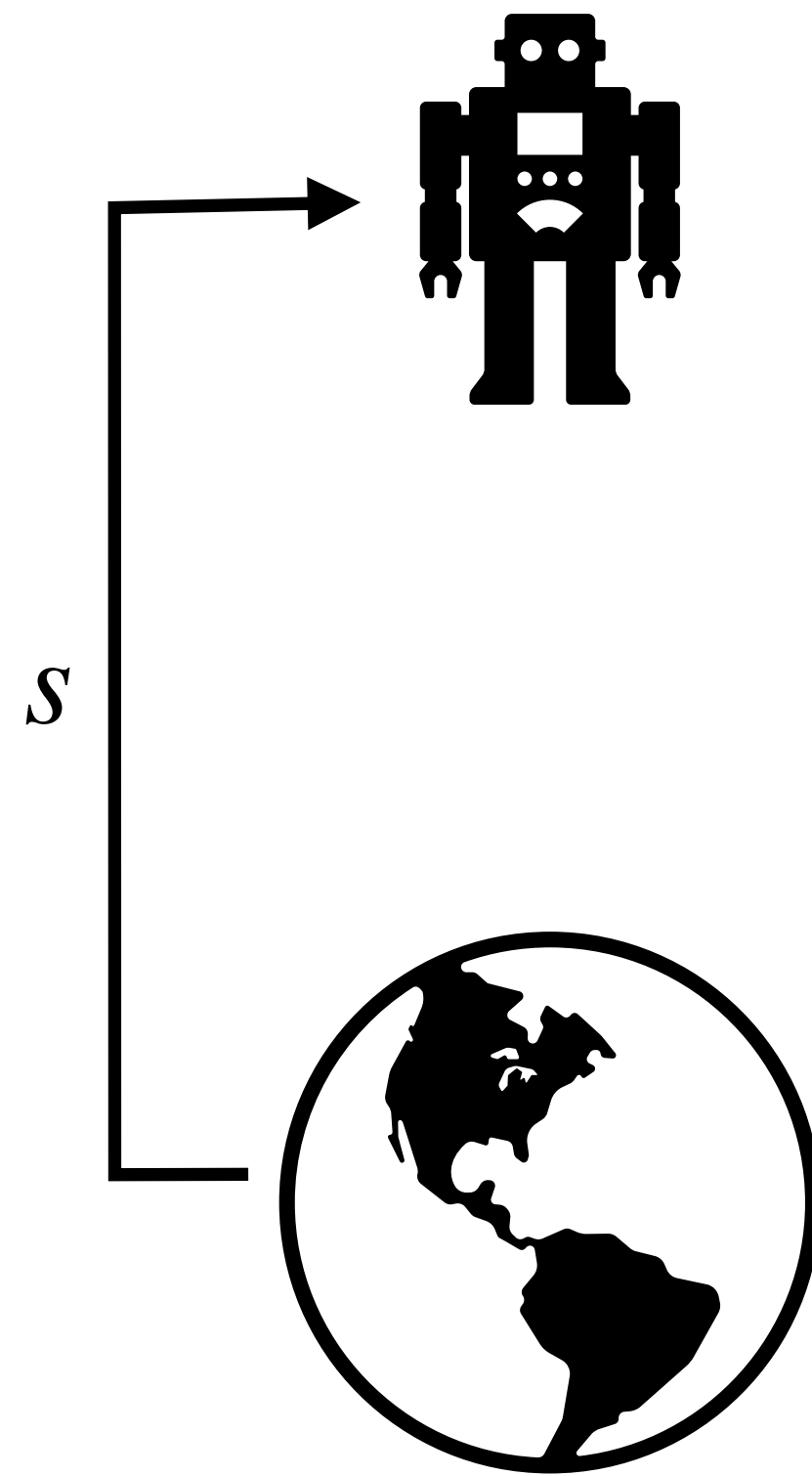
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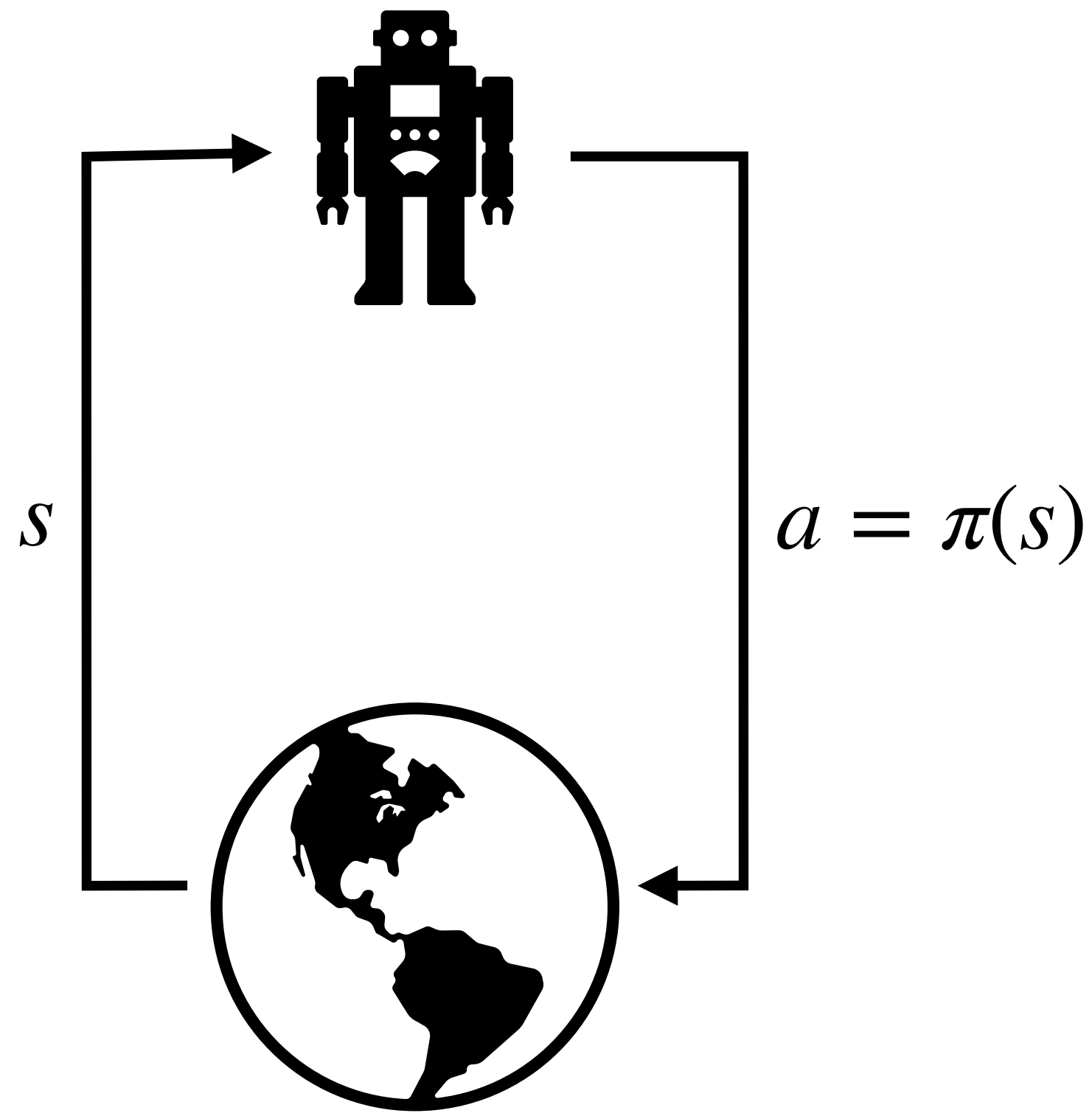
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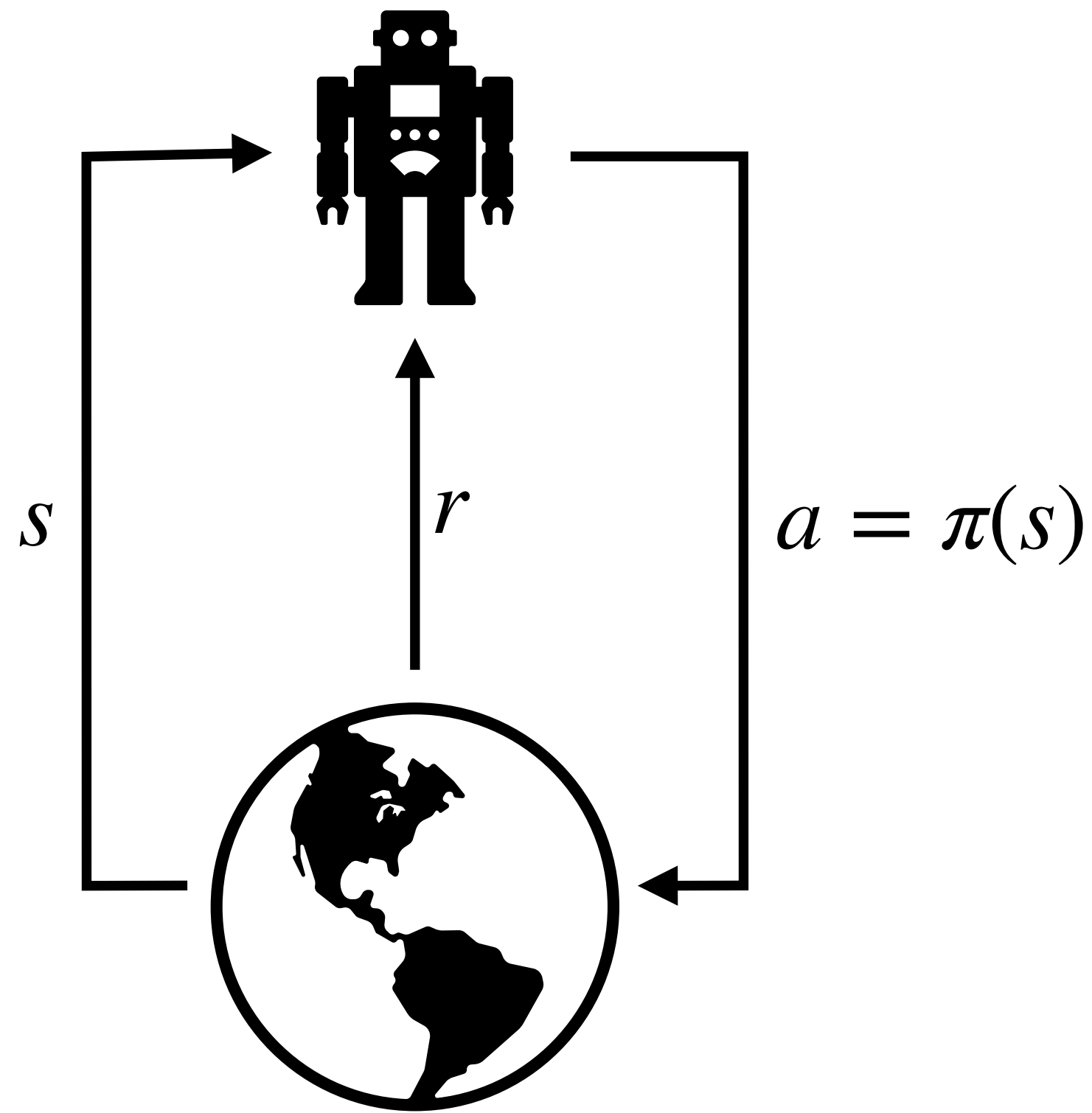
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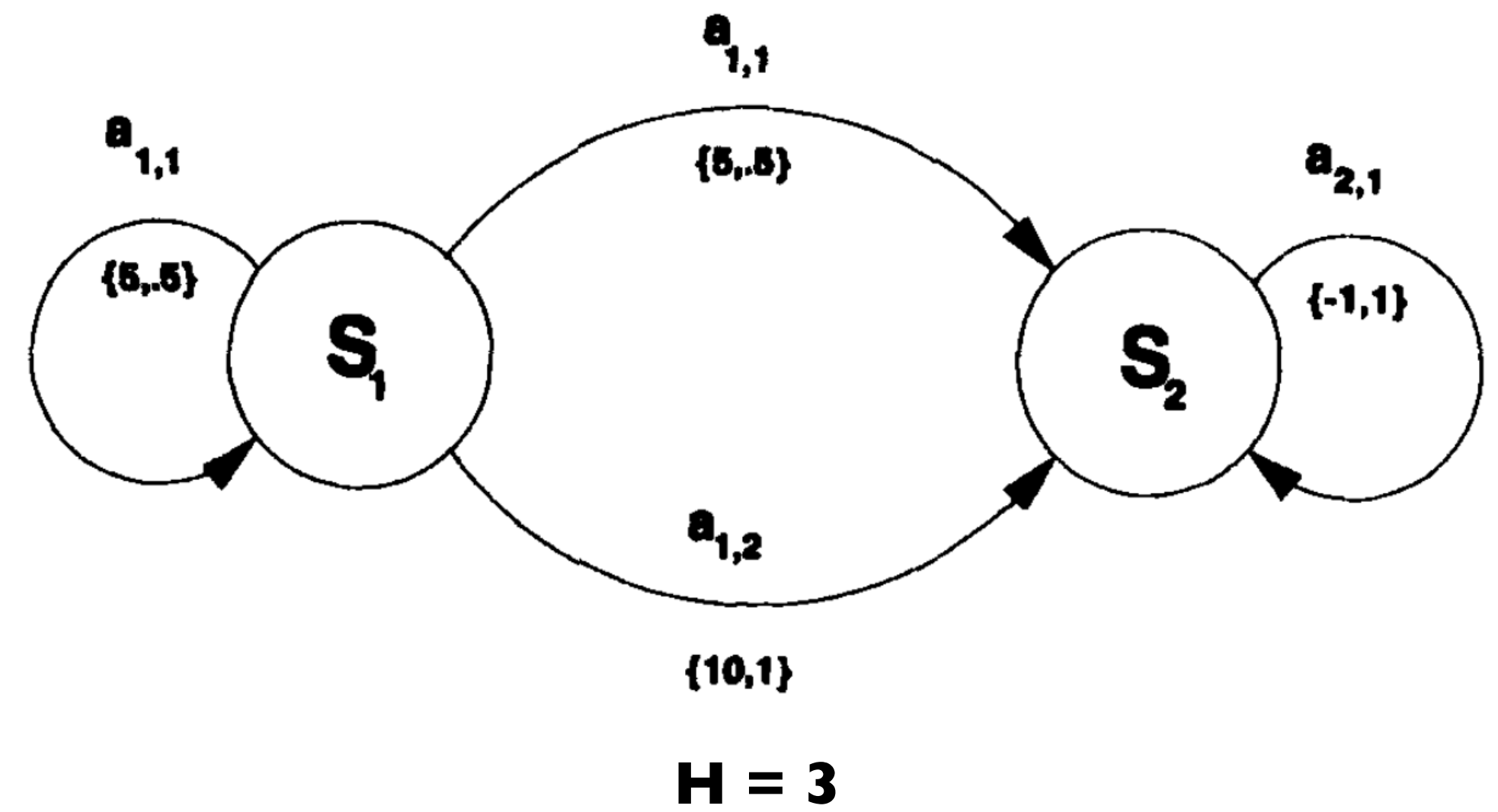
Models sequential decision making in uncertain environments



# Model: MDPs

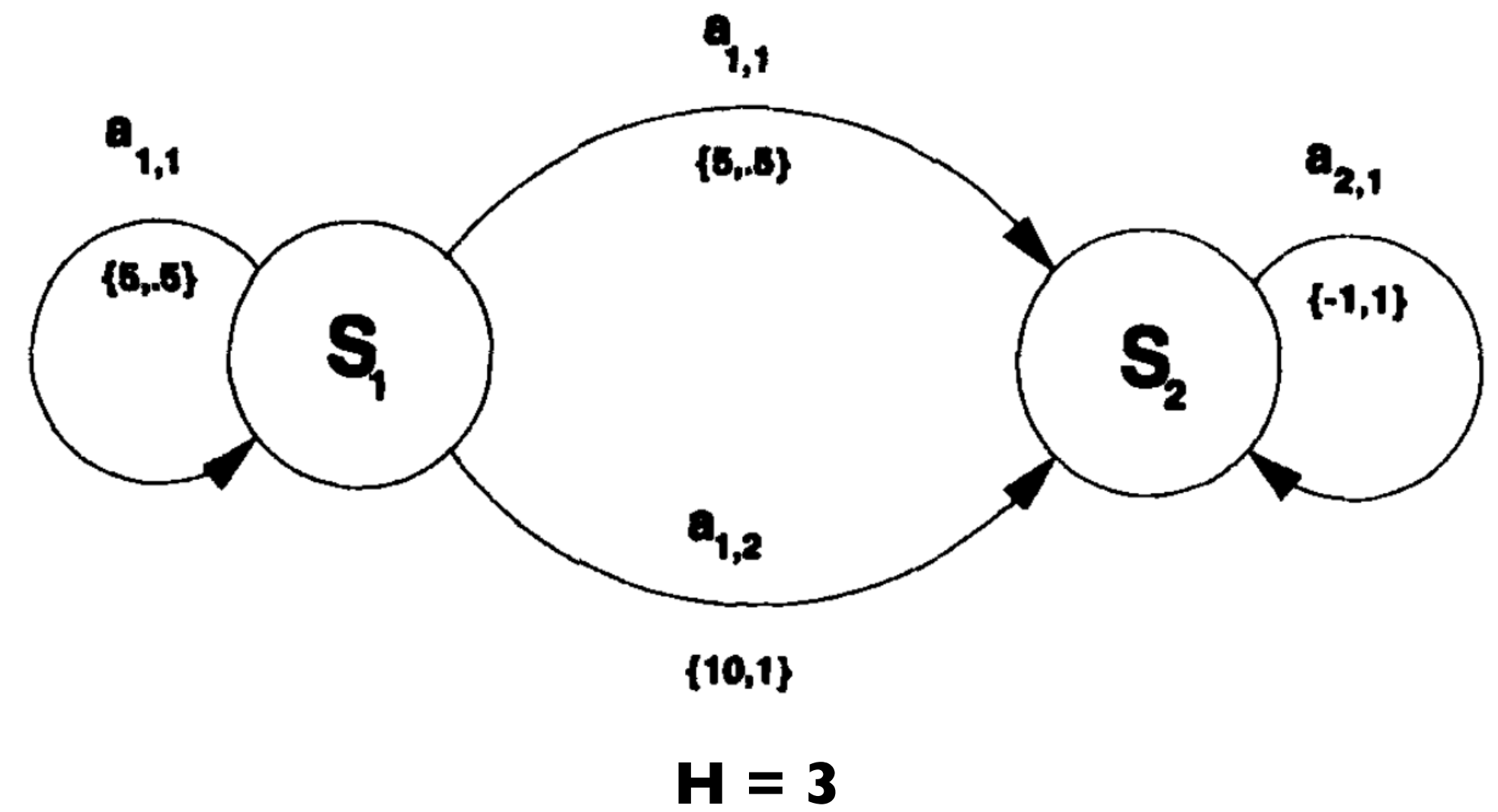
**H = 3**

# Model: MDPs



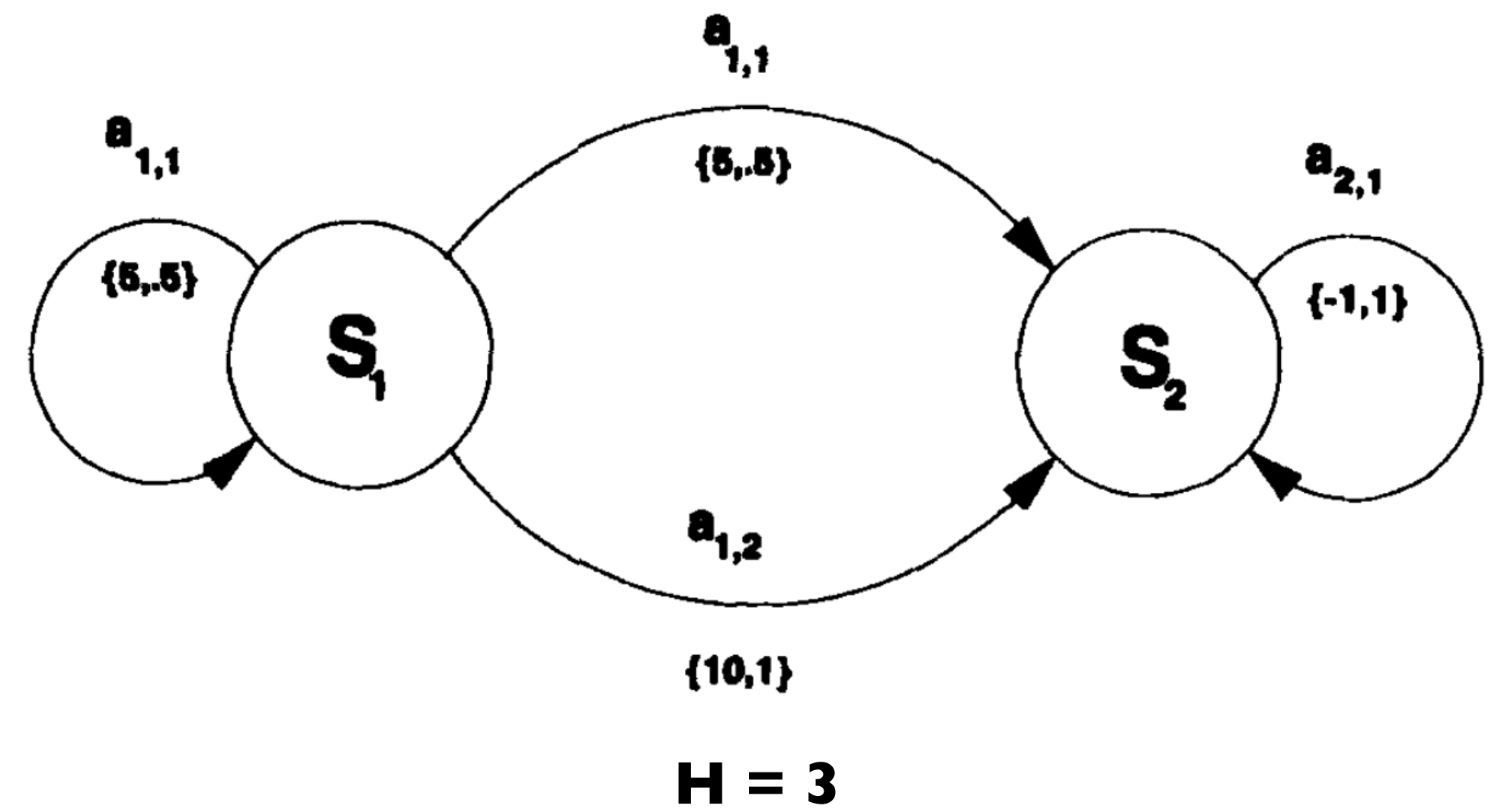
# Model: MDPs

- States,  $S$



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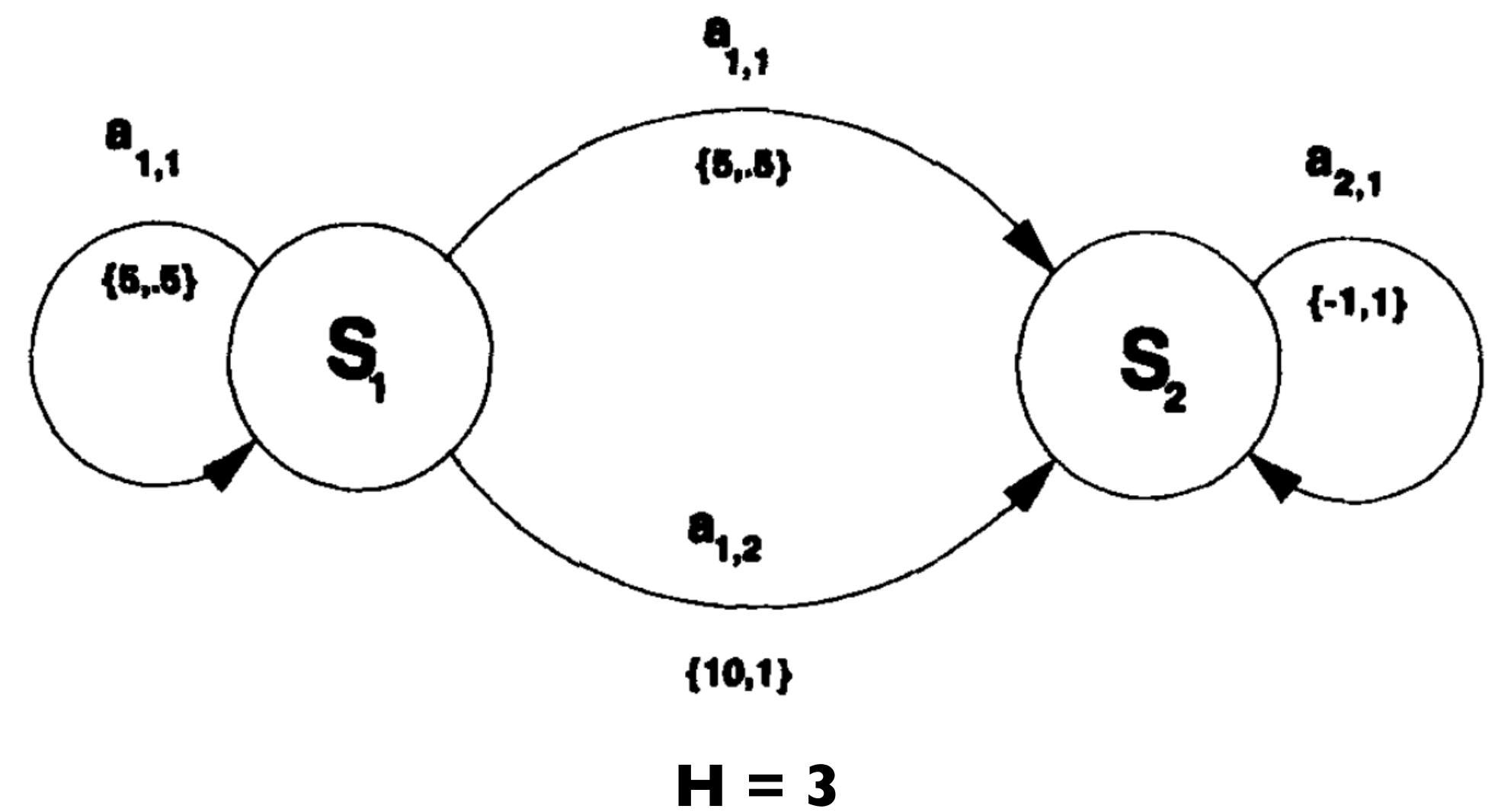
- States,  $\mathcal{S}$
- Actions,  $\mathcal{A}$





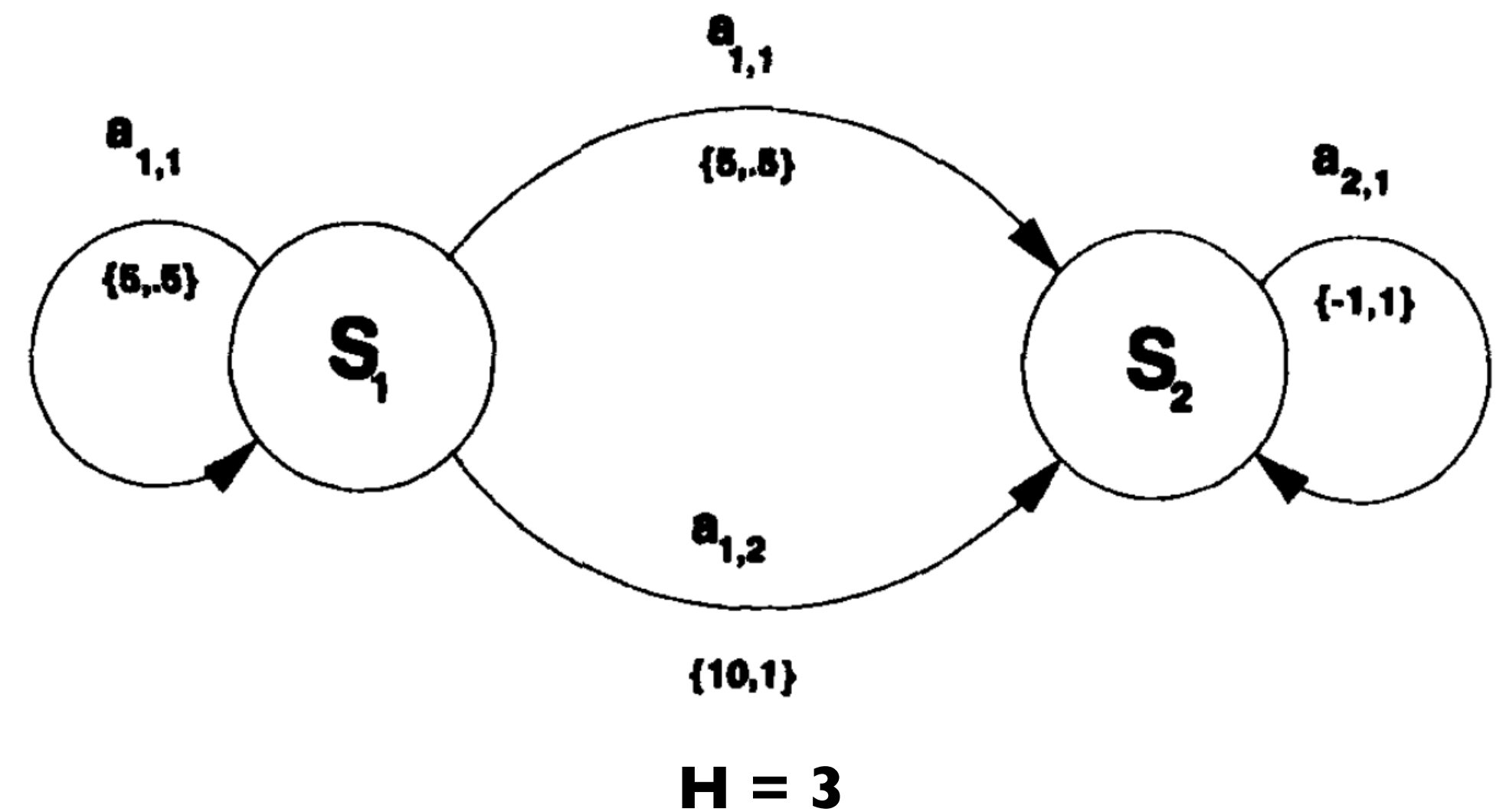
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- States,  $S$
- Actions,  $A$
- Rewards,  $r(s, a)$



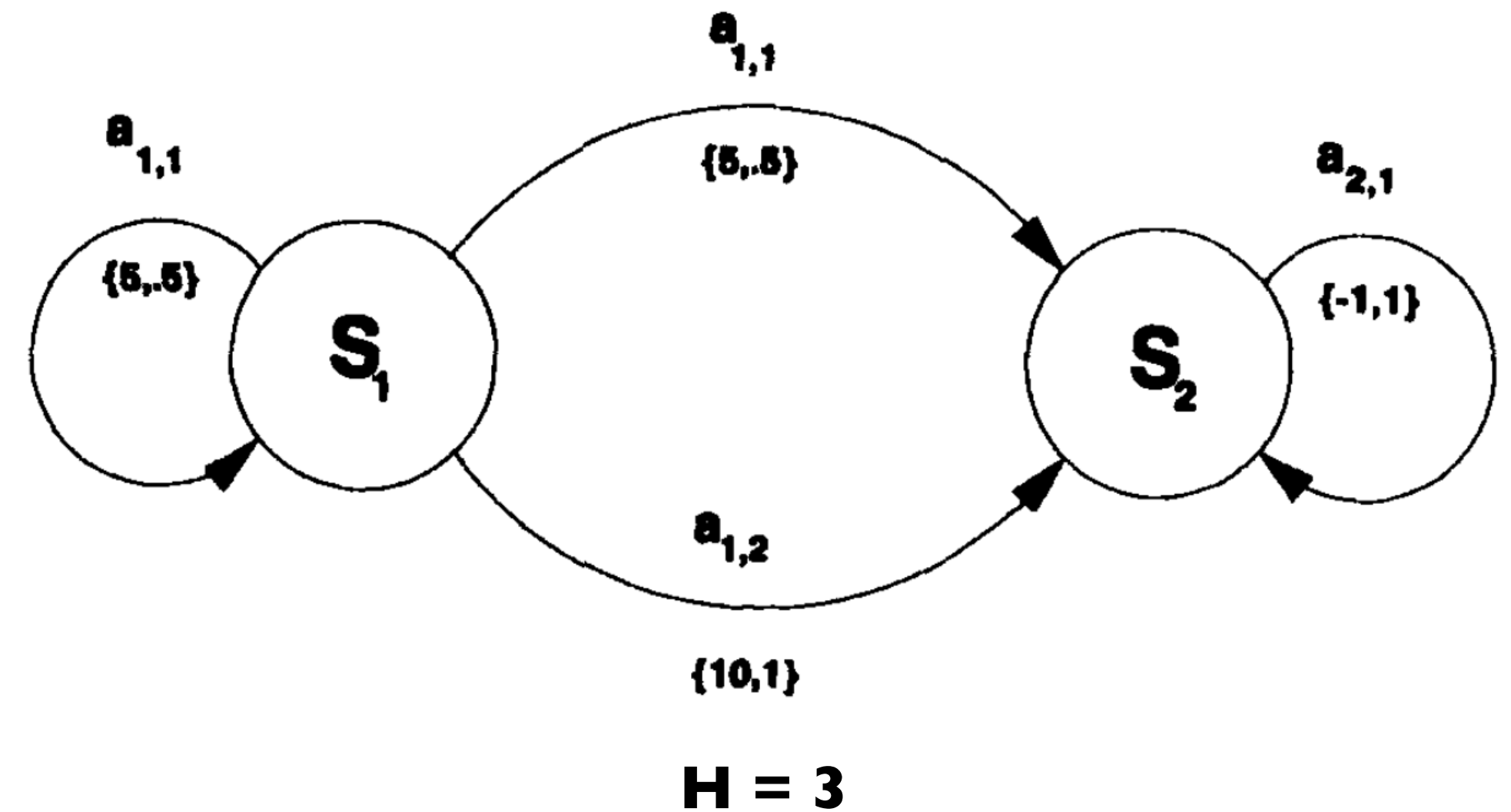
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# Model: MDPs

- States,  $\mathcal{S}$
- Actions,  $\mathcal{A}$
- Rewards,  $r(s, a)$
- Transition Probabilities,  $P(s' | s, a)$
- Time Horizon,  $H$



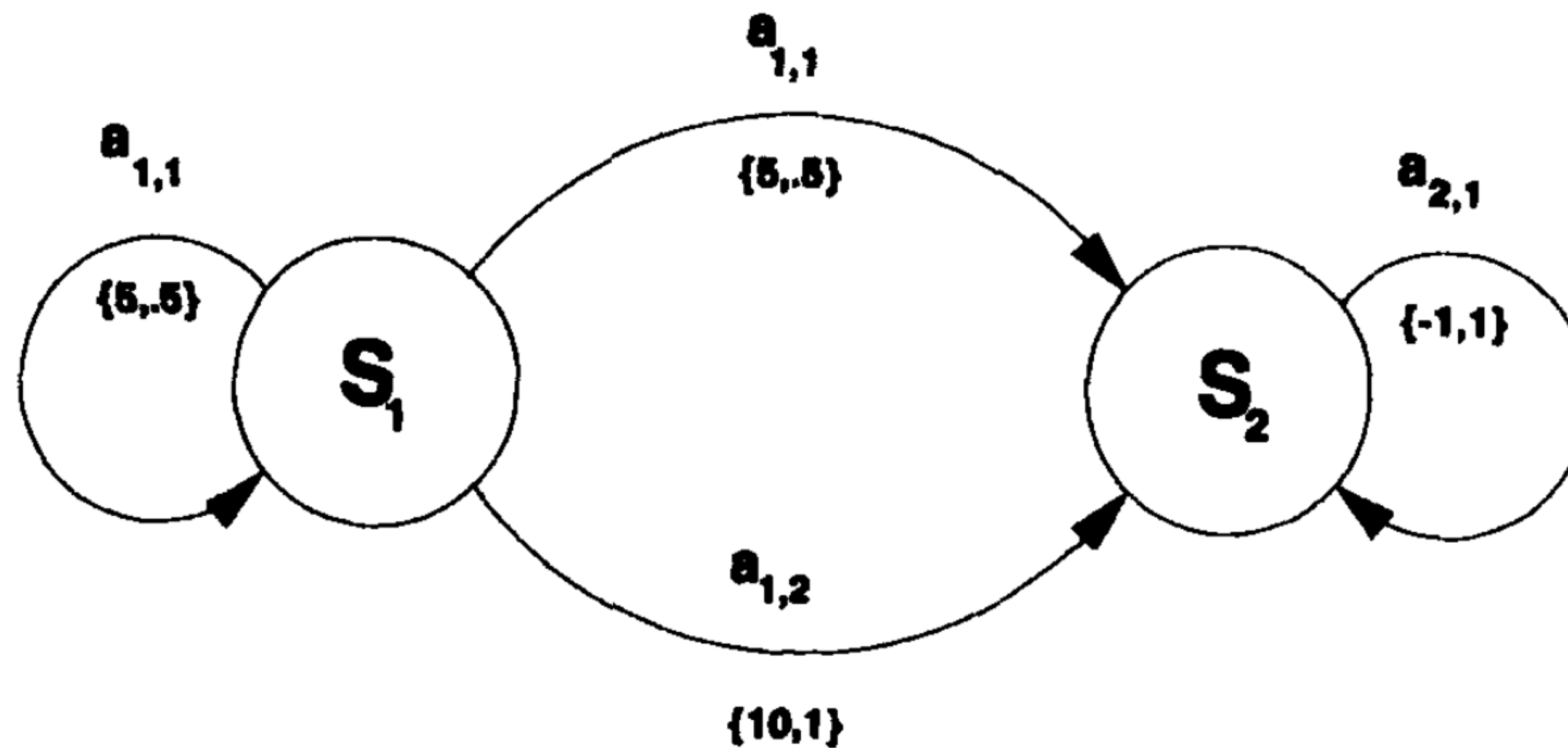
# Policies

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A *policy* is a plan of what action to take in each state.

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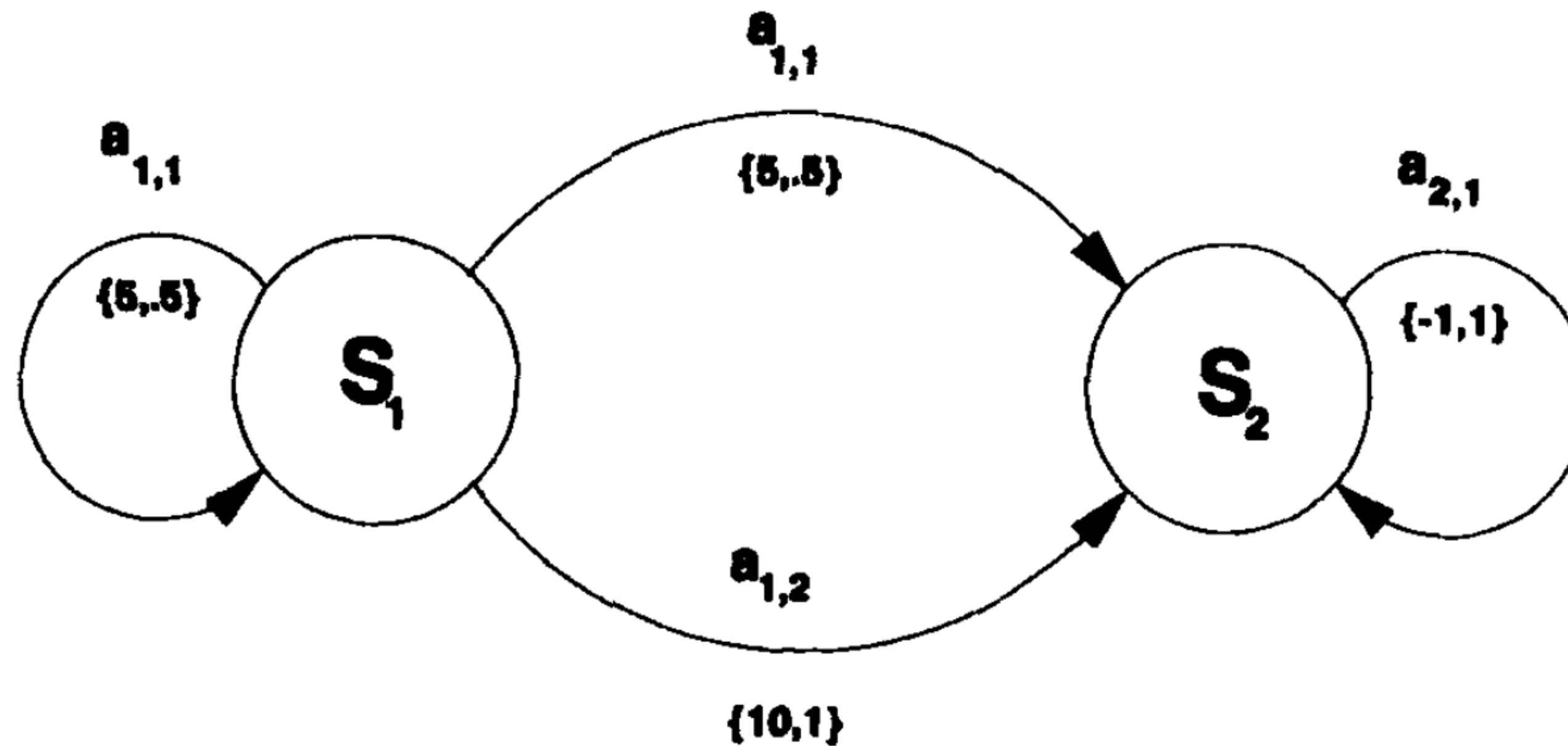
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$$\pi(s_1) = a_{1,2} \quad \pi(s_2) = a_{2,1}$$

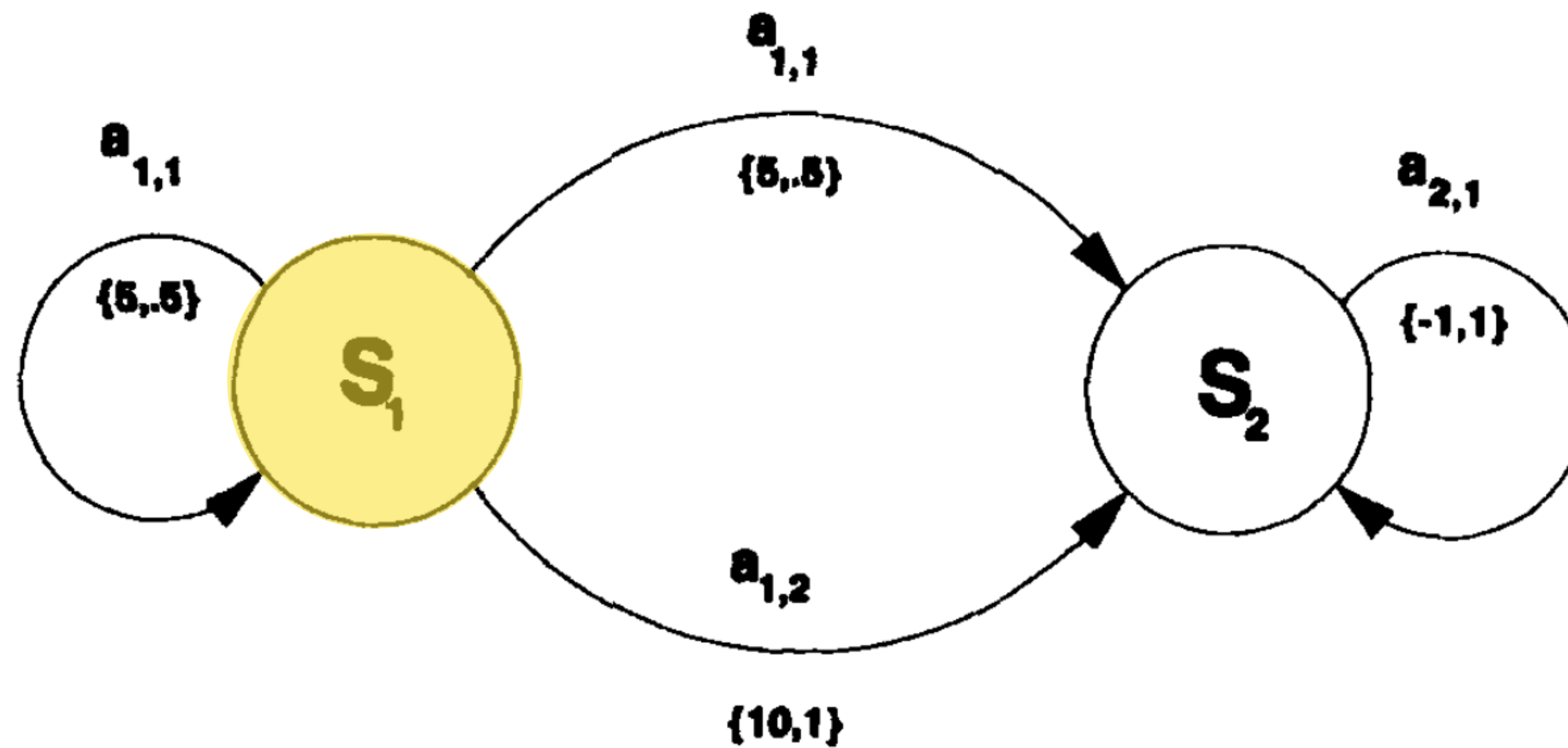


# Value

The *value* of  $M$  under  $\pi$  is:  $V^\pi(s) = E_\pi \left[ \sum_{h=1}^H r_h(s, a) \mid s_0 = s \right]$ .



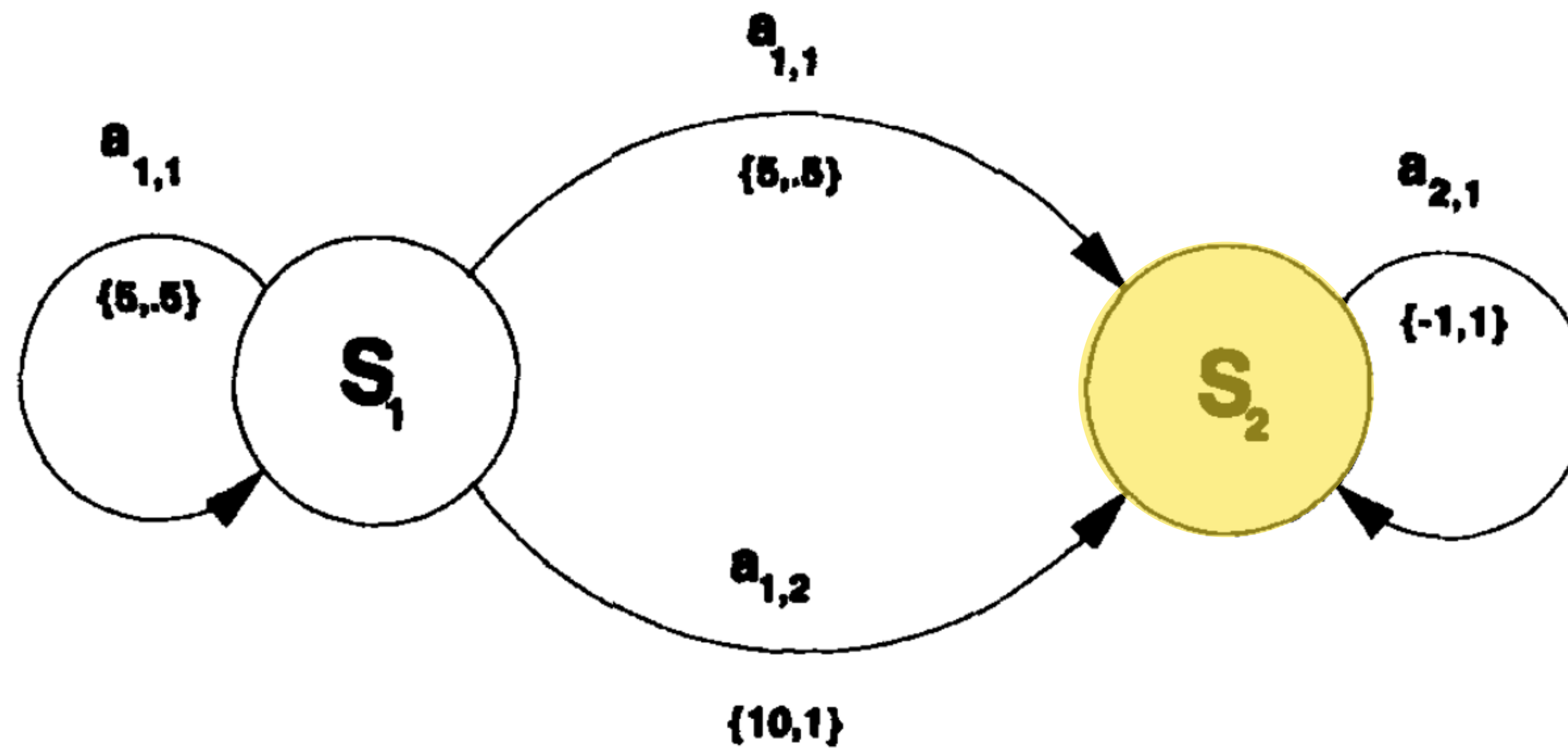
# Value



$$\pi(s_1) = a_{1,2}$$

Reward = 10

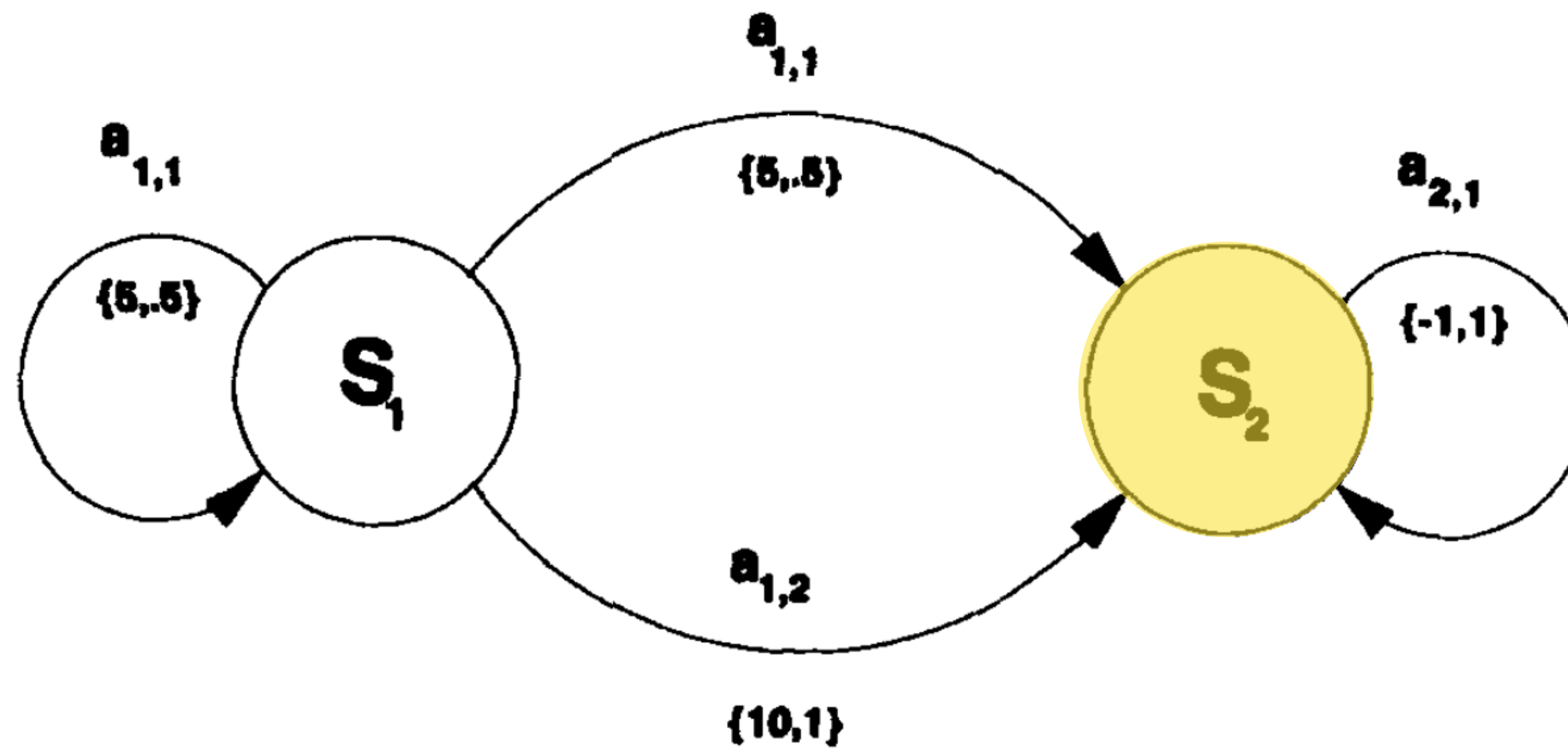
# Value



$$\pi(s_2) = a_{2,1}$$

Reward = -1

# Value



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# Value

$$V^{\pi}(s_1) = 10 - 1 - 1 = 8$$

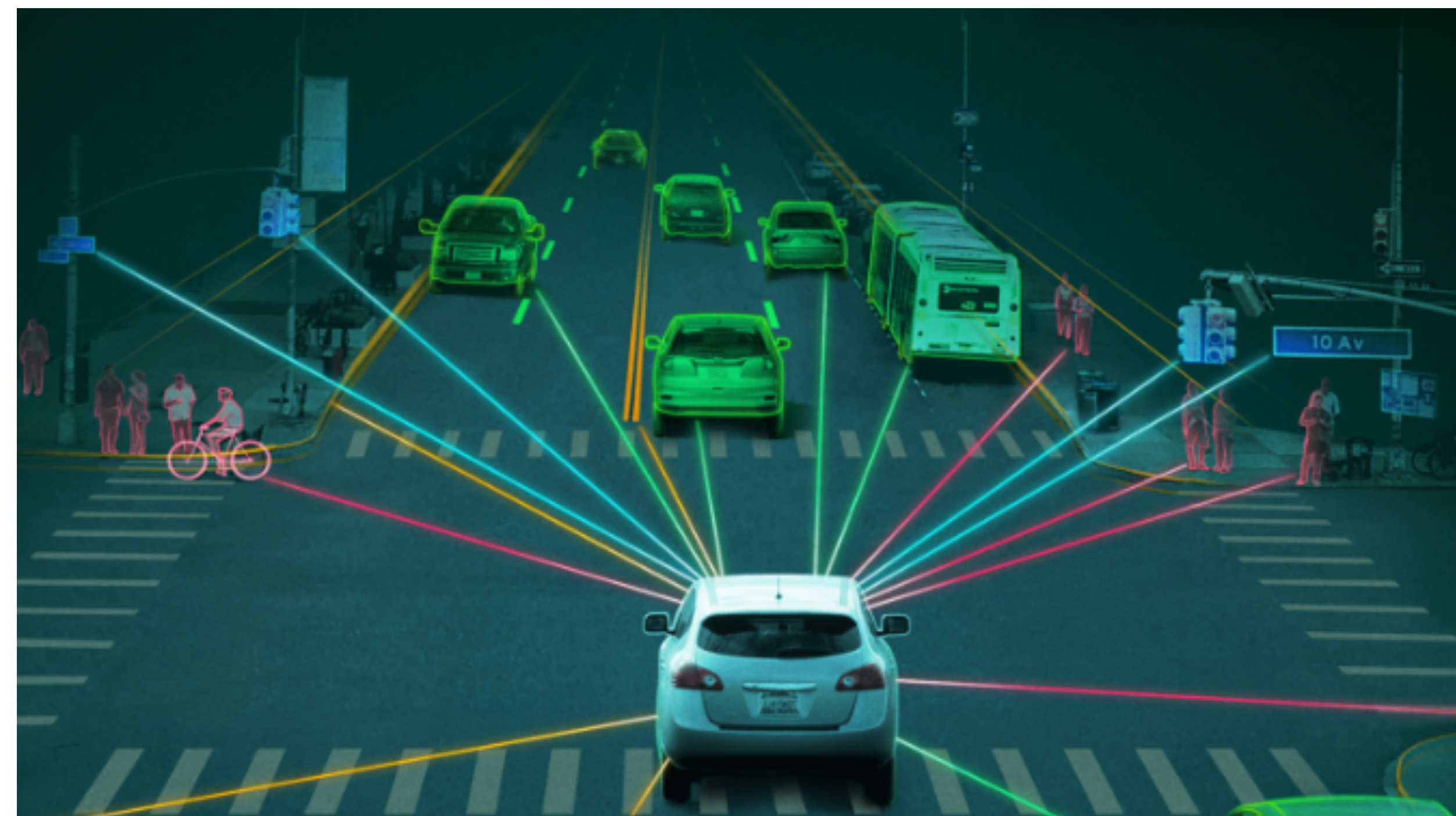
# Optimal Policies

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$$\pi^* = \sup_{\pi} V^{\pi}(s_0)$$

# Example MDP

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# Example MDP

## Disaster Relief with Autonomous Vehicles

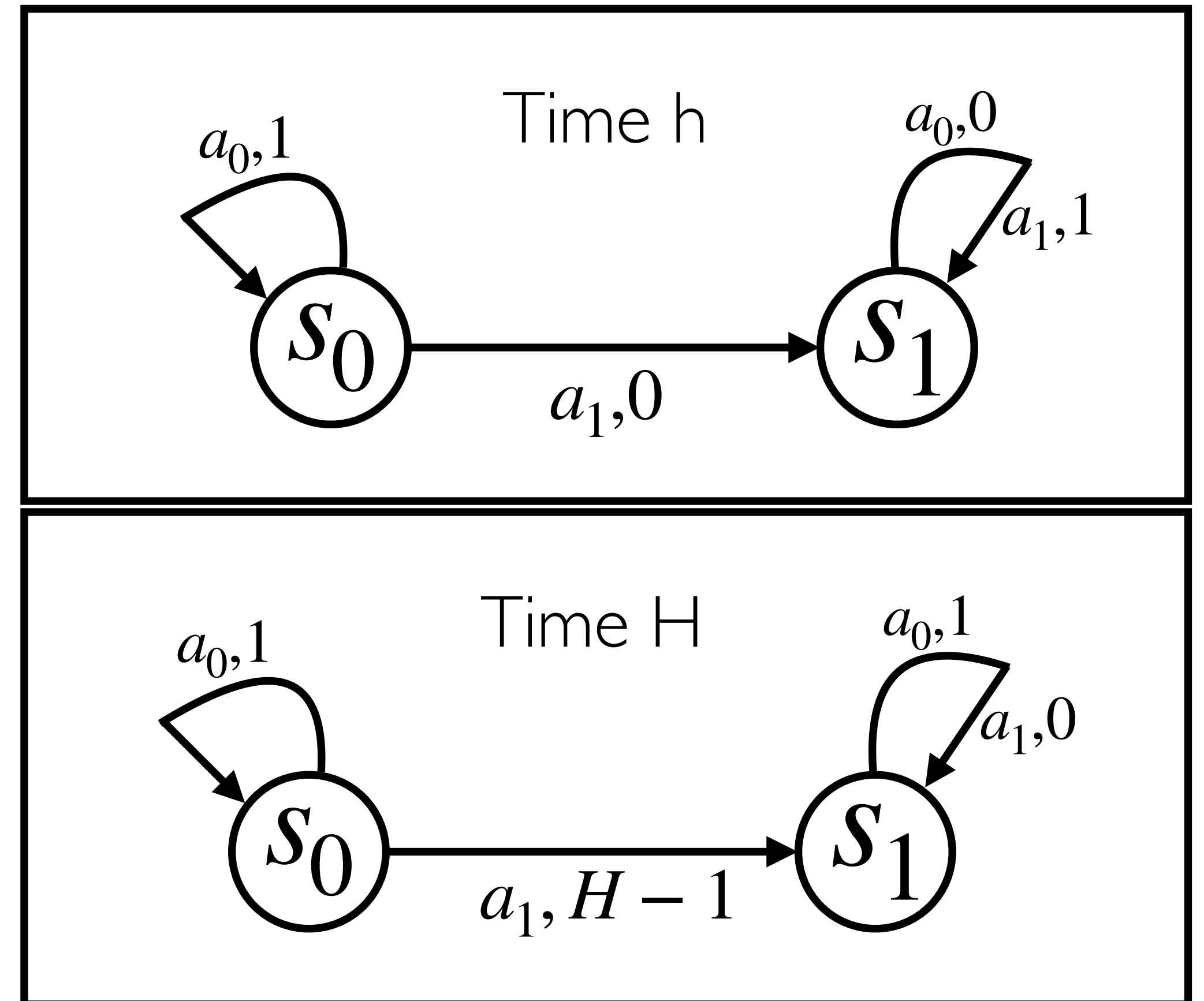
- State Space is  $\mathbb{R}^2$
- Action Space is  $[-1,1]^2$
- New location is  $s + a$
- Reward for finding people in need.



# Performance of Optimal Policies

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$M$



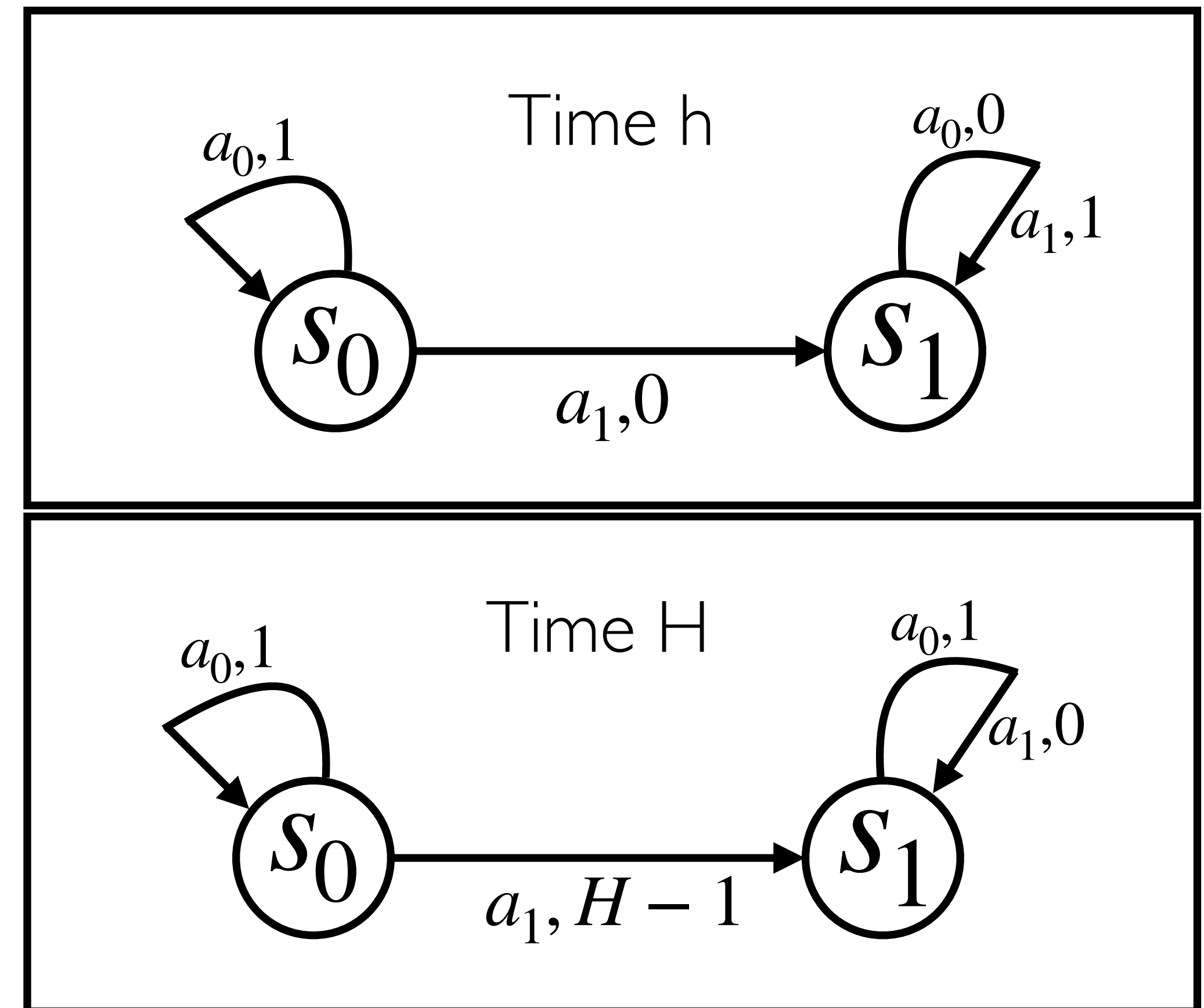
# Performance of Optimal Policies

Unique optimal policy  $\pi^*$  is:

$\pi^*$

t/S	s_0	s_1
h	a_0	a_1
H	a_1	a_0

$M$



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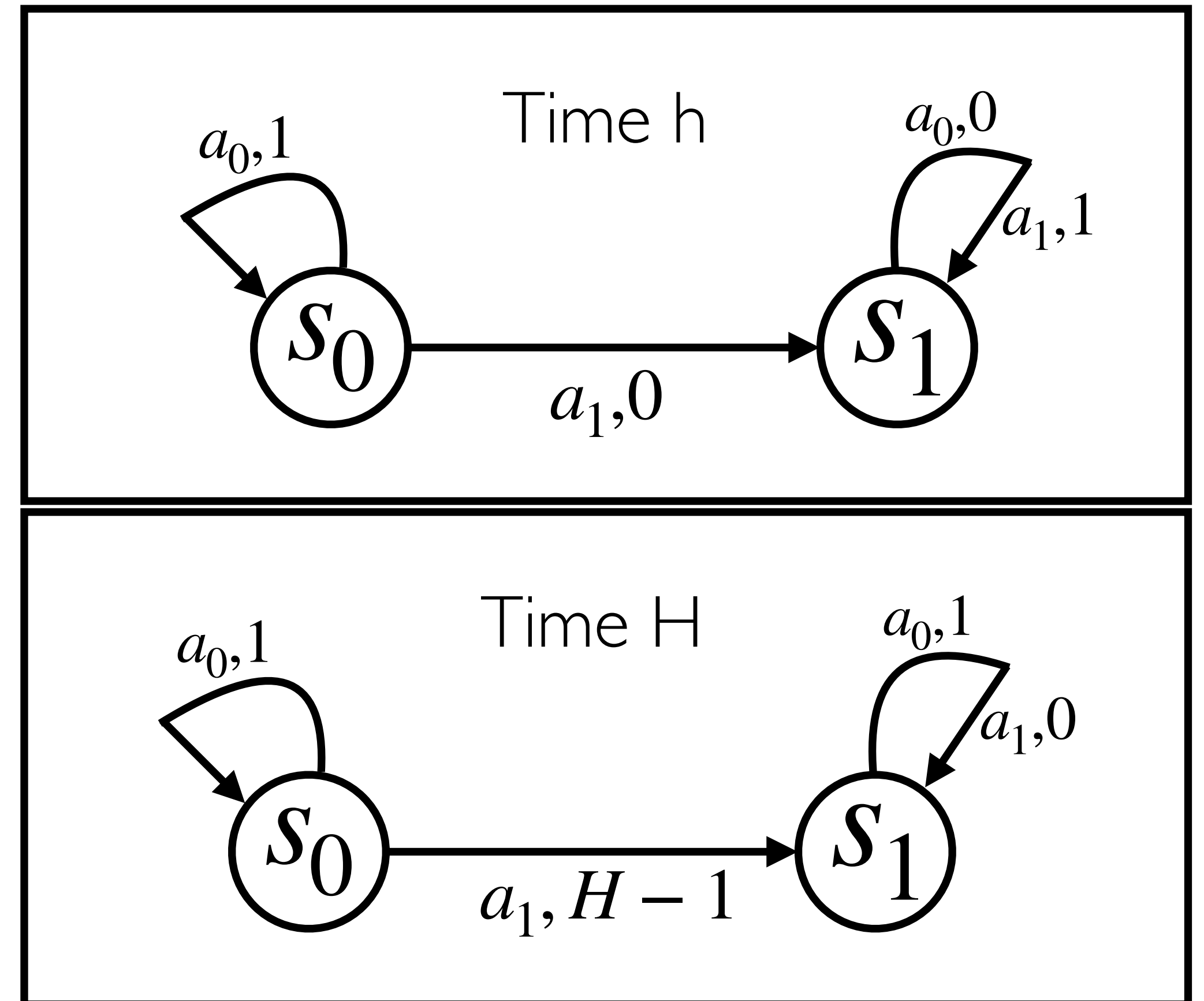
$\pi^*$

t/S	s_0	s_1
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The optimal policy achieves value:

$$V_M^{\pi^*} = 2(H - 1)$$

$M$



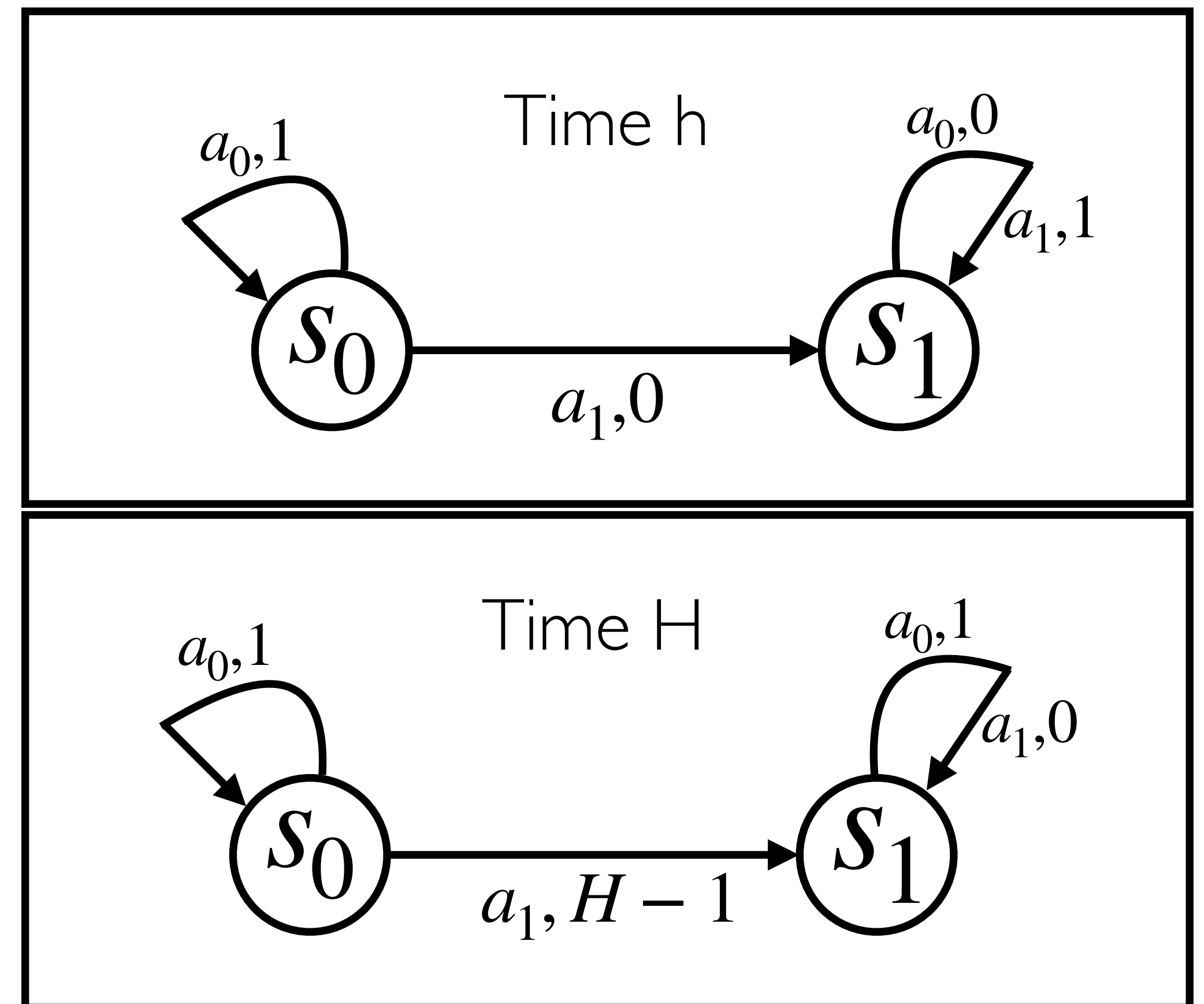
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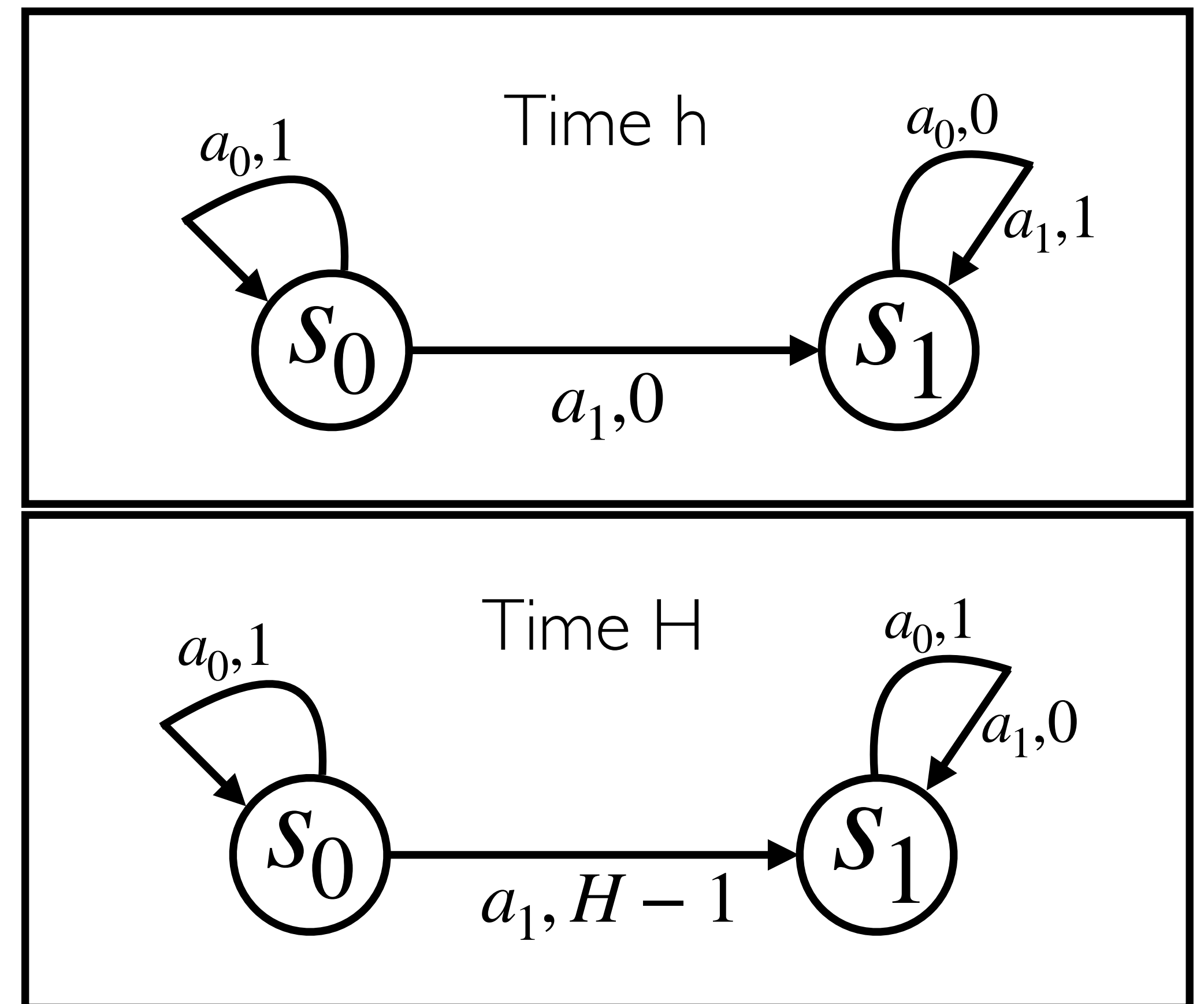




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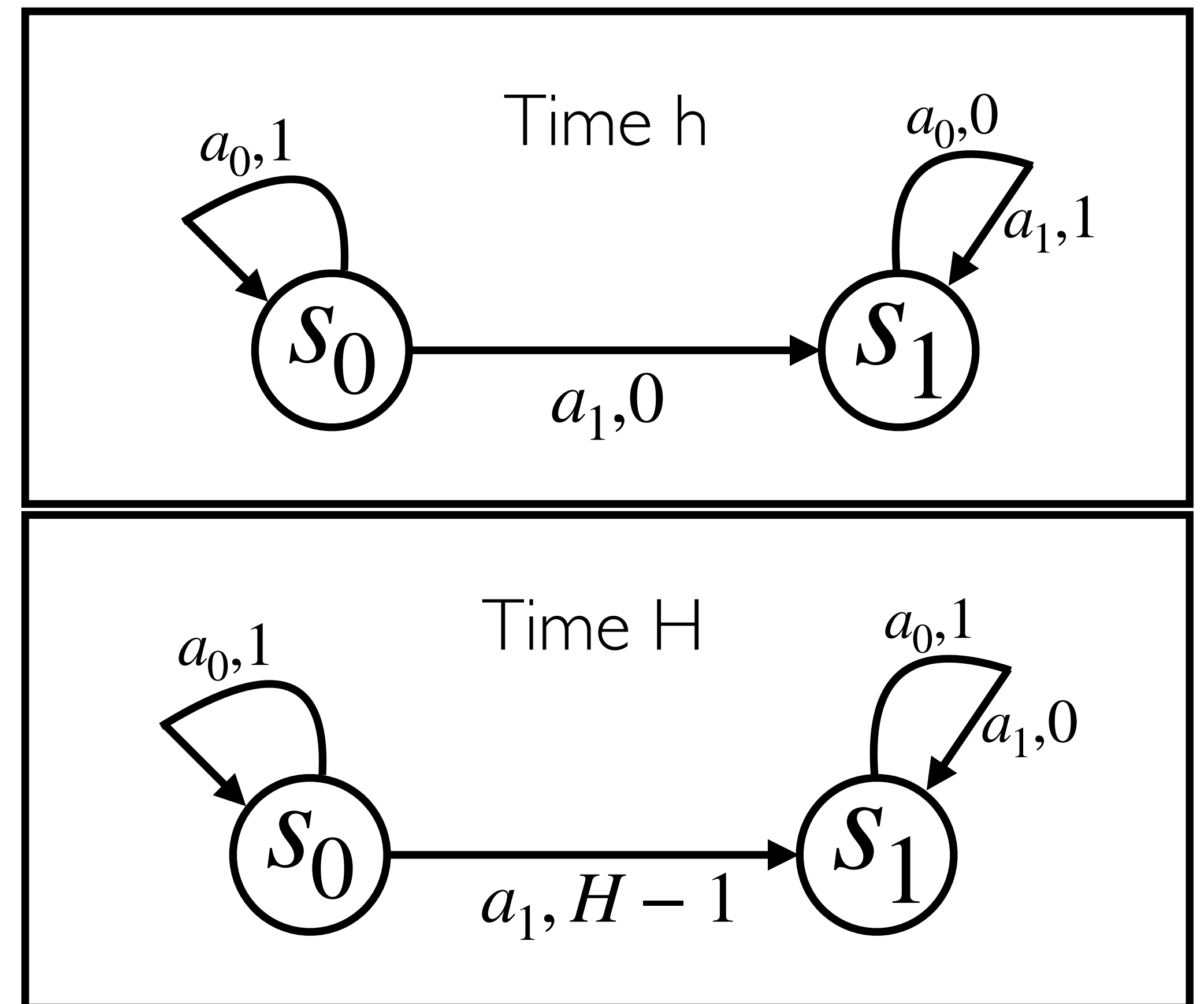
- If first state is actually  $s_1$  or M receives  $a_1$  instead,  $\pi^*$  at best gets  $1/2$  of its value.



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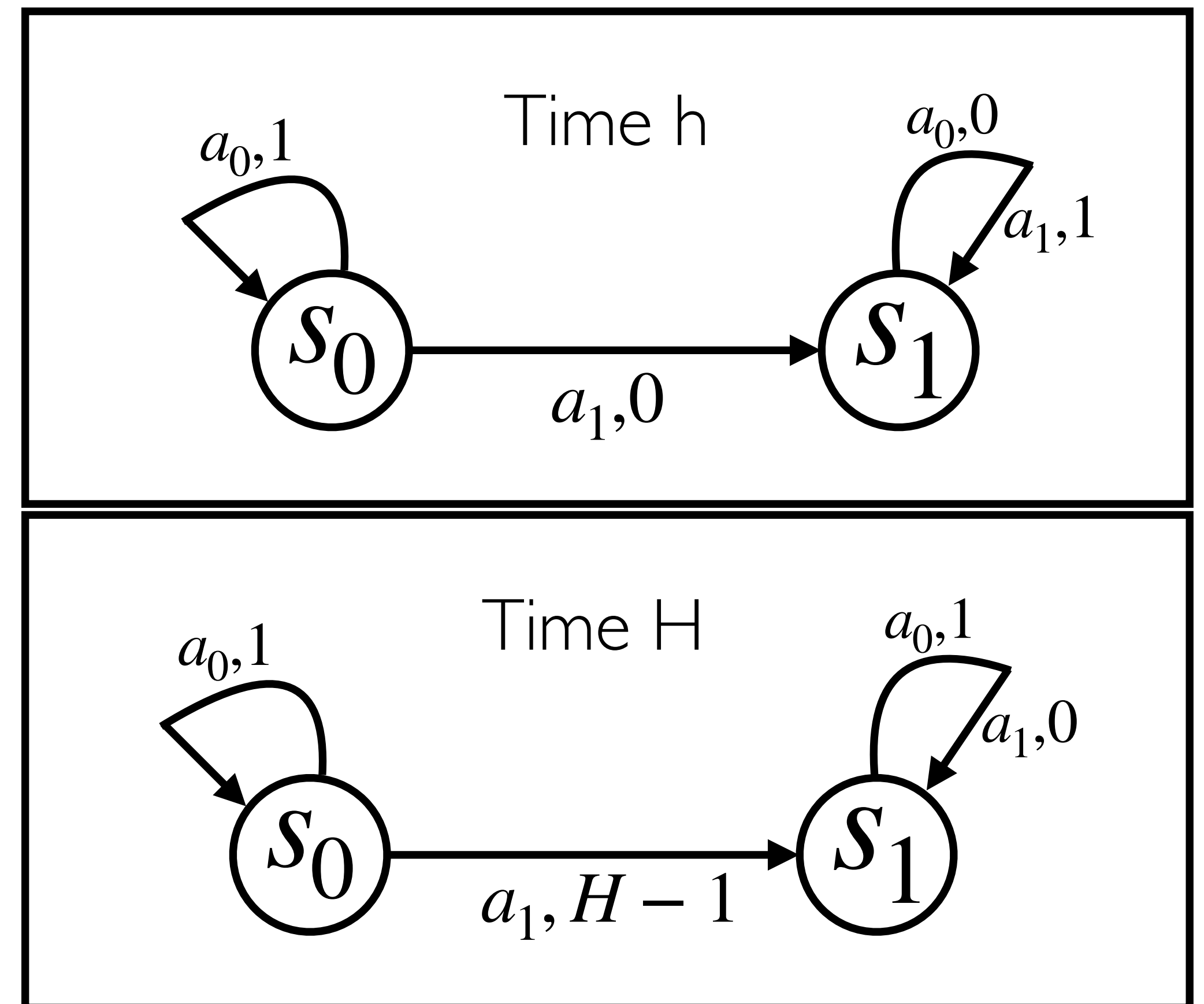
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$\pi^* \circ \nu$

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H	a_0	a_1



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- Strategies to compute **robust** policies are needed.
- Inspiration for field of **adversarial RL**.

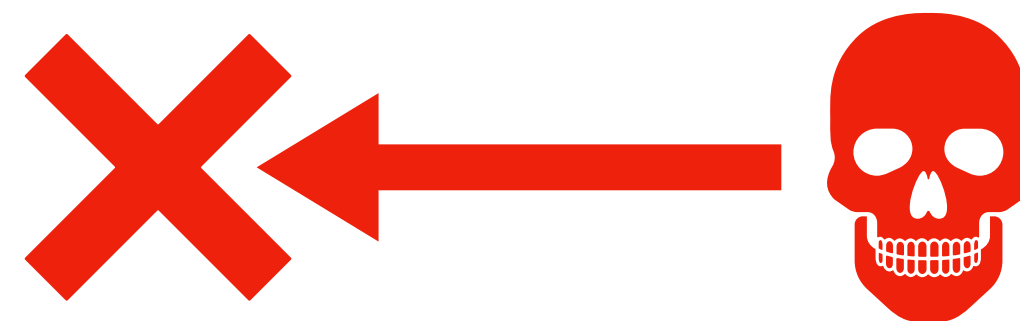
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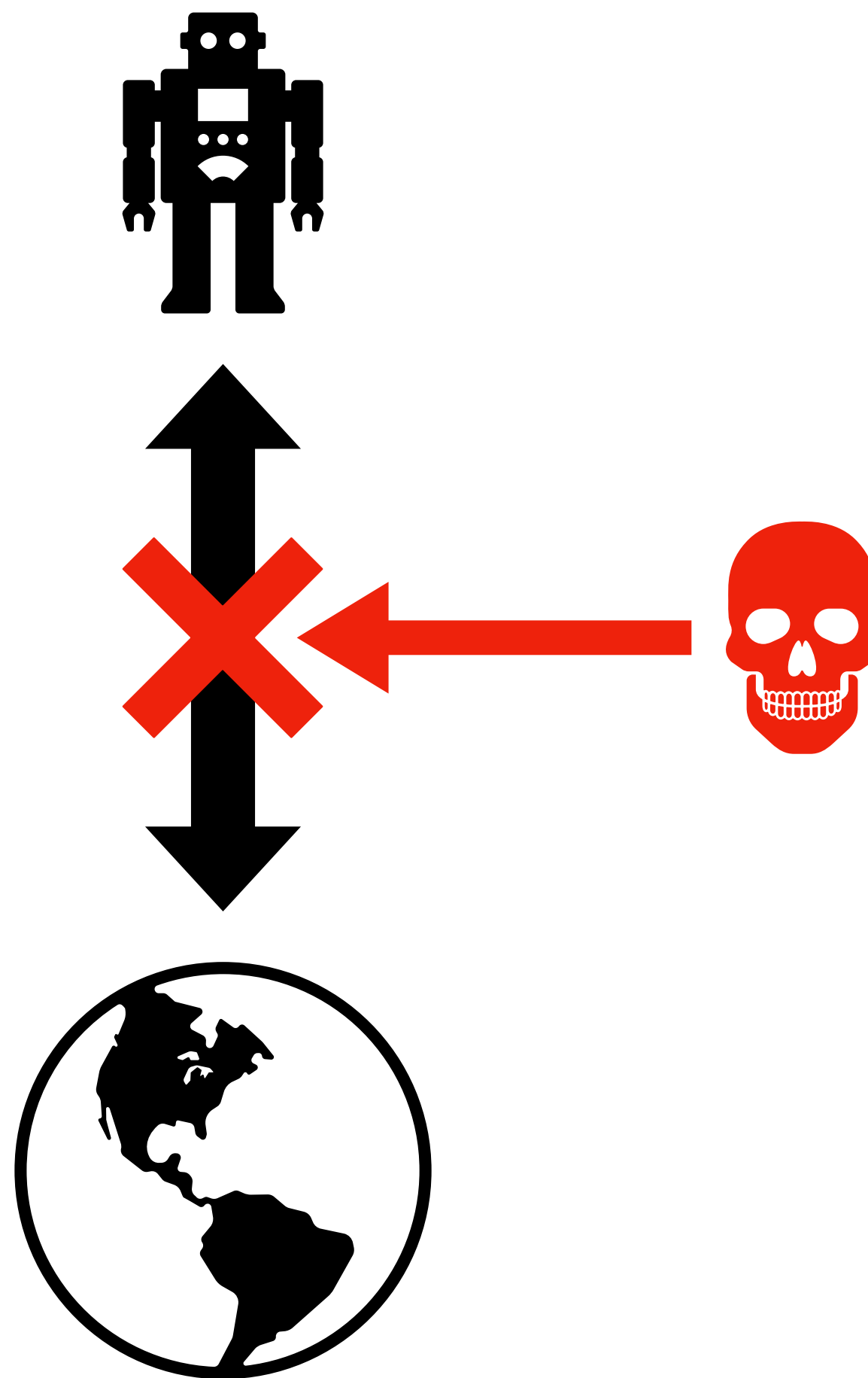
# Adversarial RL

An external attacker can  
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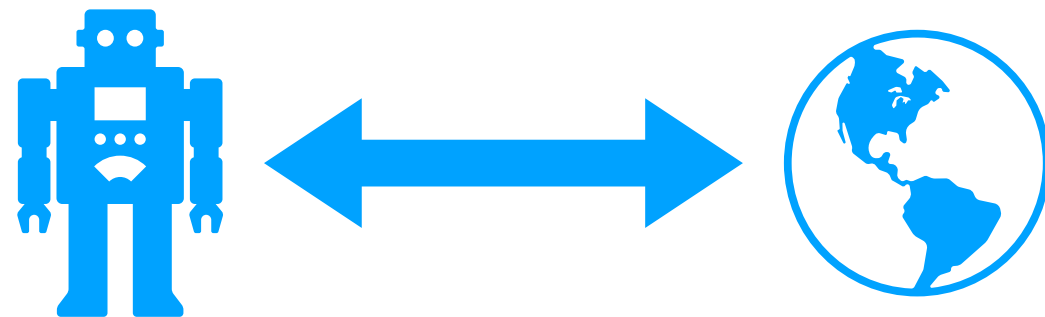
# Attack Paradigms

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**Training Time**

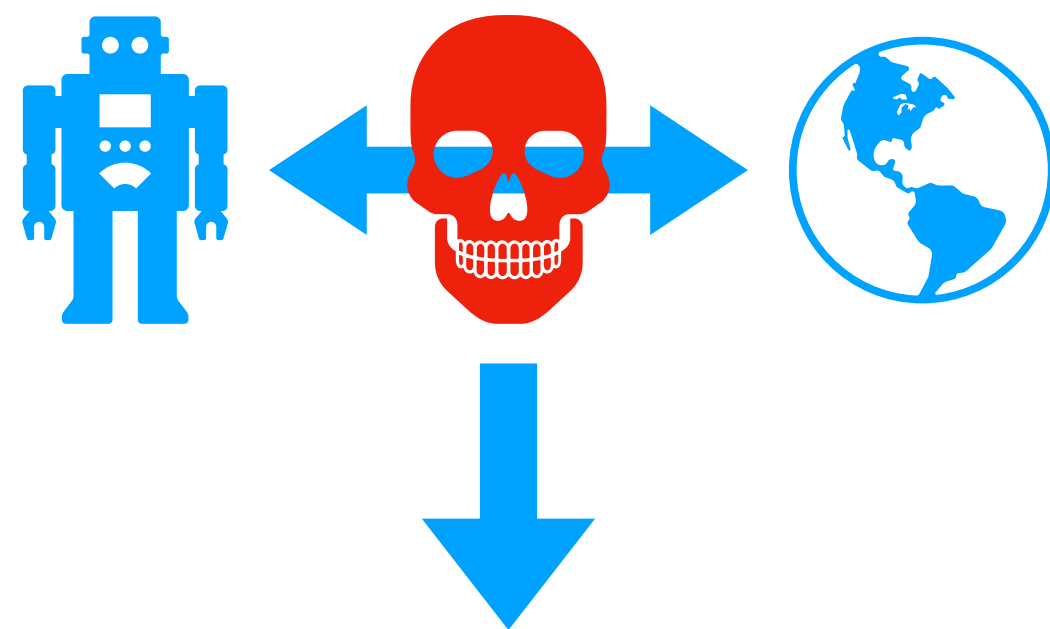
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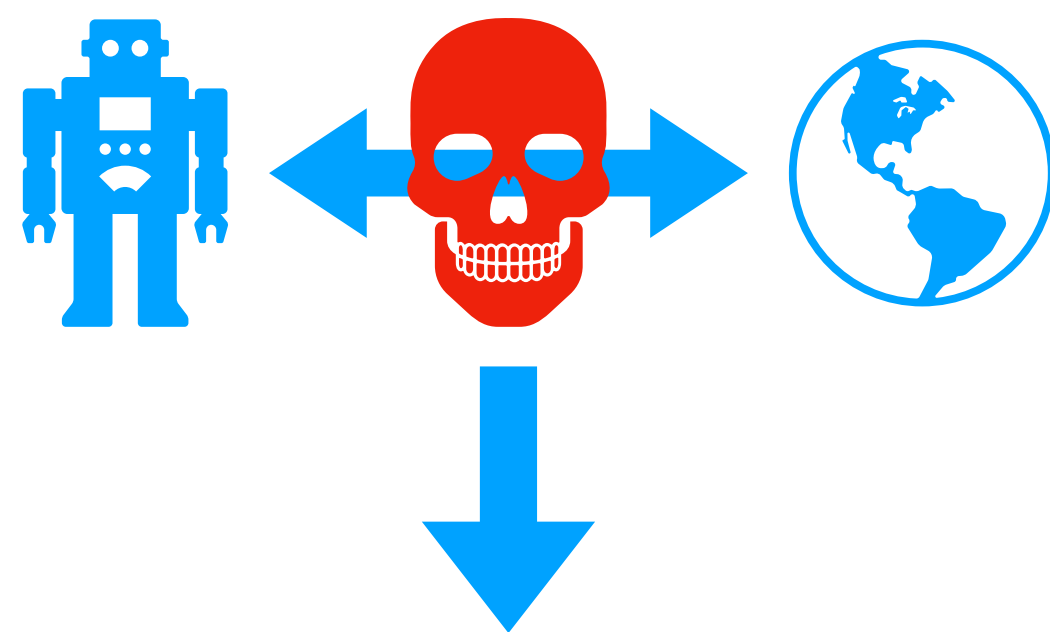
Training Time



Learn bad  $\pi^\dagger$

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Training Time



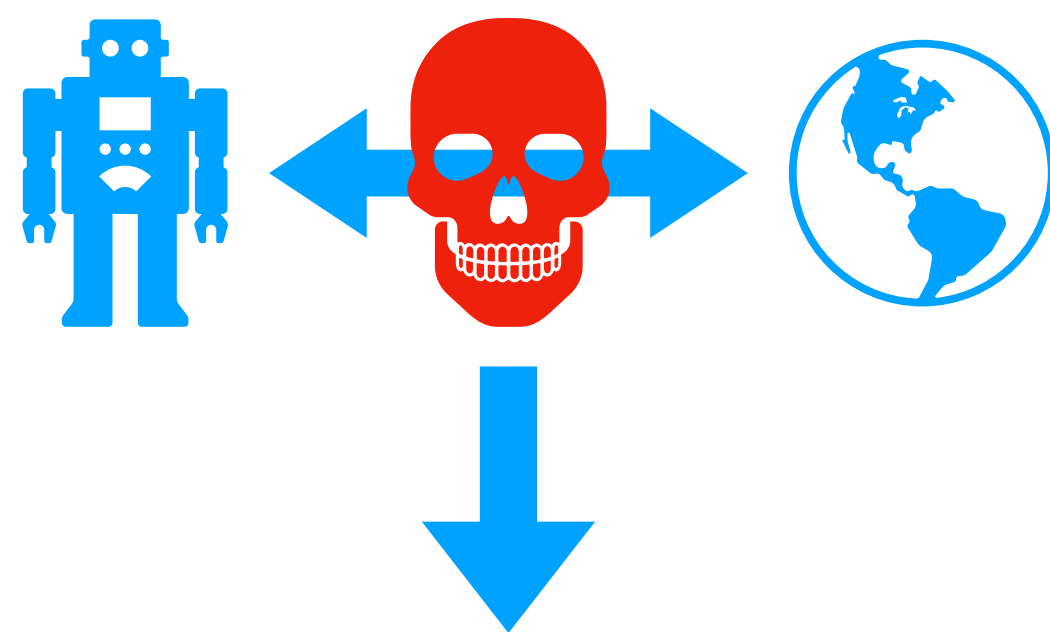
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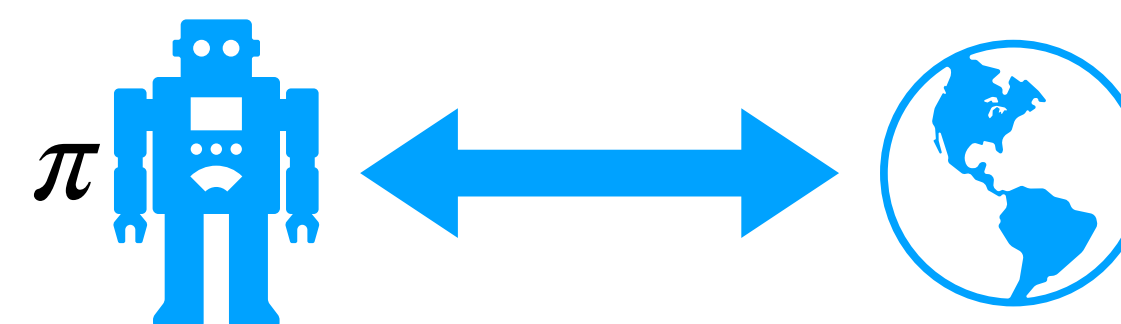
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Training Time



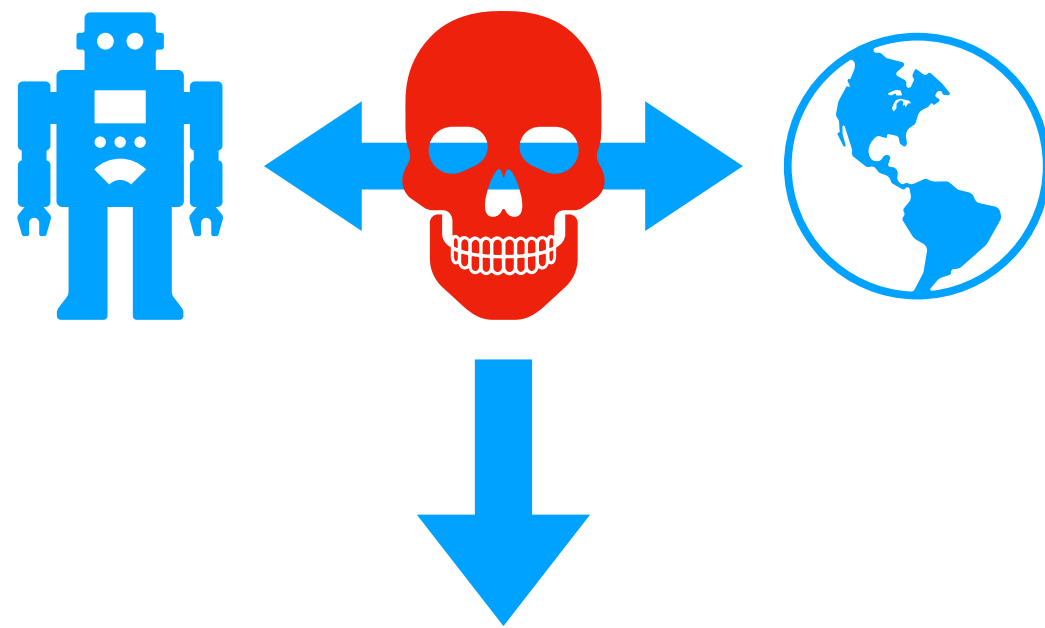
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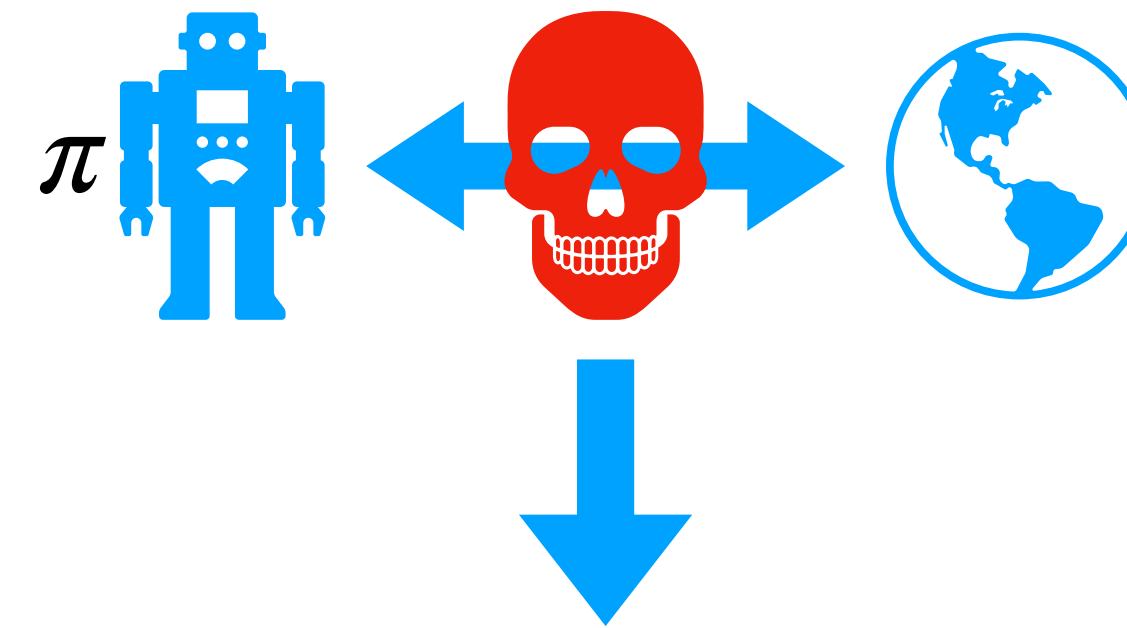
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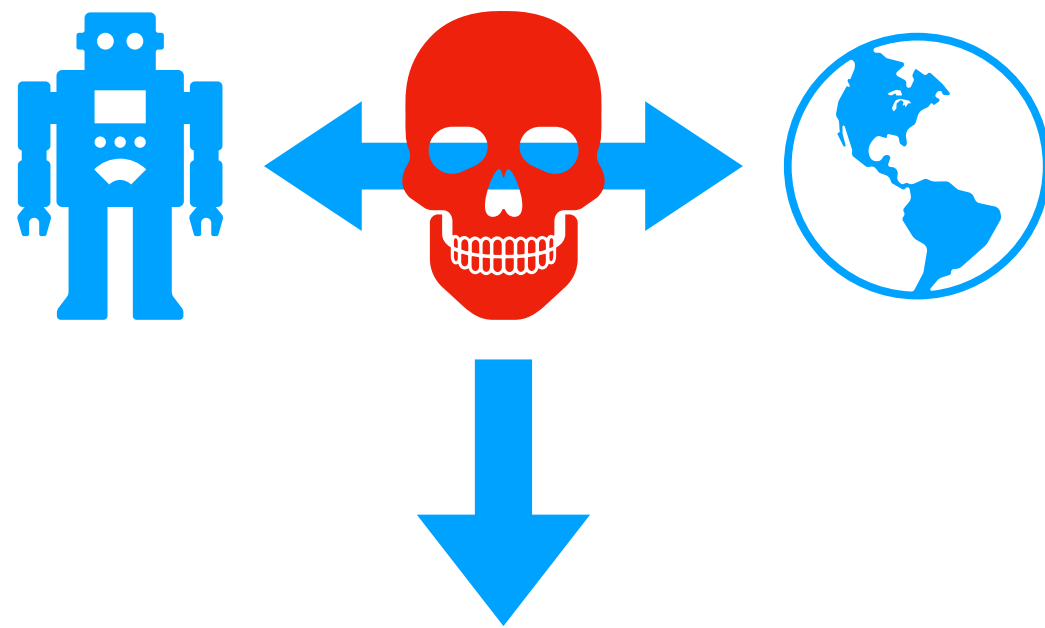
Test Time



Cause bad outcomes

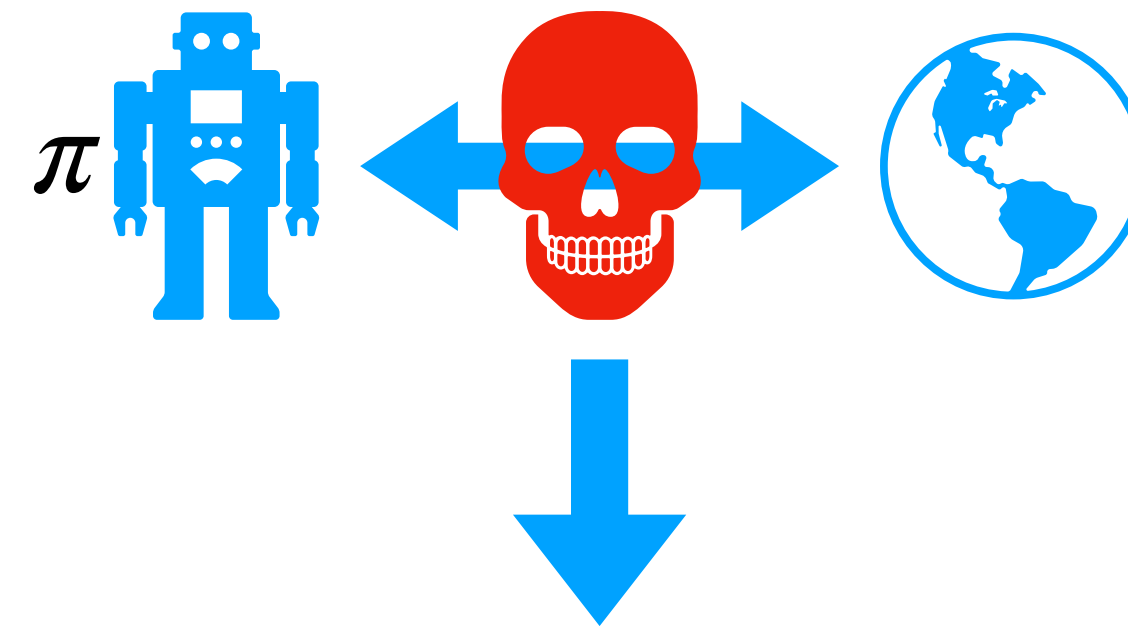
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Learn bad  $\pi^\dagger$

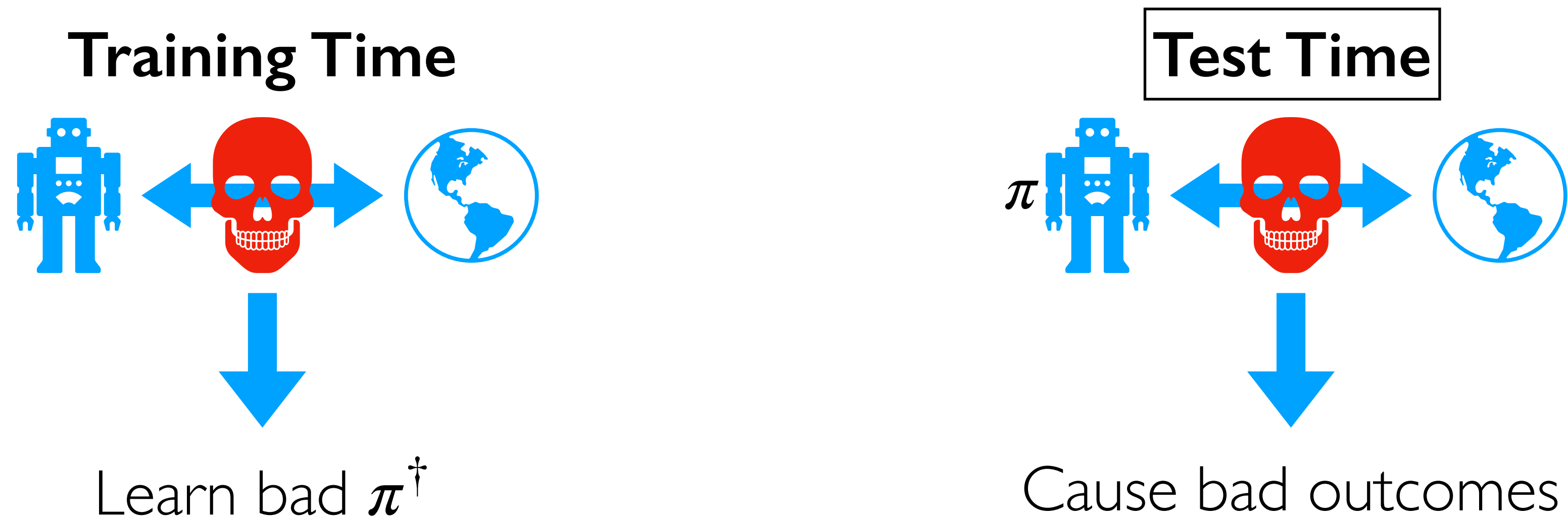
Test Time



Cause bad outcomes

Trojan

# Attack Paradigms



## Trojan

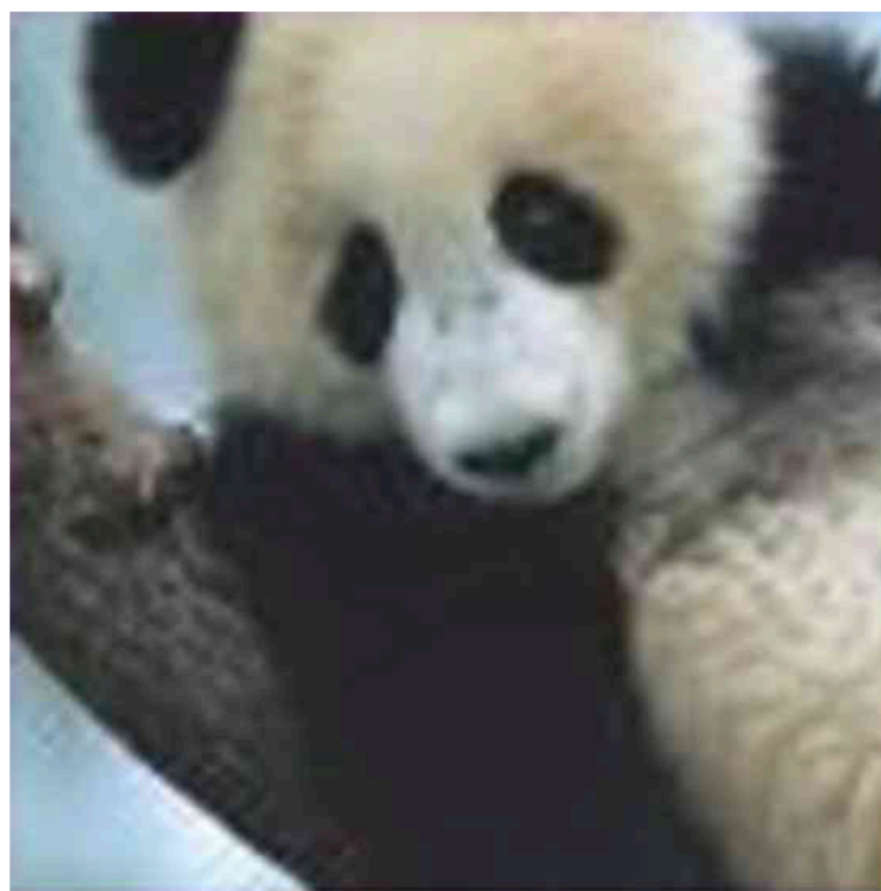
Hybrid: poison training to make policy easily test-time attackable

# Panda Example

# Panda Example

In [Explaining and Harnessing Adversarial Examples](#), Goodfellow and his team added a small perturbation to the image of a panda, as seen below. The result was surprising. Not only did the classifier mark the panda as a gibbon, but did so with high confidence.

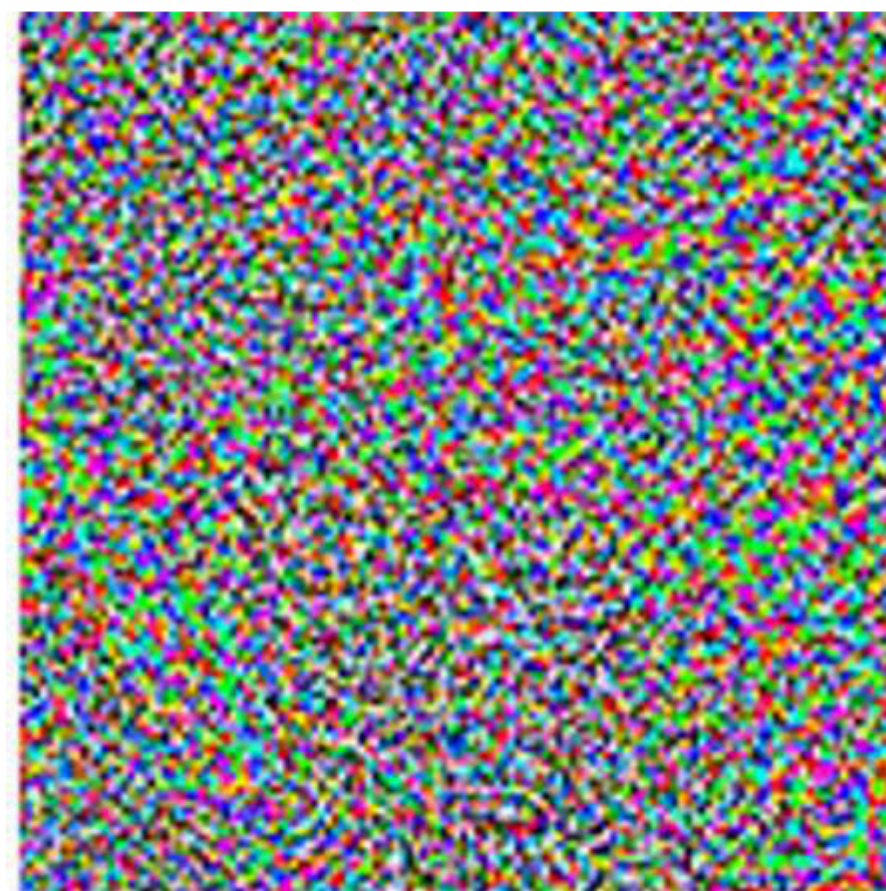
As you can see, a barely noticeable disturbance that appears normal to us can easily deceive an ML model into predicting an incorrect class.



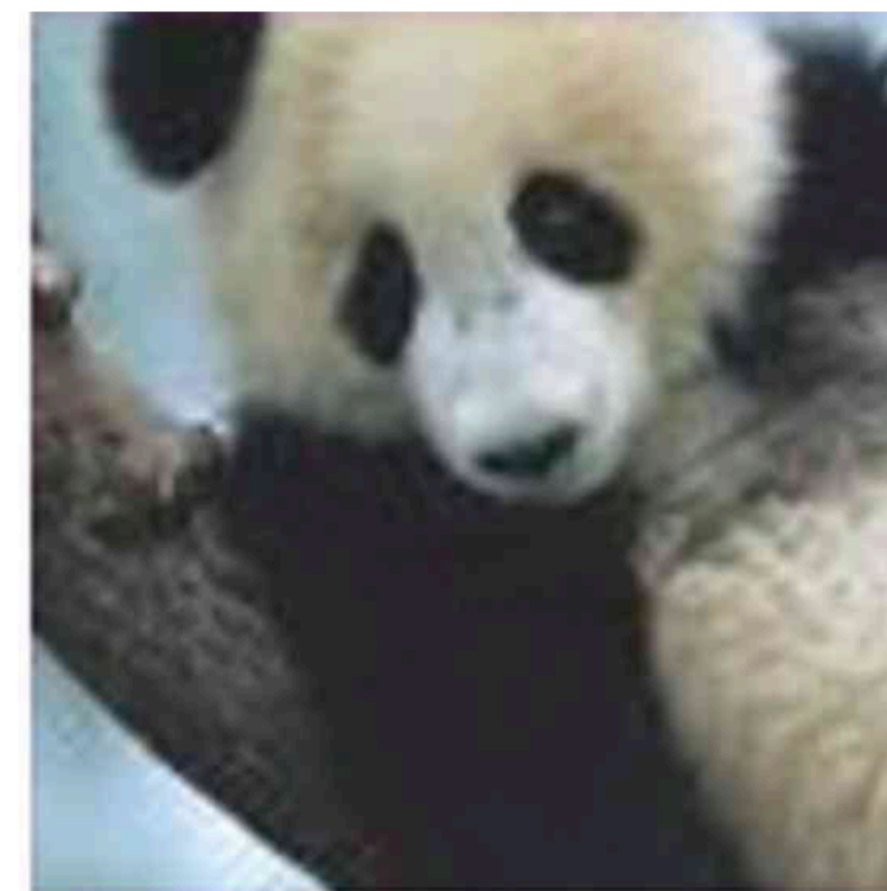
“panda”

57.7% confidence

+  $\epsilon$



=



“gibbon”

99.3% confidence

Source: [Goodfellow et al, 2014](#)

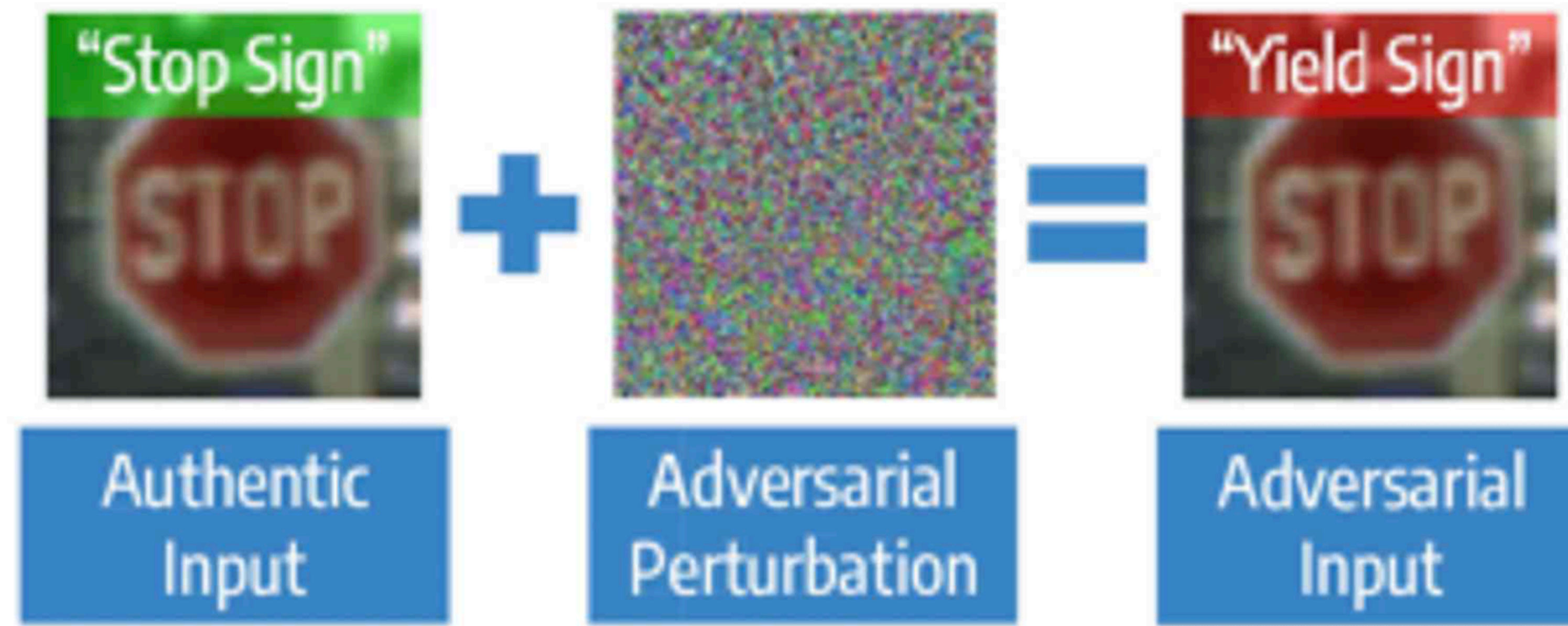


# Car Crashing

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While the panda turned gibbon in the eyes of a machine is a harmless example of an adversarial attack, there are other forms of danger we must watch out for.

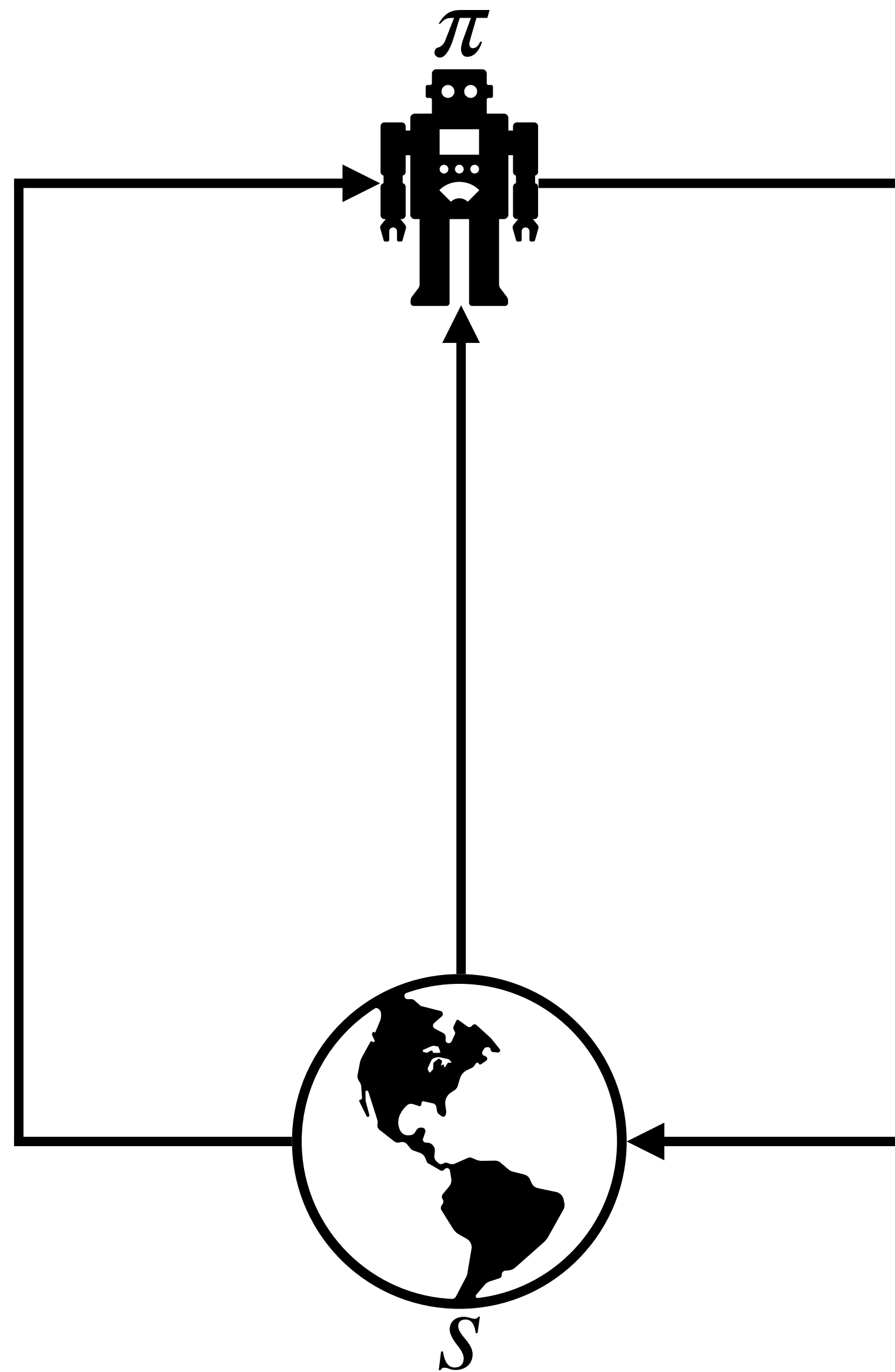
For instance, adversarial examples can also be used to [hijack the ML models behind autonomous vehicles](#), causing them to misclassify 'stop' signs as 'yield', as seen below.



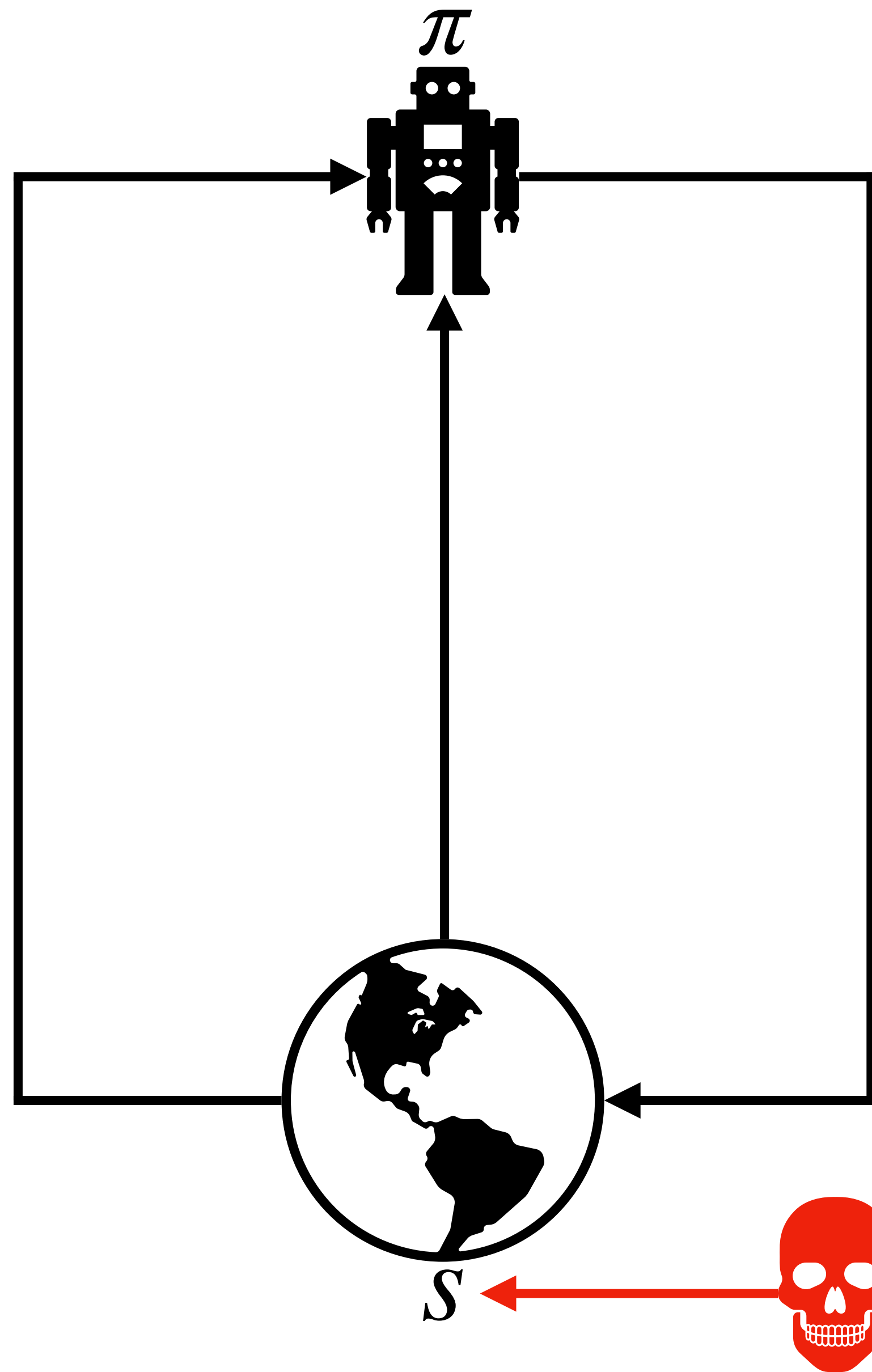
Source: [Kumar et al, 2021](#)



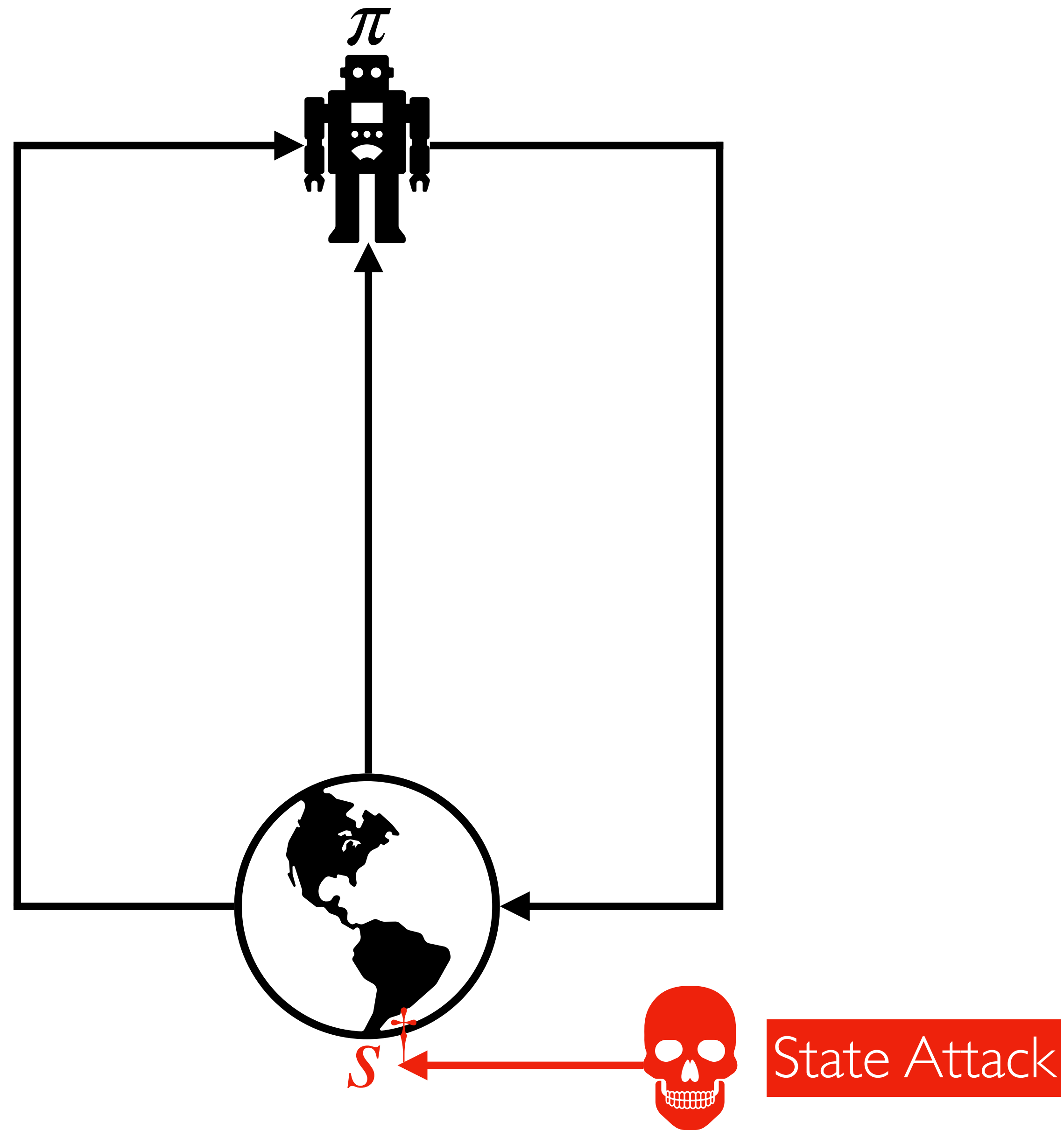
# Attack Surfaces



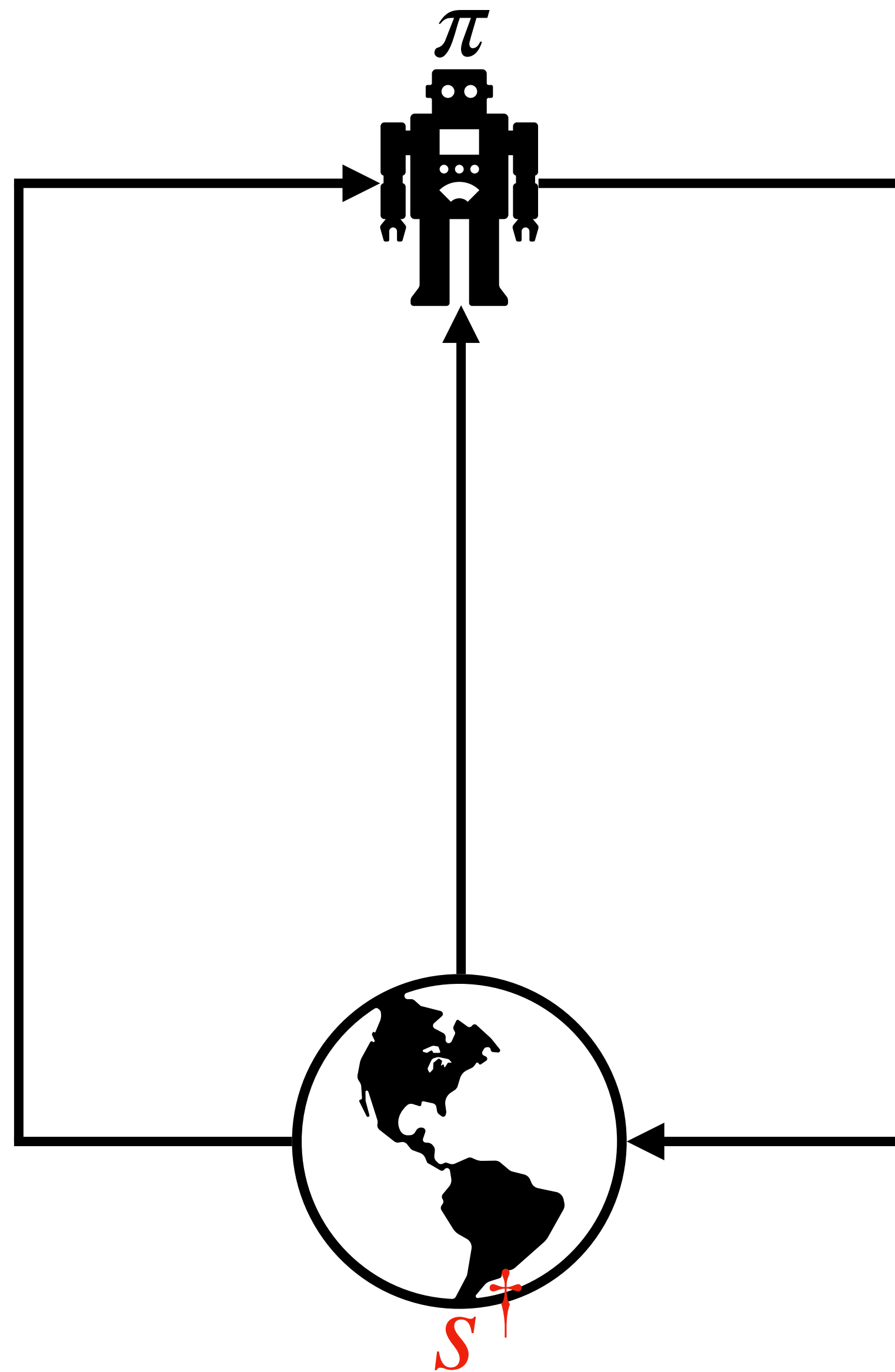
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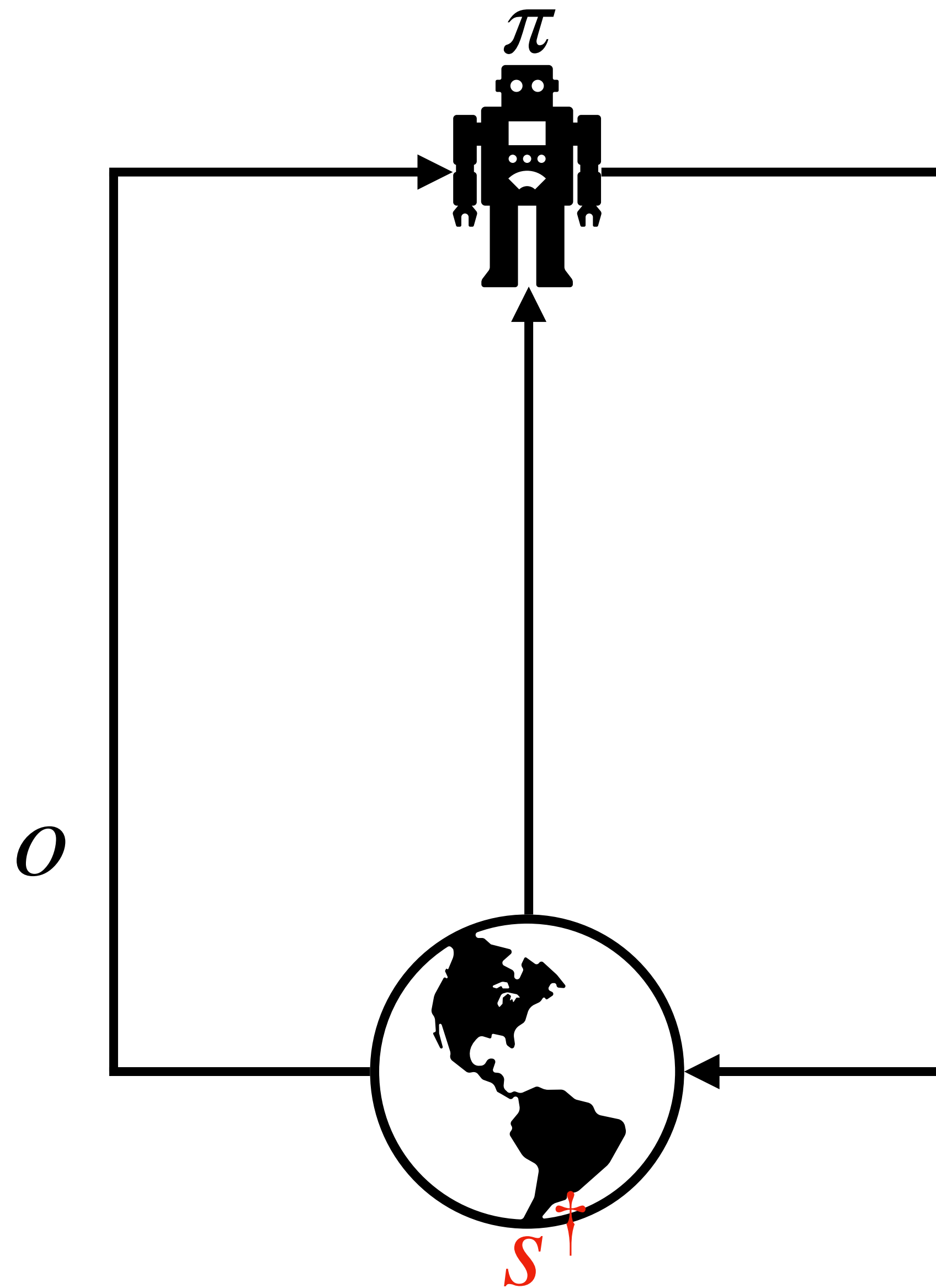
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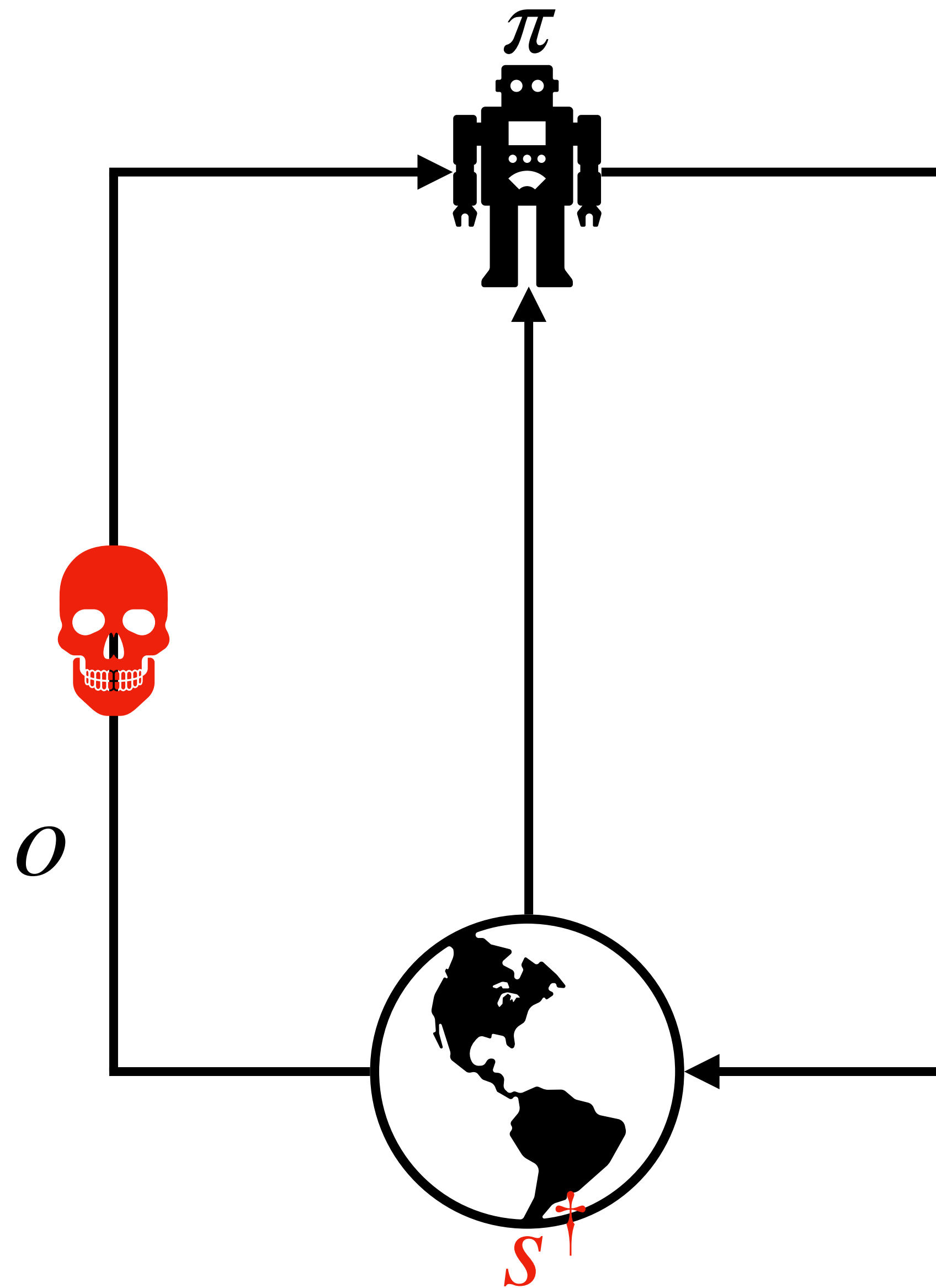
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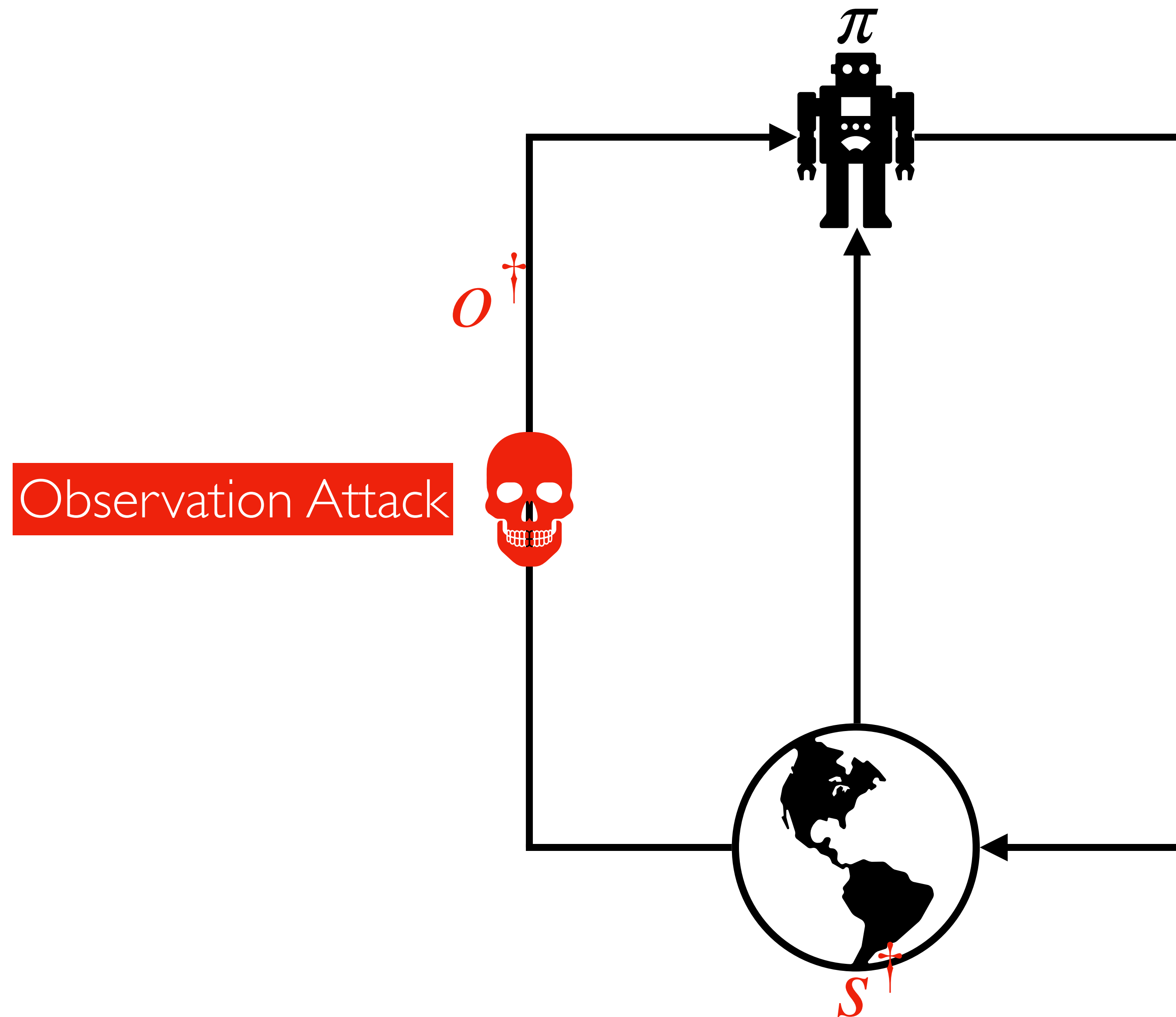
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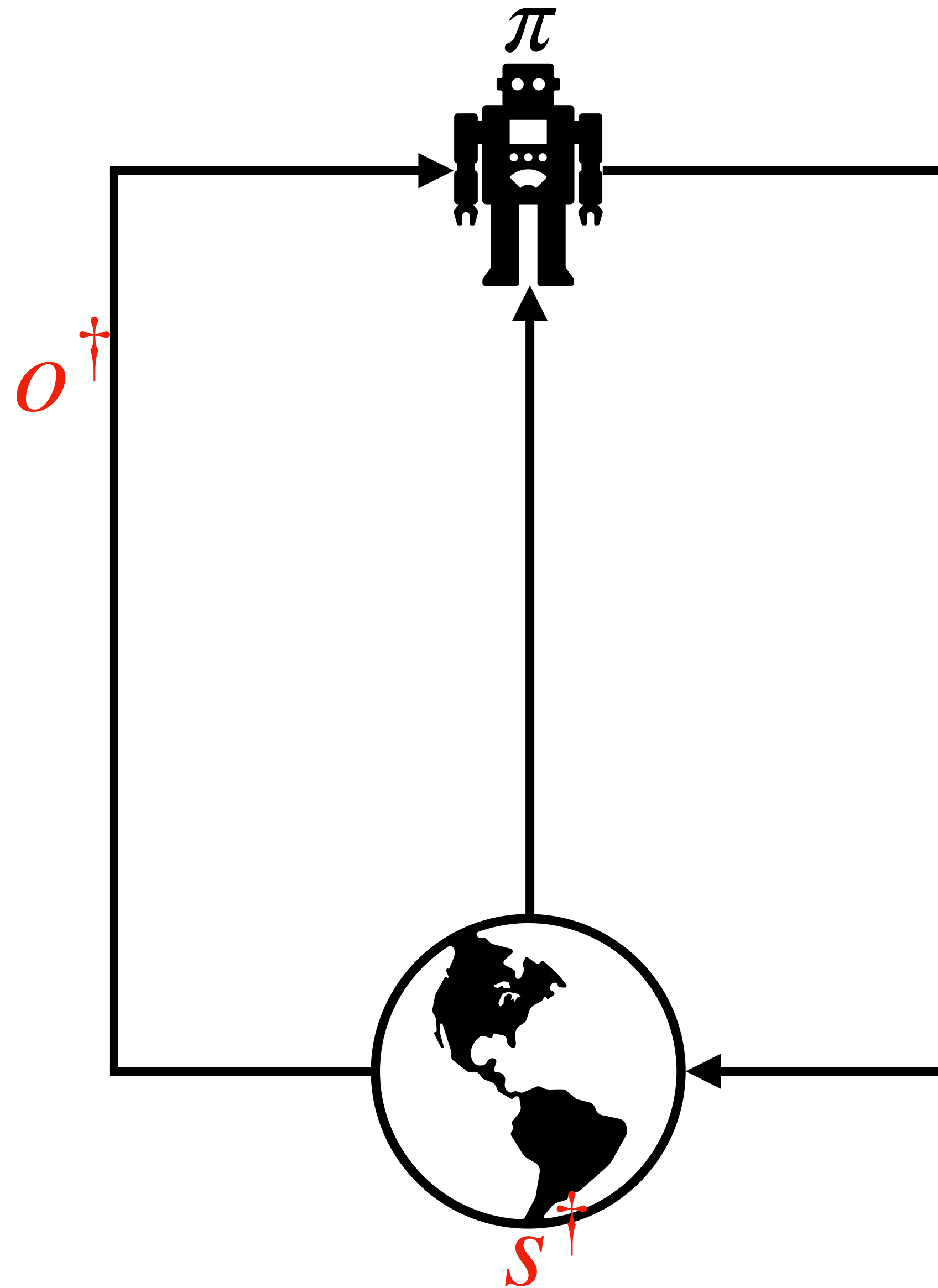
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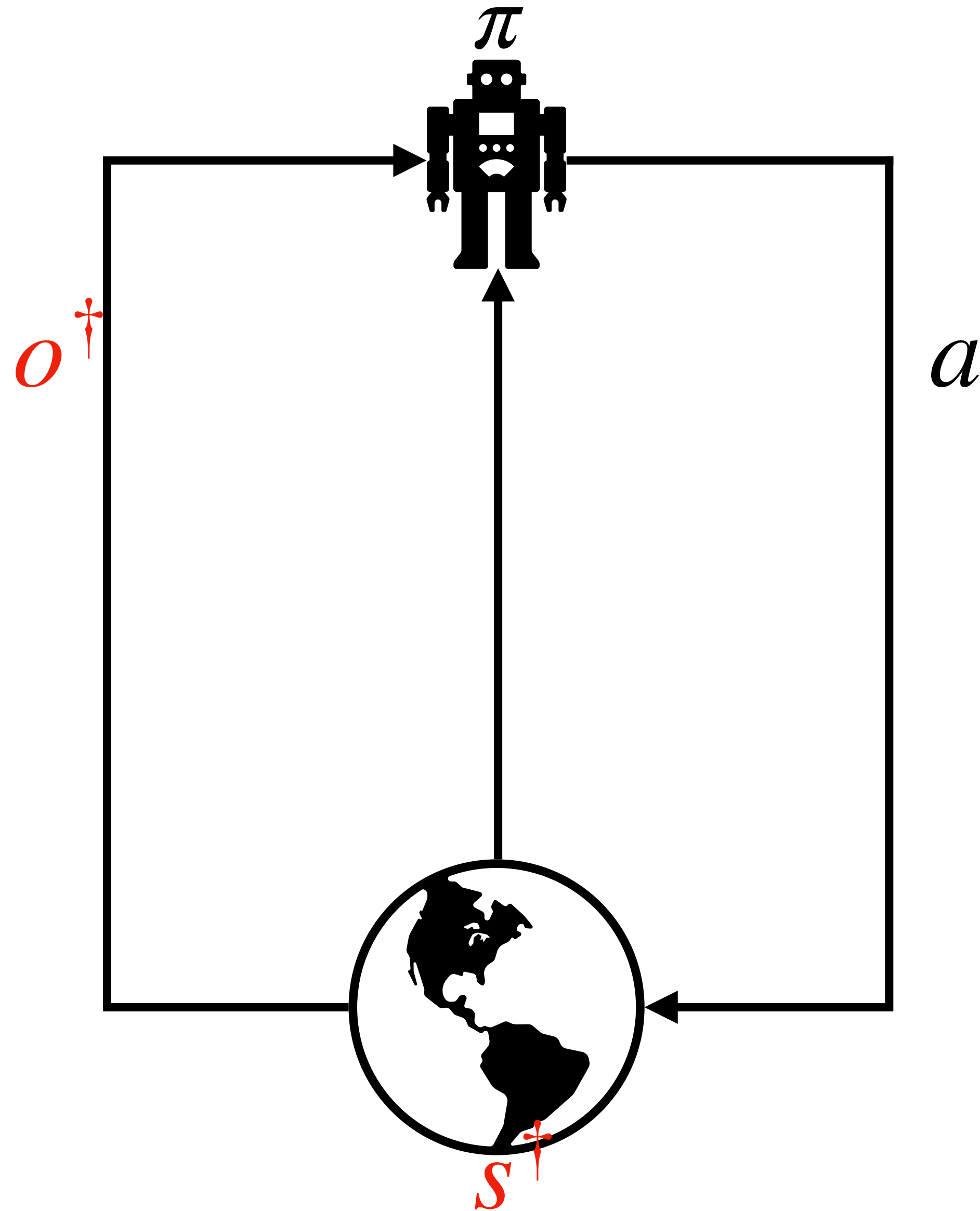


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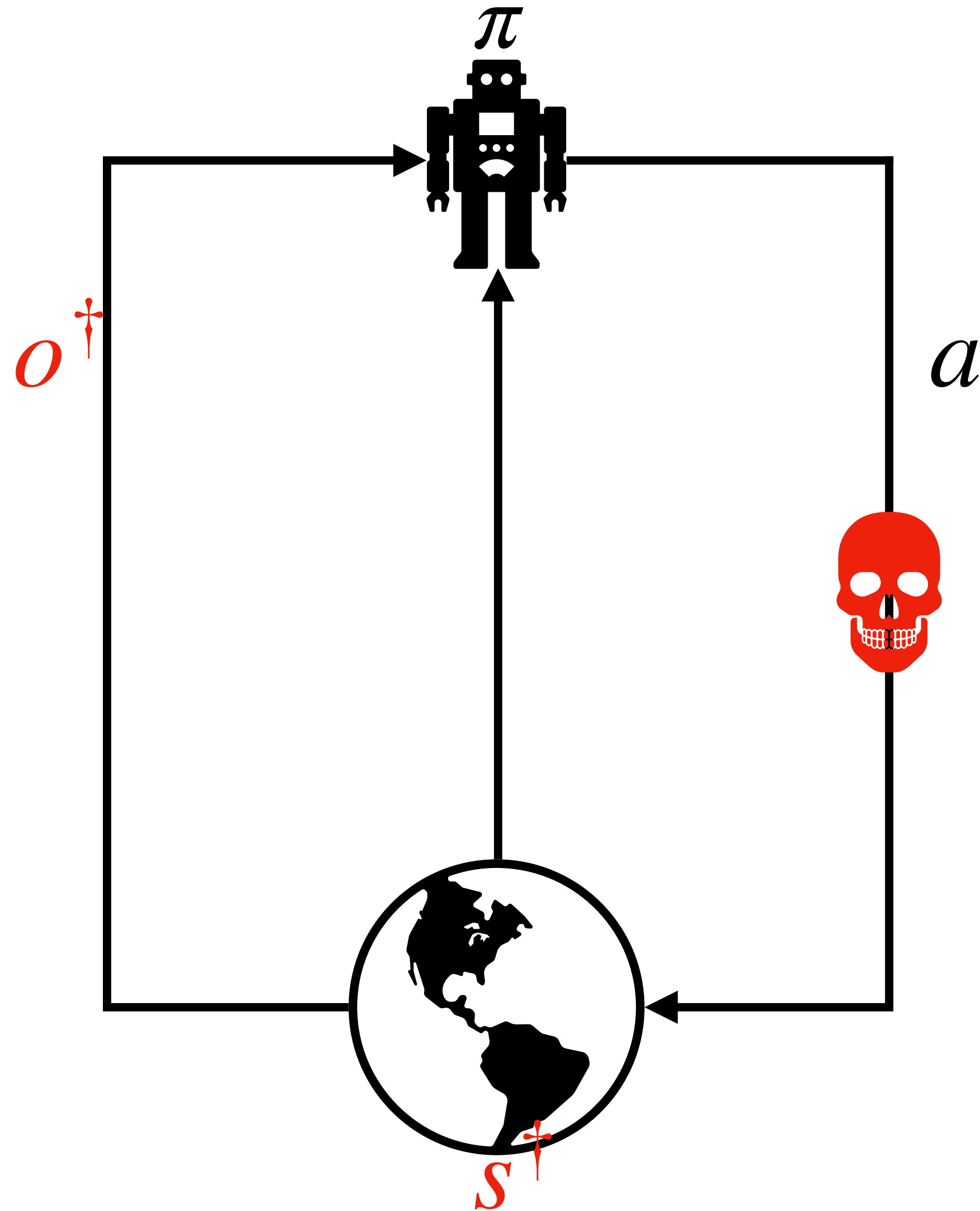




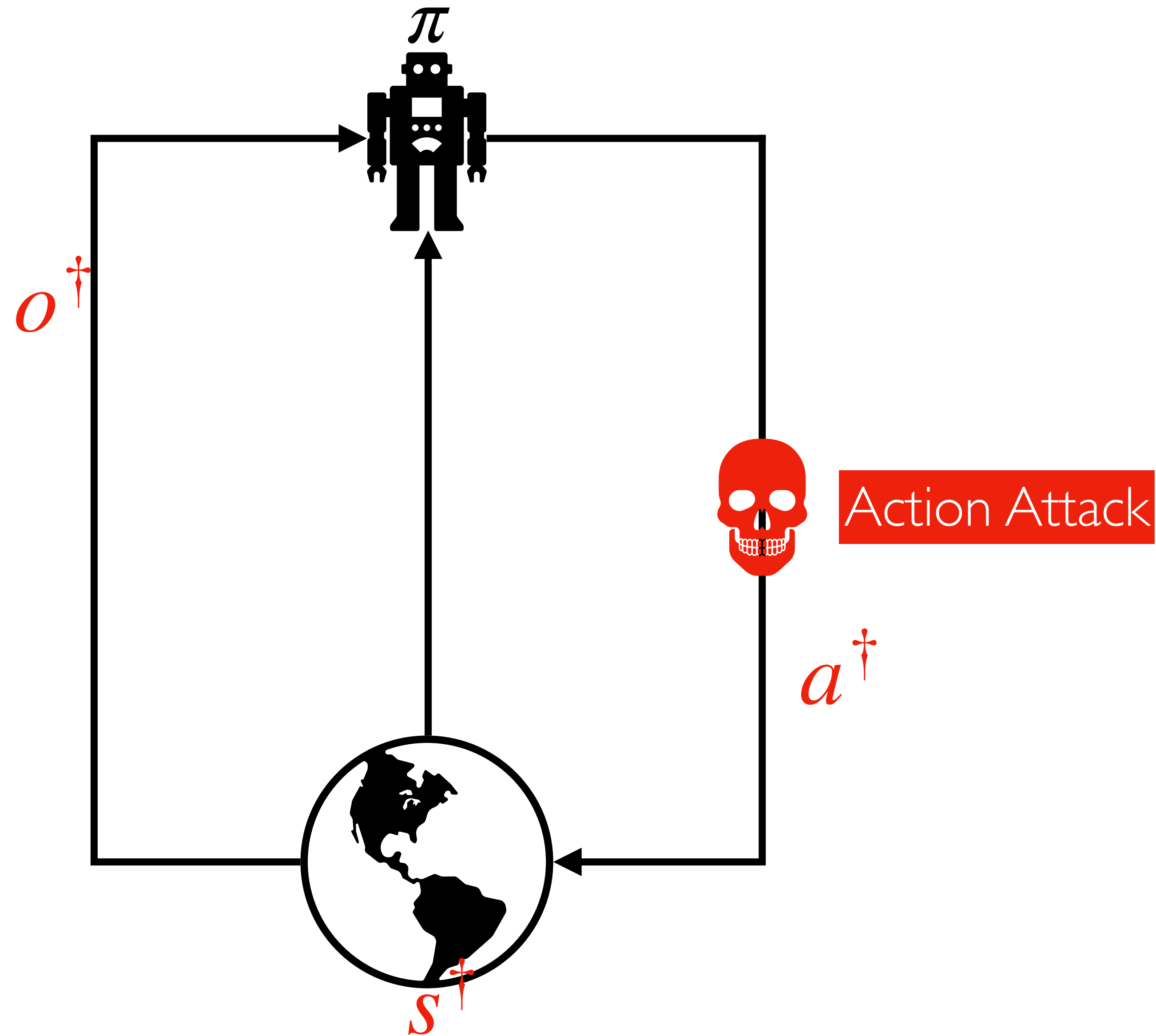
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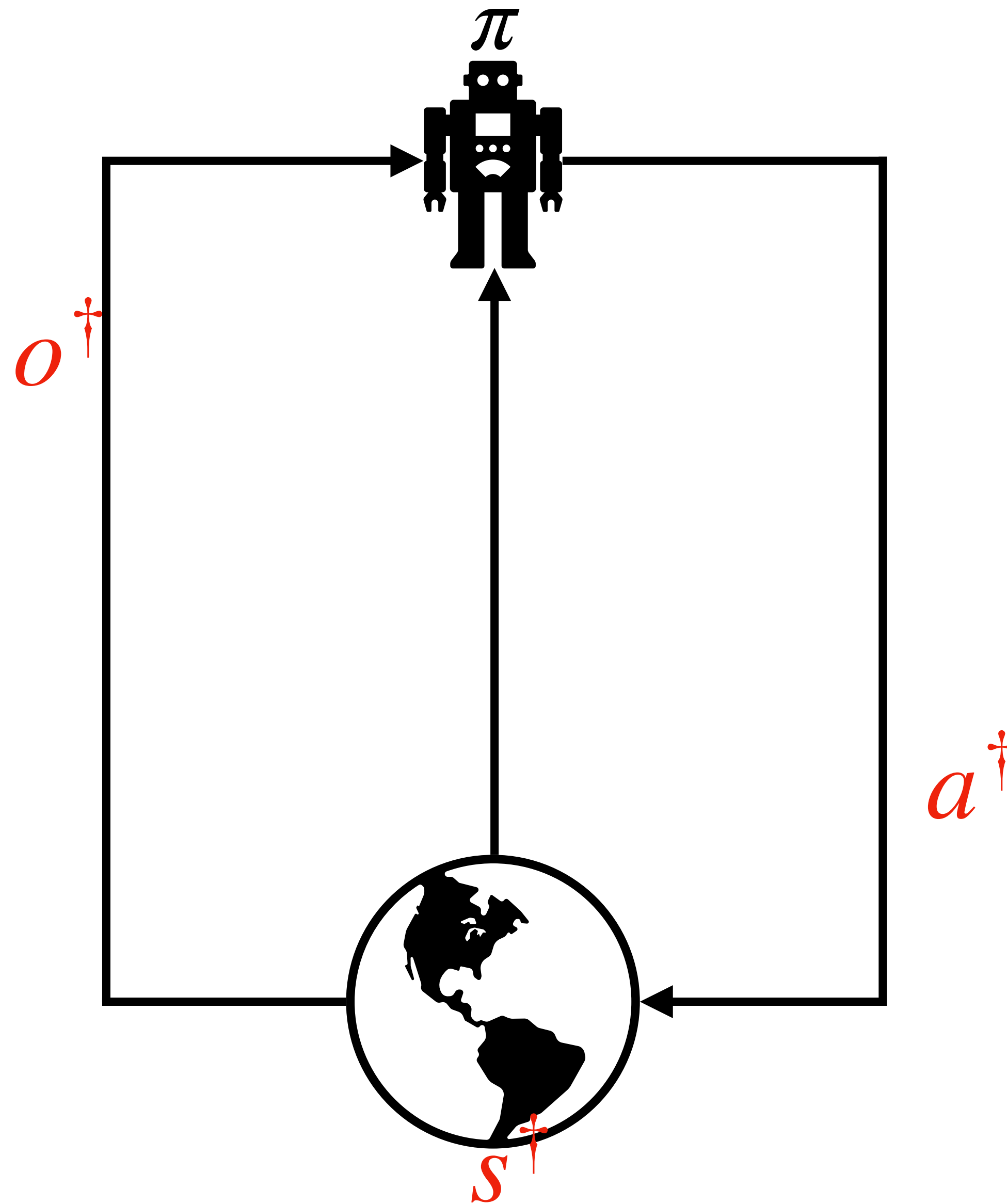
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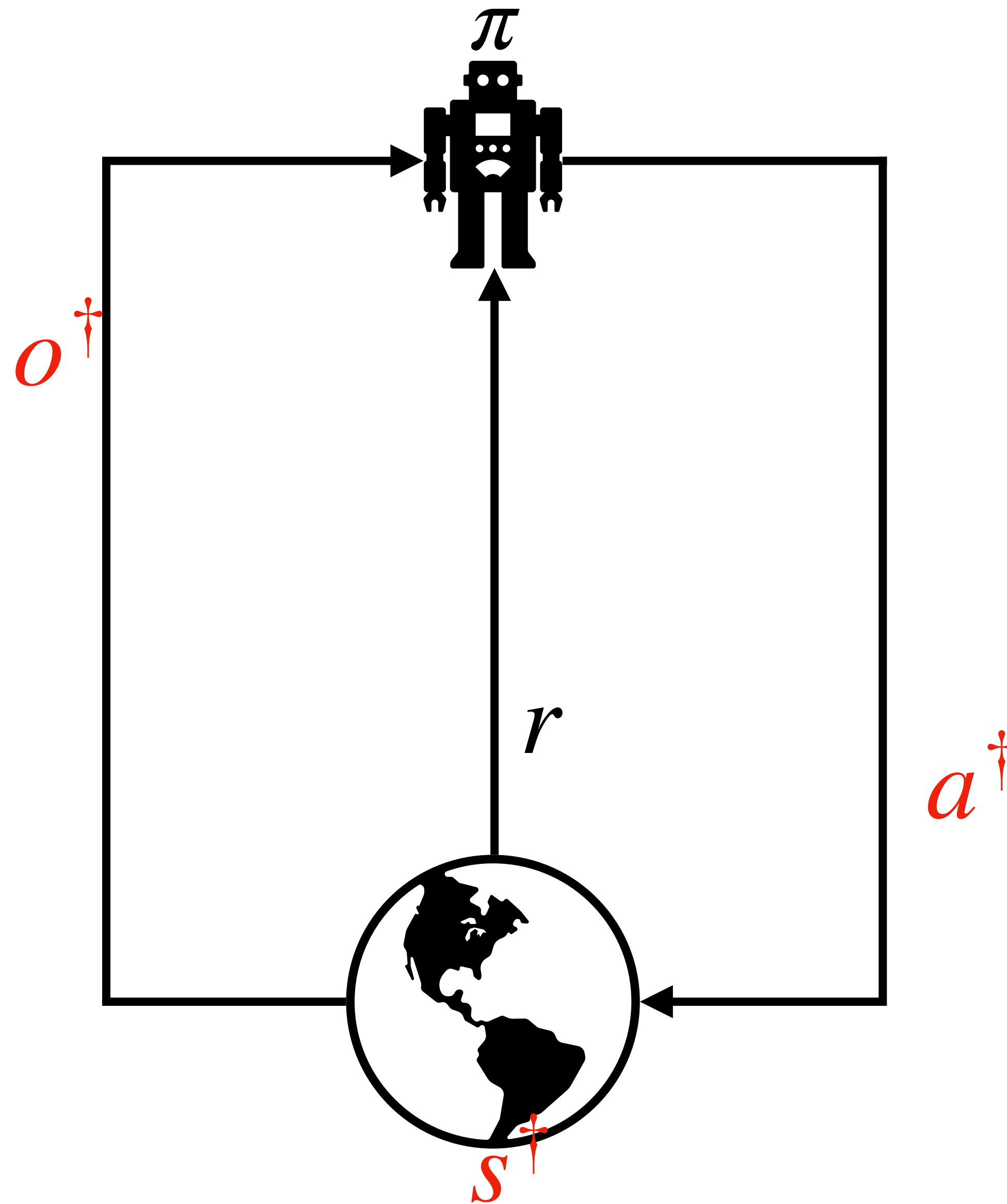
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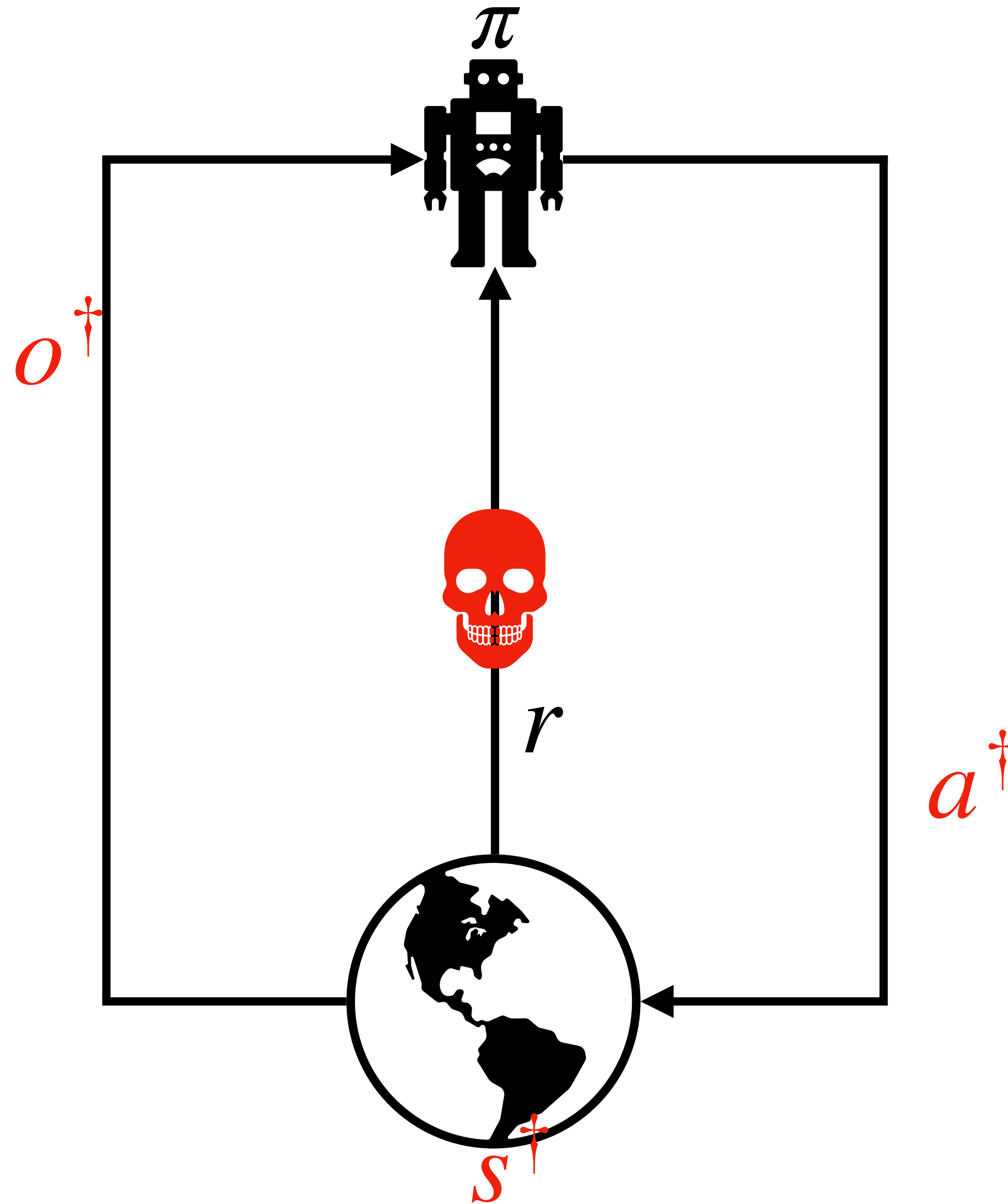
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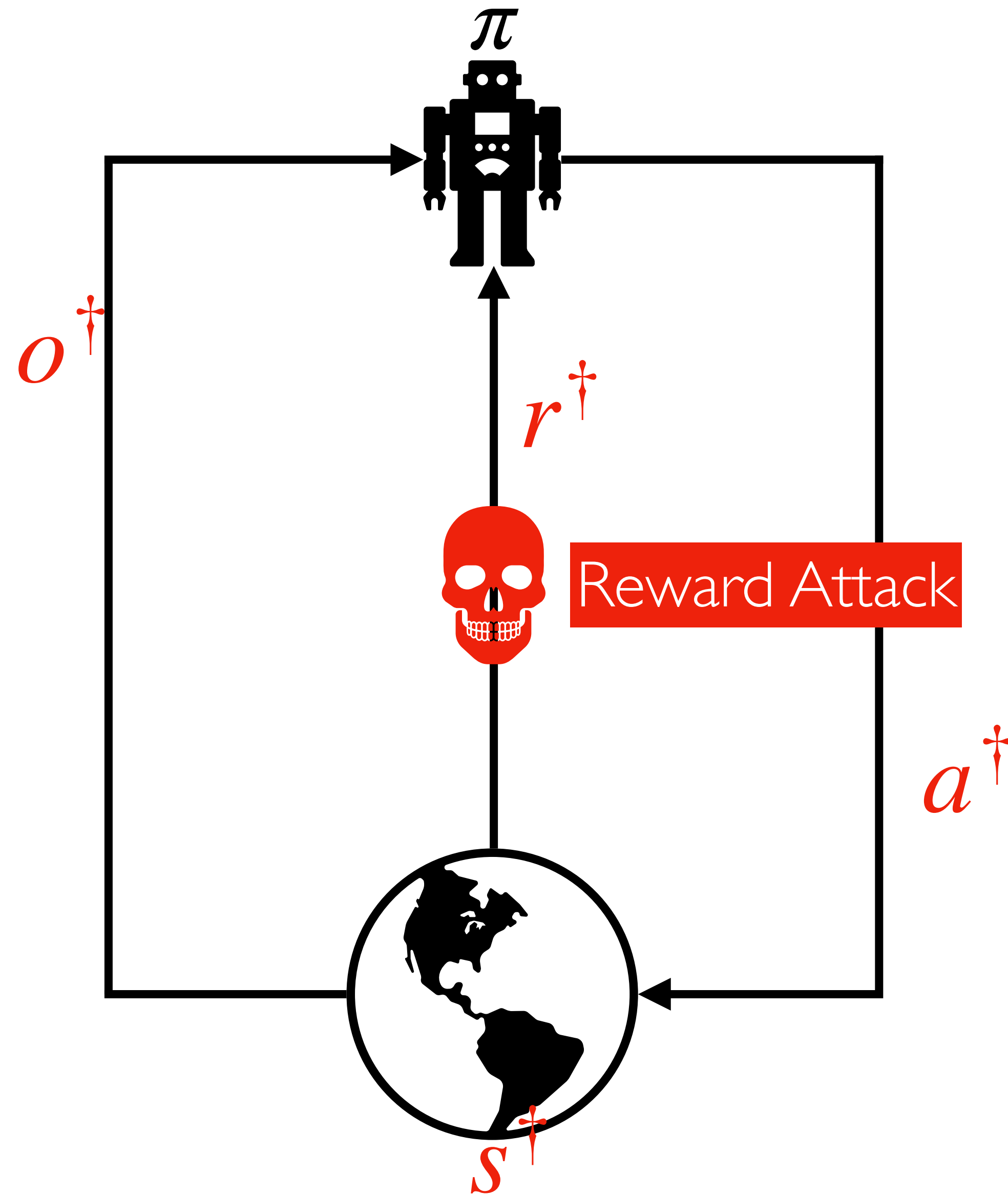
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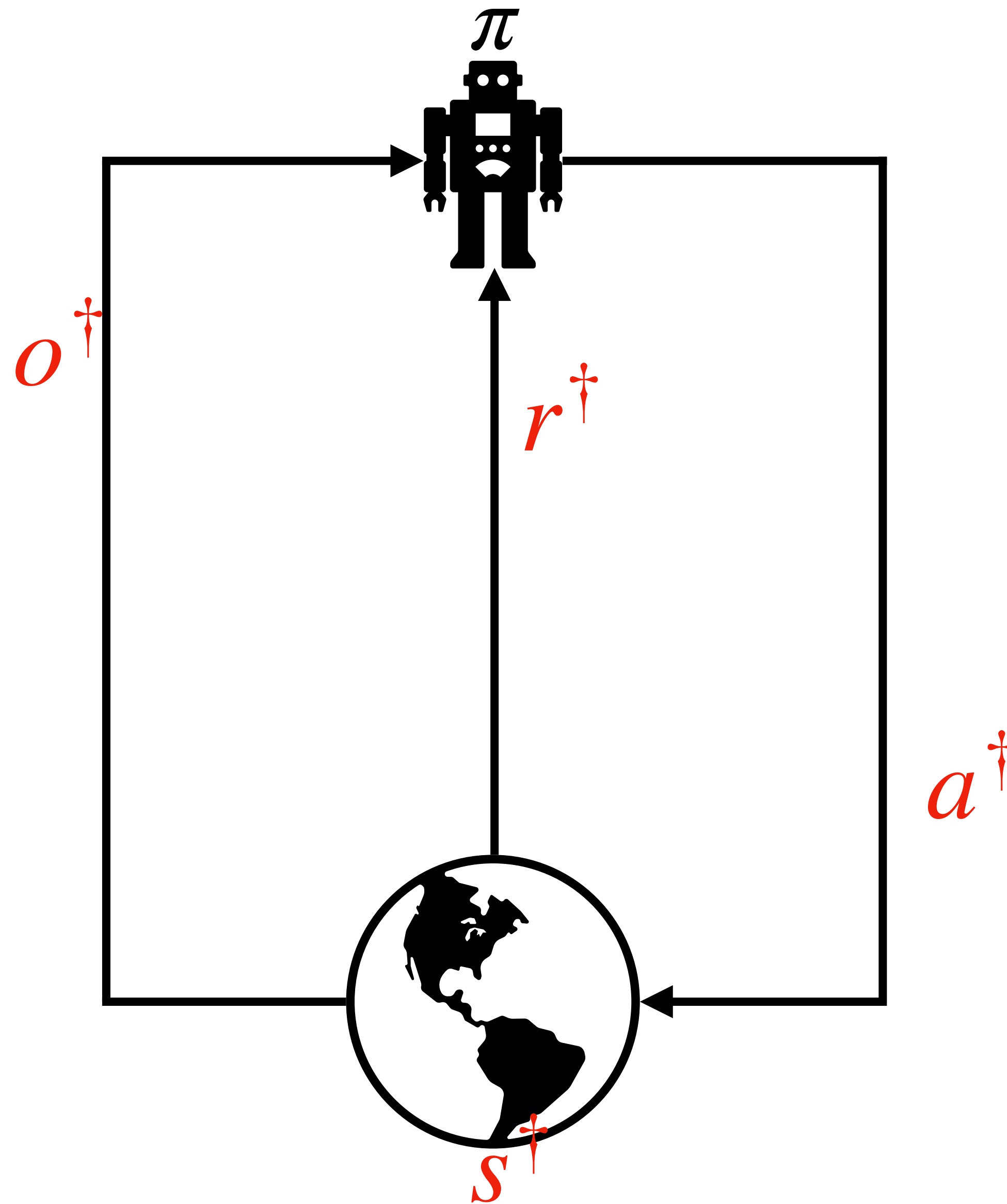
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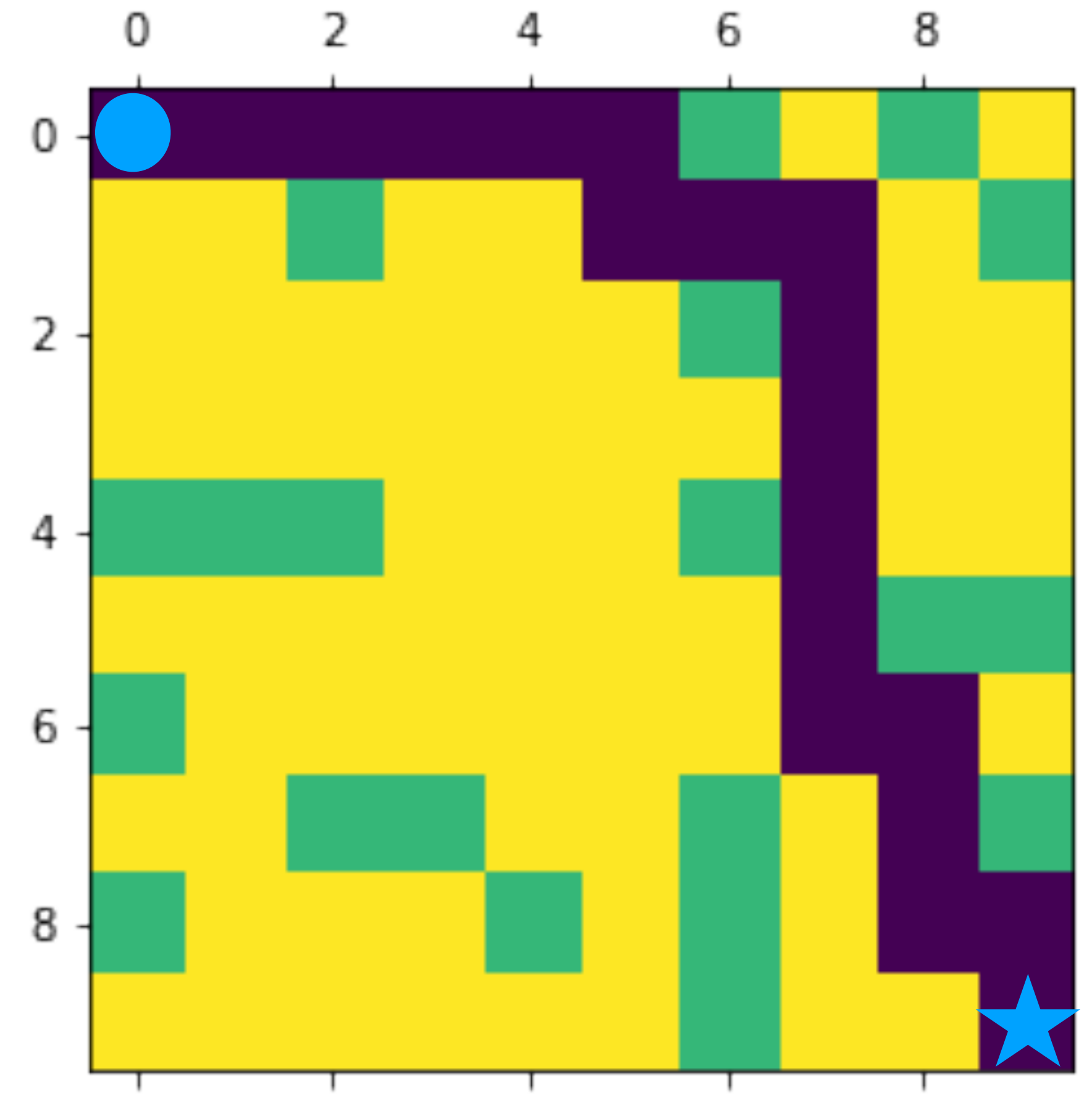
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The attacker can manipulate any element of the interaction tuple  $(s, a, r)$ .

# Maze Environment

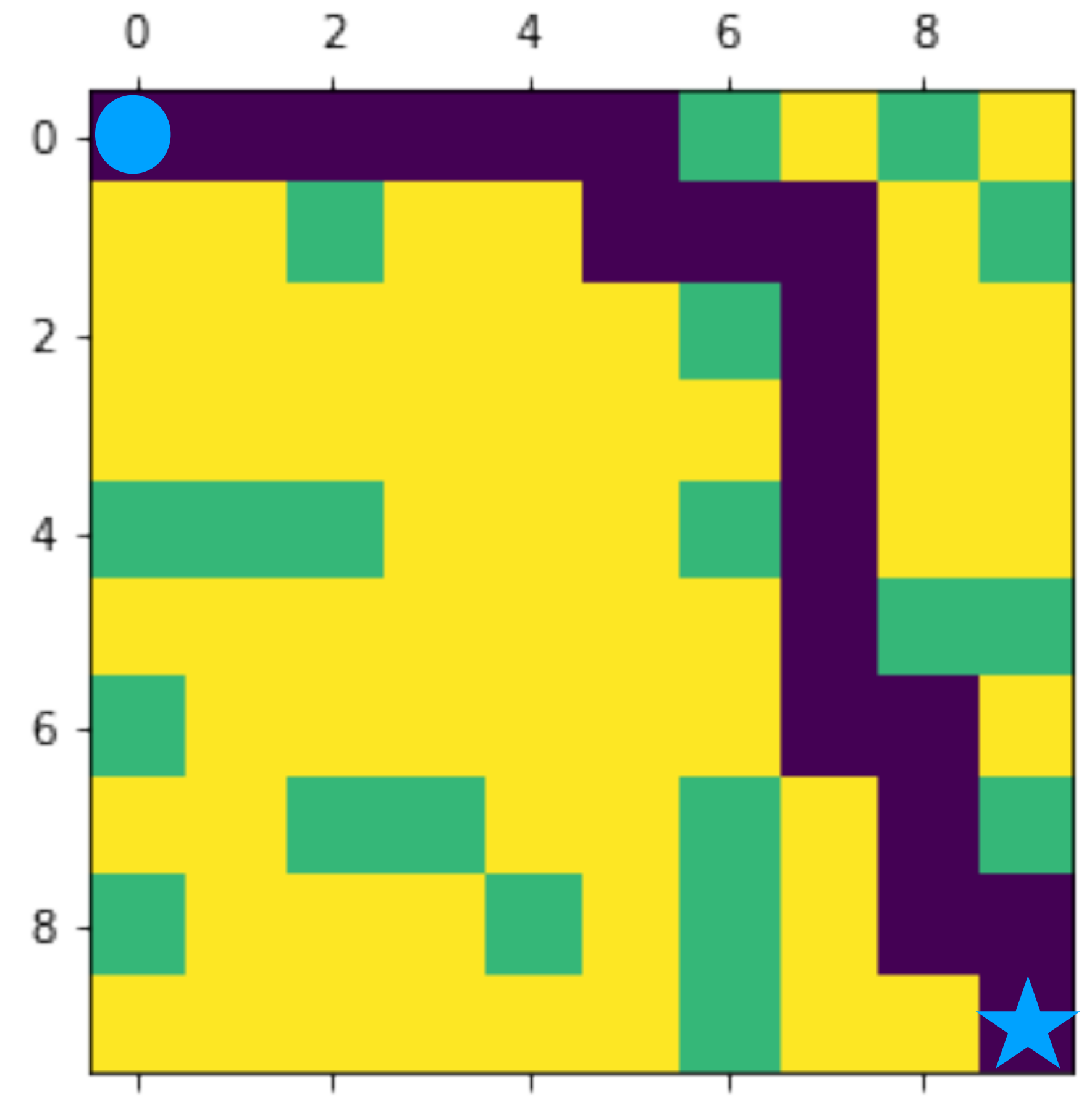
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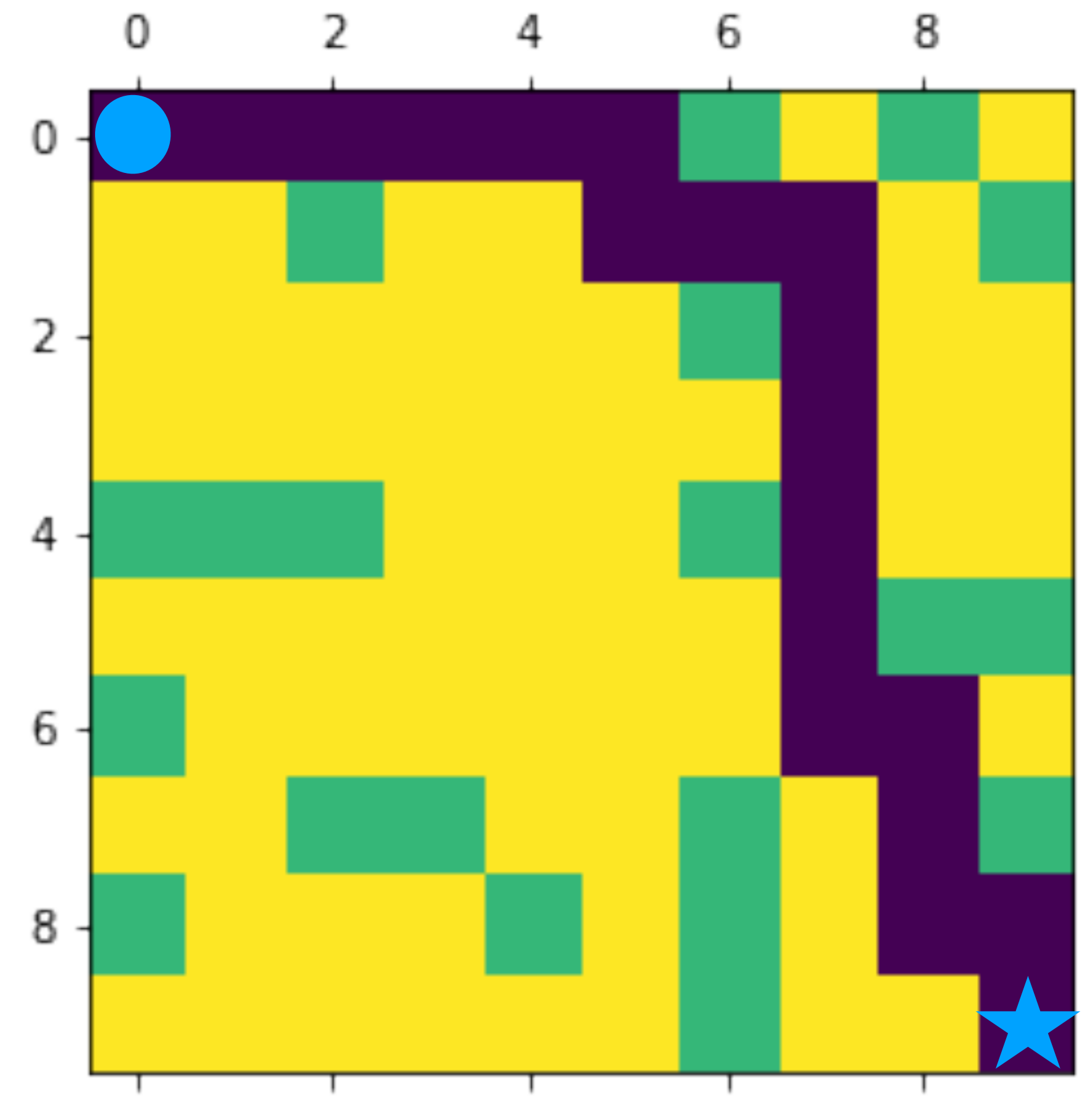
# Maze Environment

- Green Squares are obstacles.



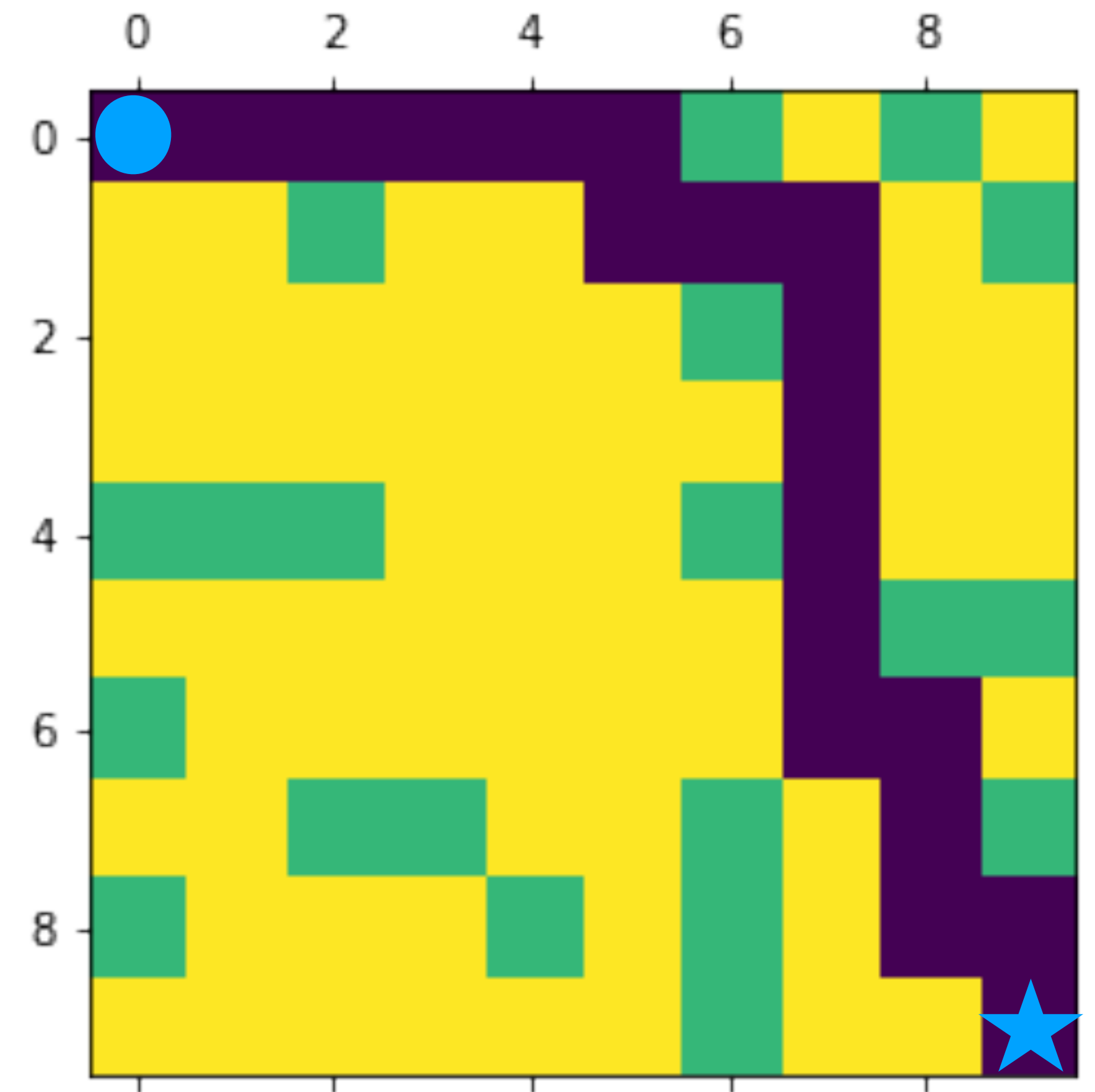
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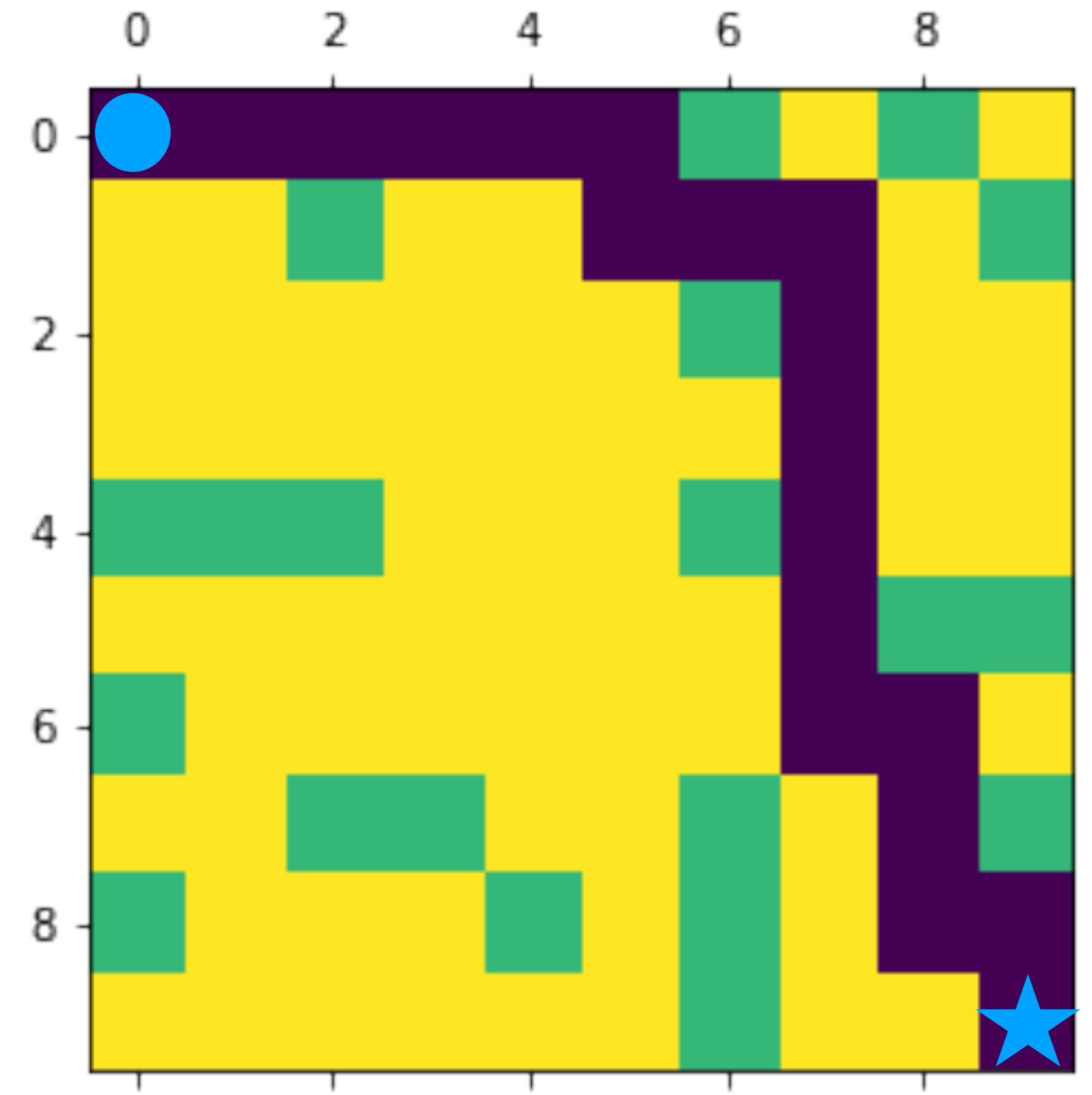
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- Green Squares are obstacles.
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- The agent starts at top left corner.



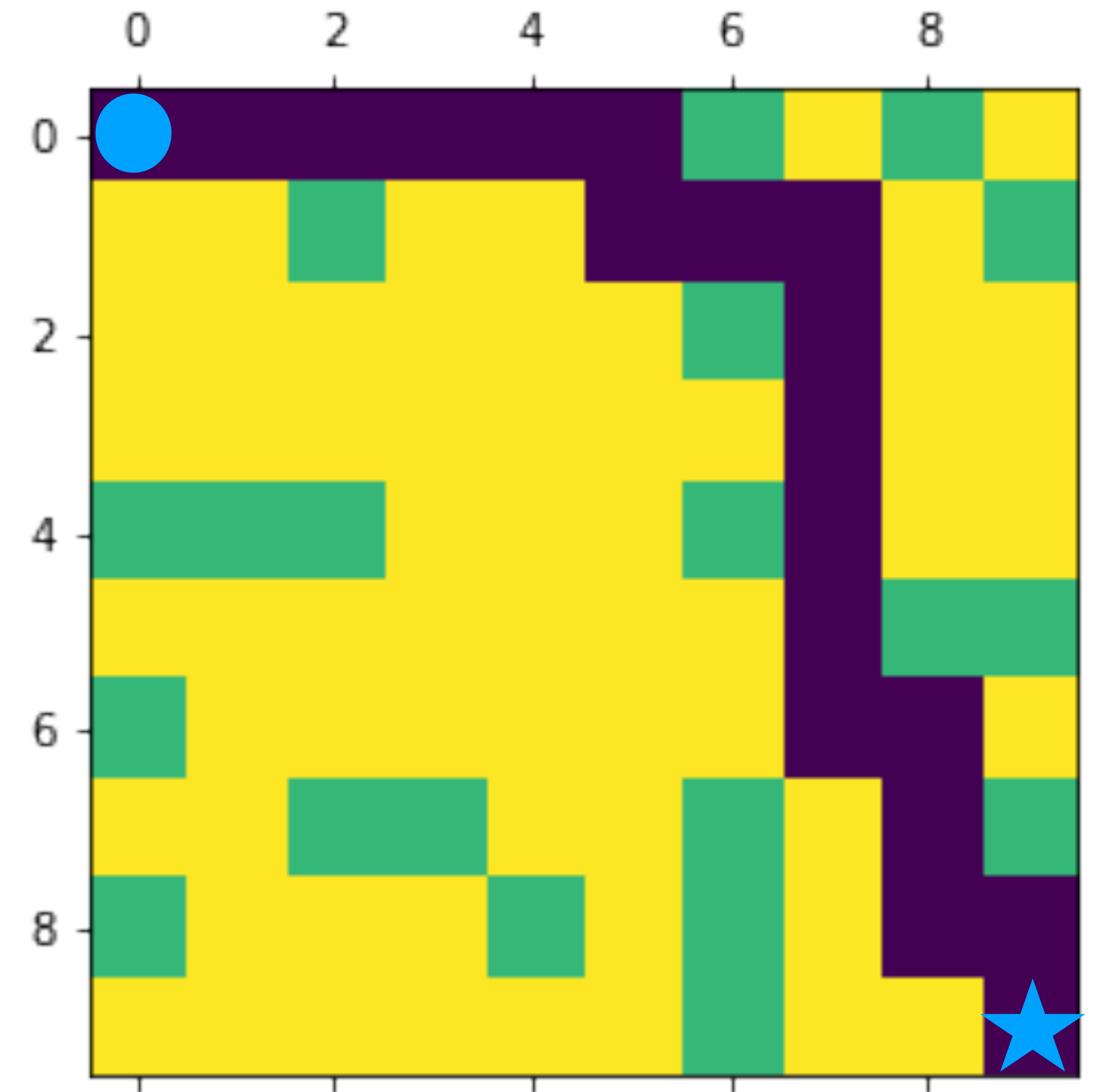
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- The agent receives reward only at the bottom right corner.
- An optimal (shortest path) policy for the agent is in purple.



# Perceived-State Attack

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- Attacker shows agent  $s^\dagger$ .

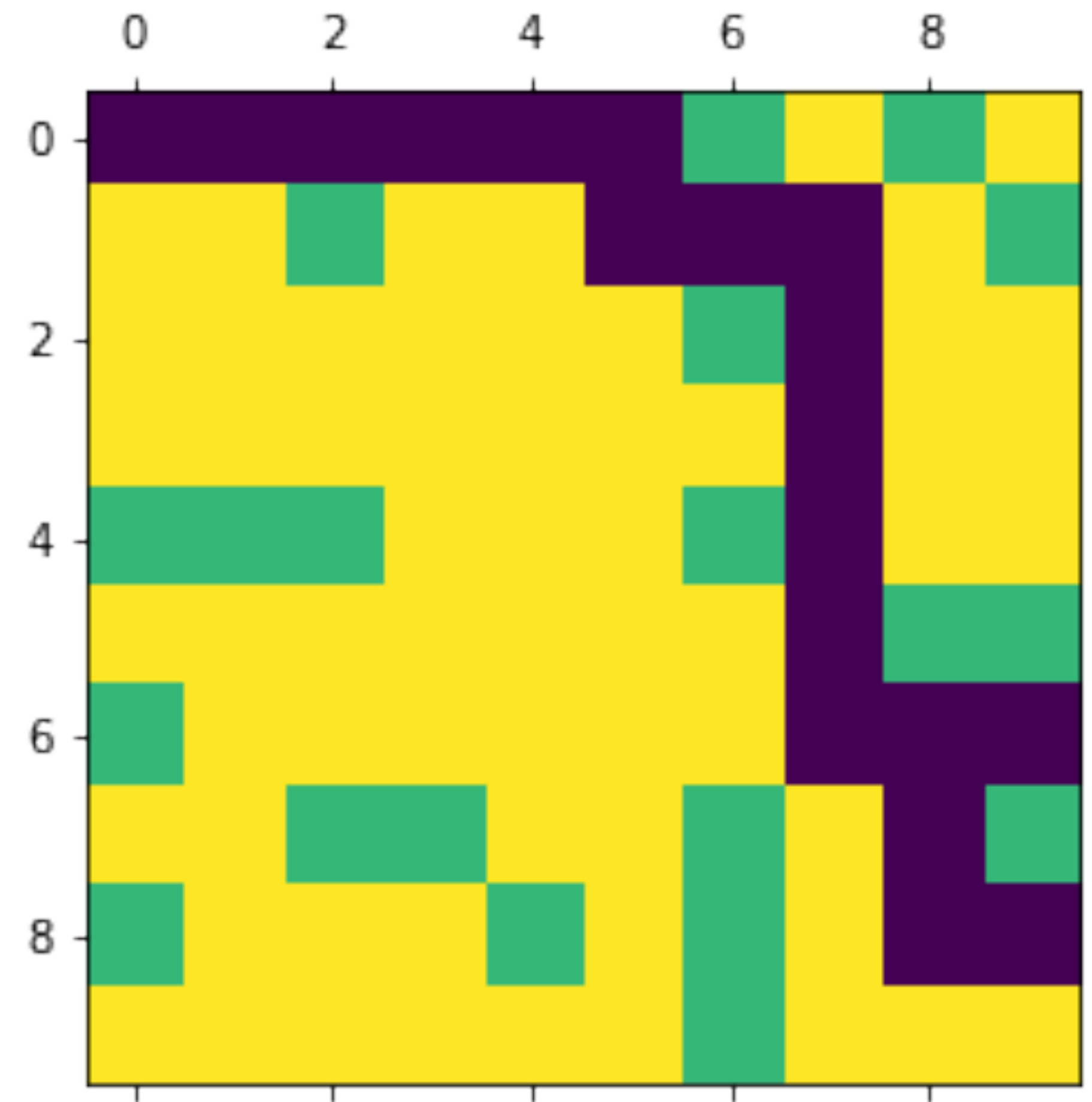
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- Attacker shows agent  $s^\dagger$ .
- Agent chooses action  $\pi(s^\dagger)$  instead of  $\pi(s)$



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Check out Shubham's full paper in  
Neurips22!  
Provable Defense against Backdoor  
Policies in Reinforcement Learning

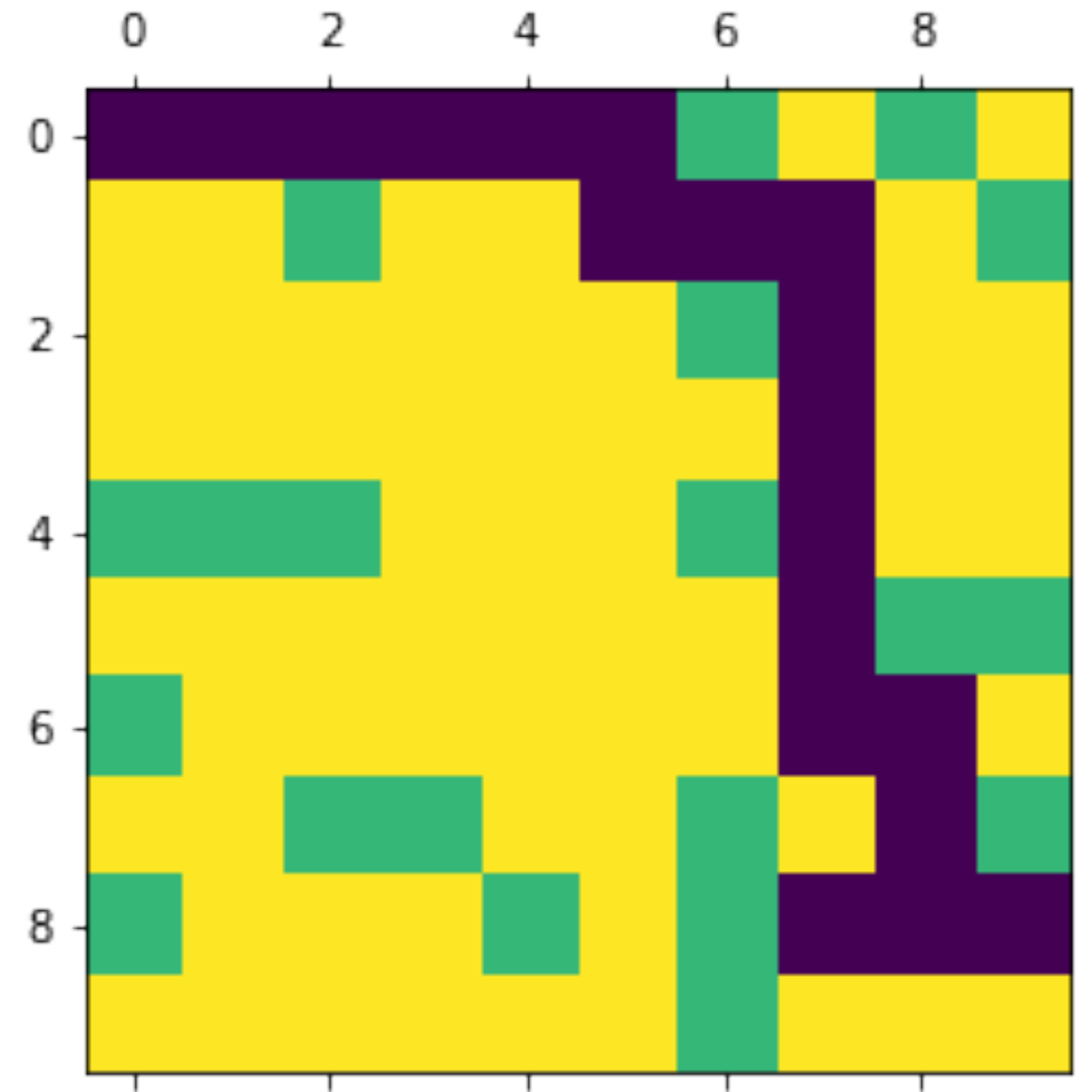
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# True-State Attack

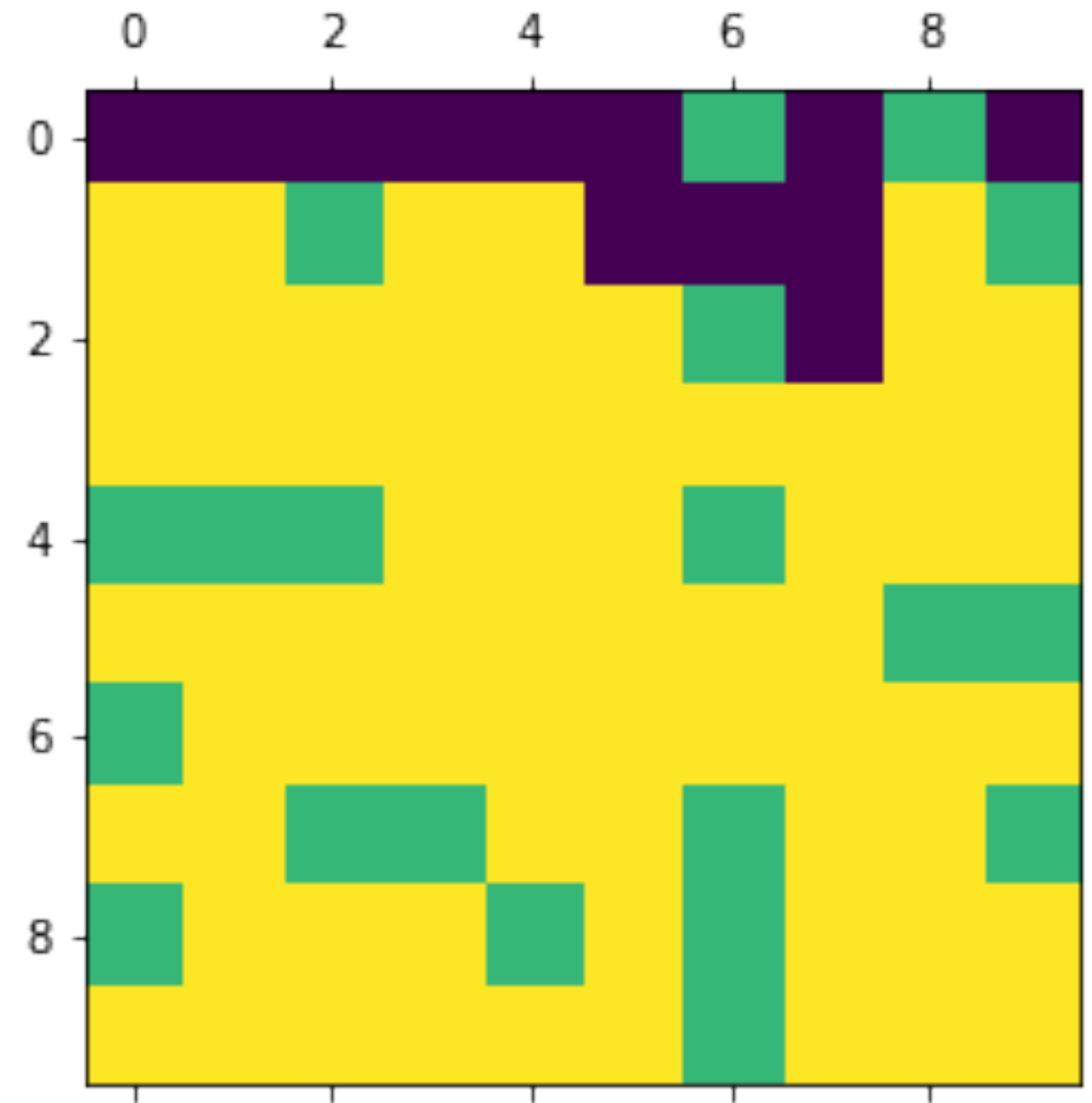


# True-State Attack

Attacker changes the environment's state to  
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Attacker changes the environment's state to  $s^\dagger$



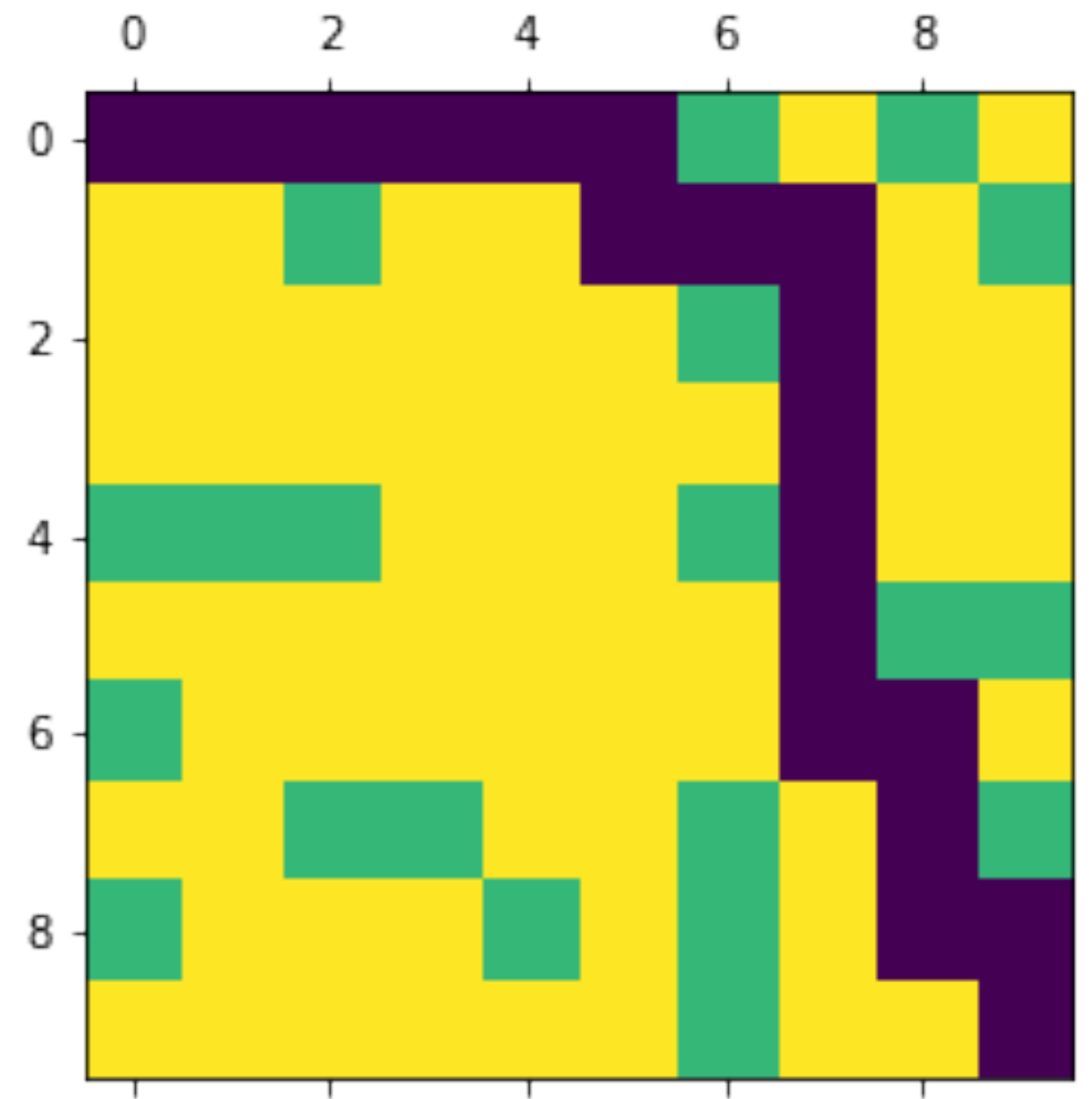
# Reward Attack

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Attacker changes the reward the agent  
receives to  $r^\dagger$

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Attacker changes the reward the agent receives to  $r^\dagger$



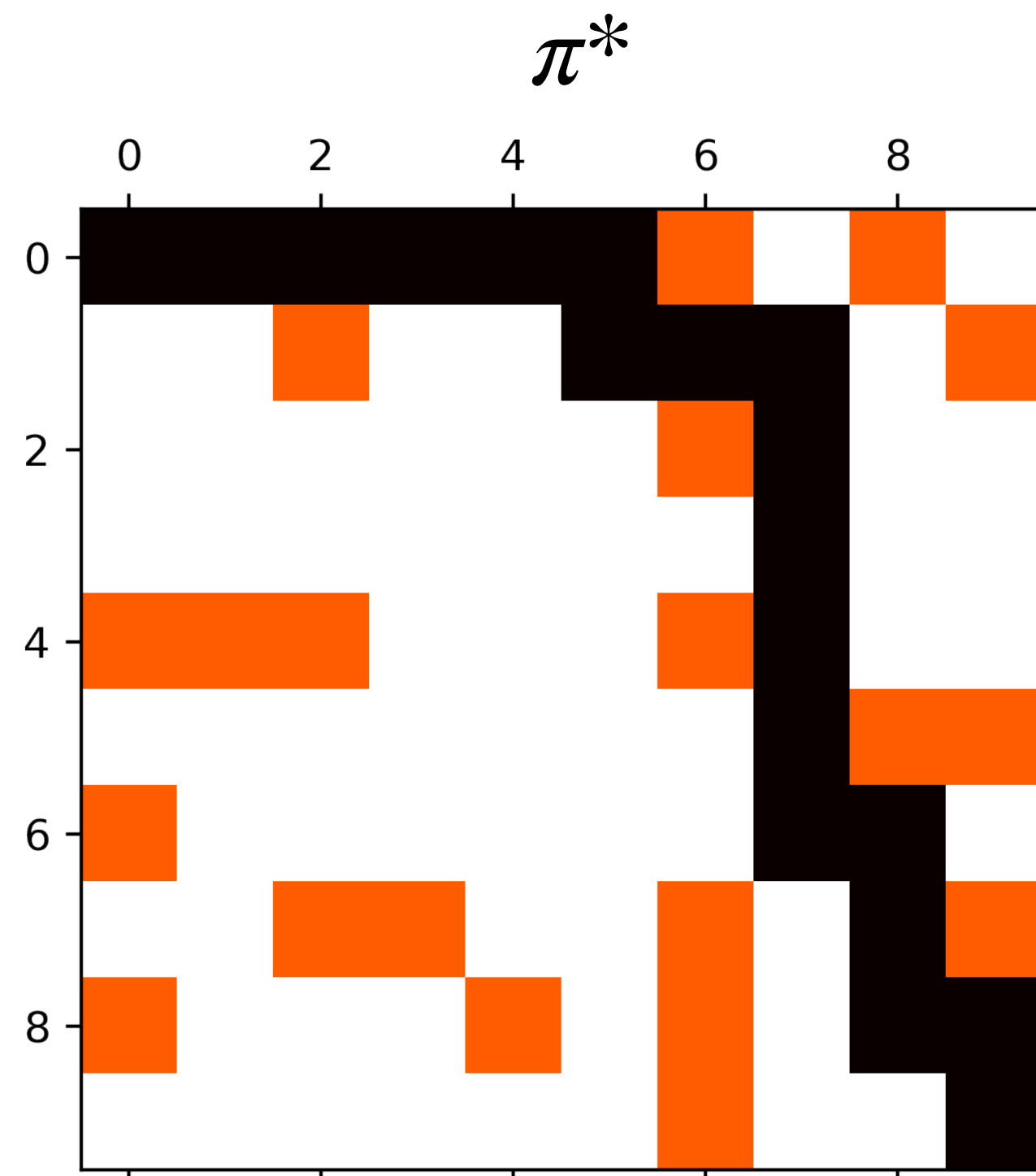
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Optimal policies may be sensitive to noise or attacks.

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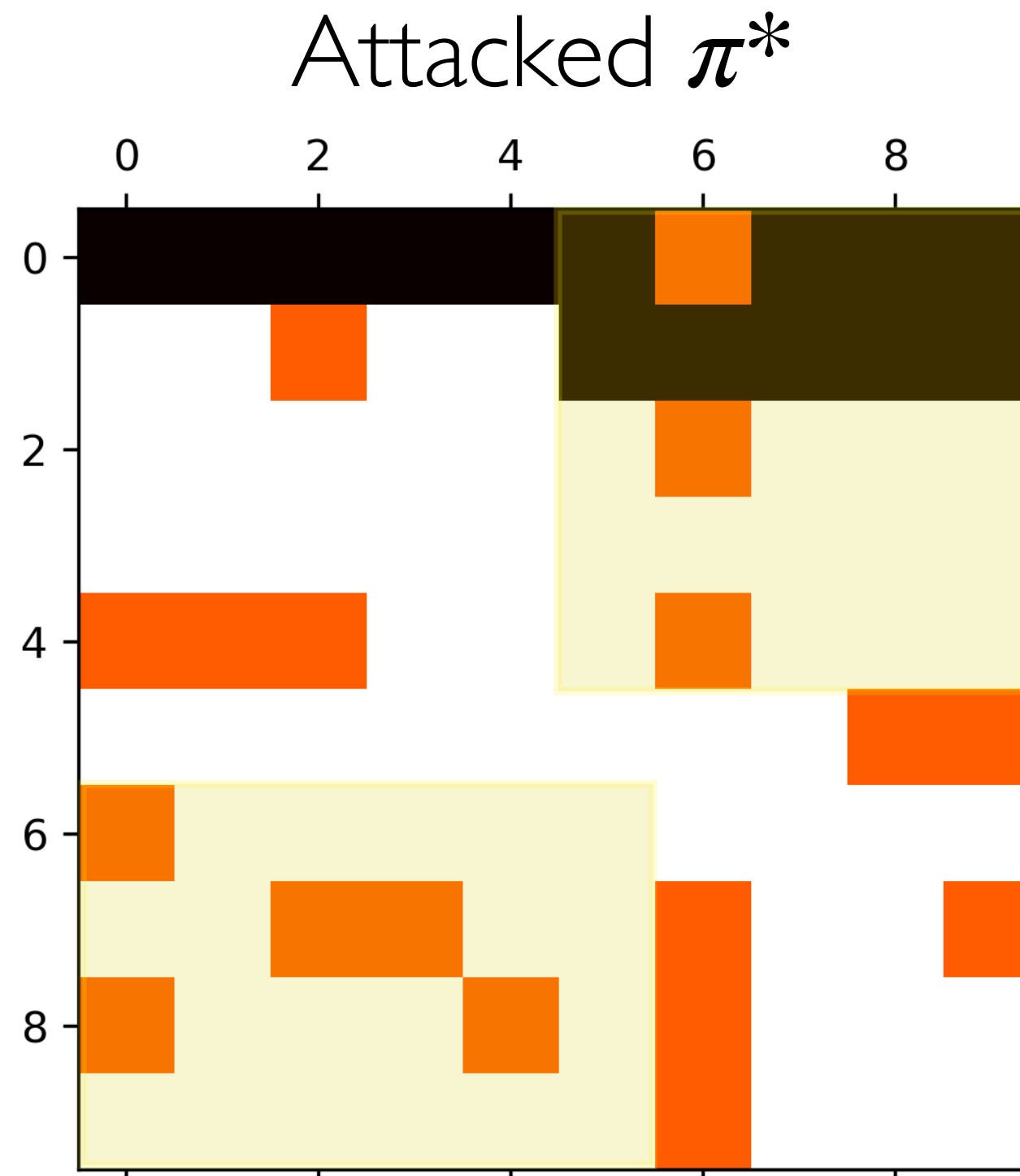
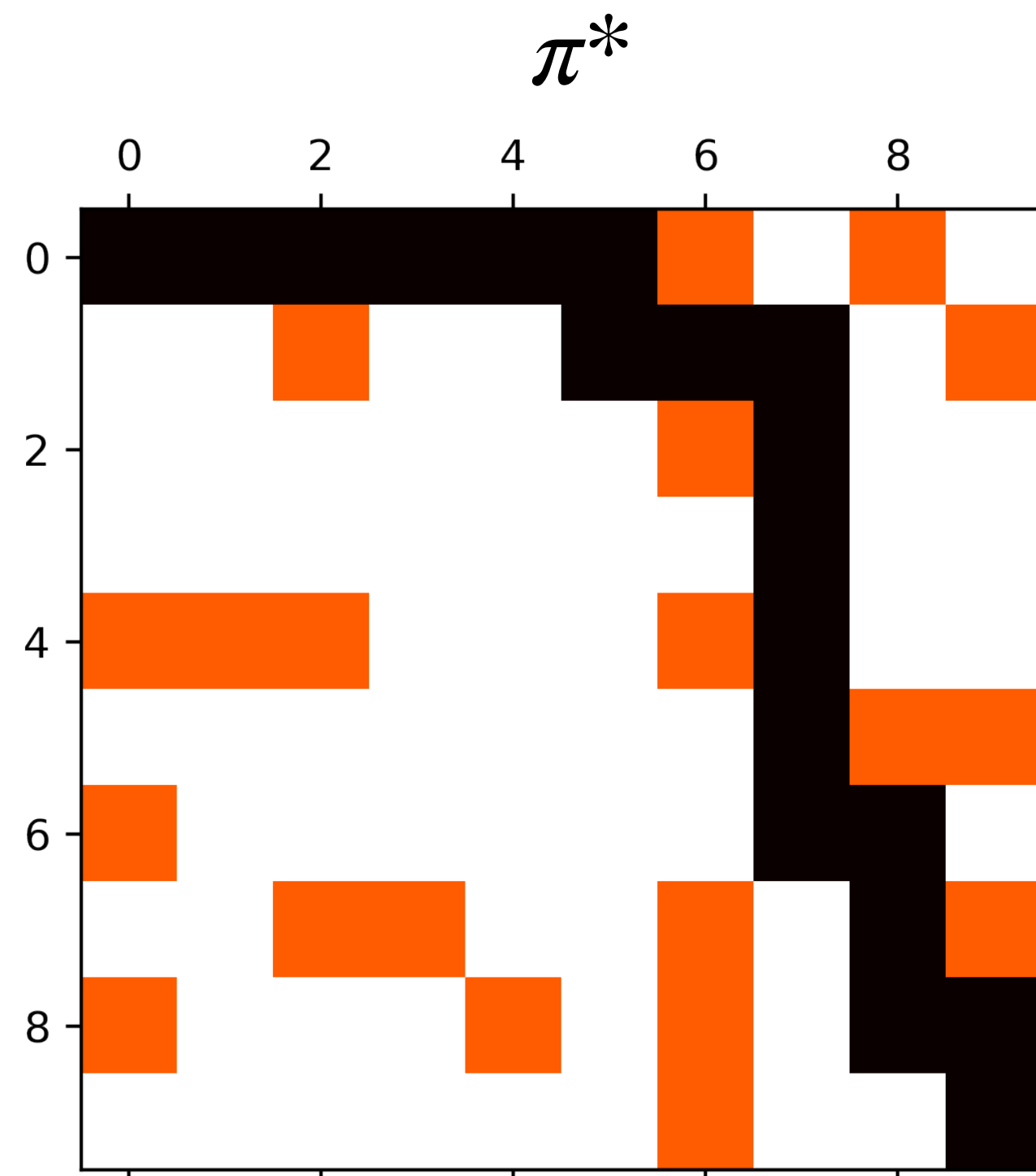
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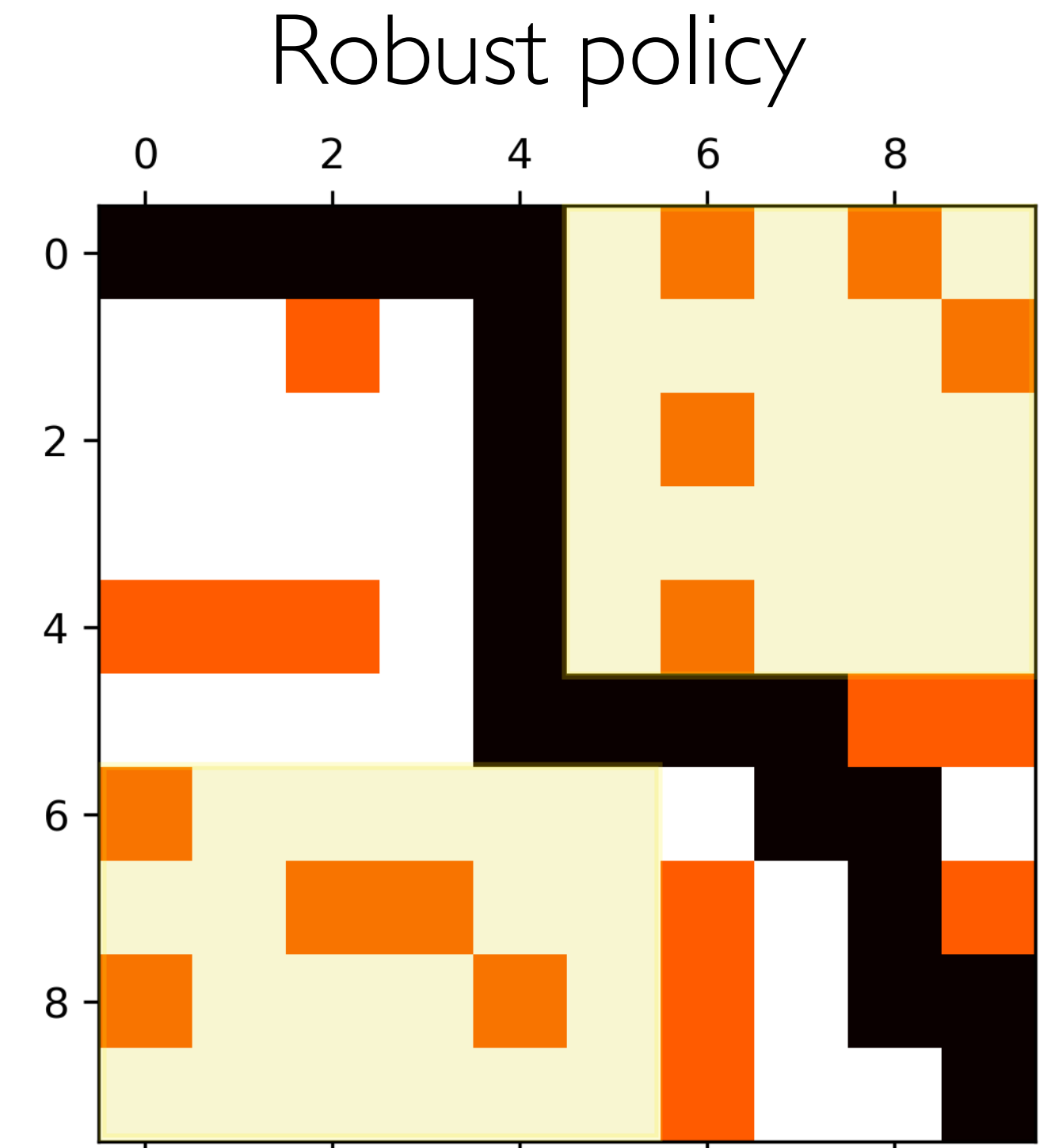
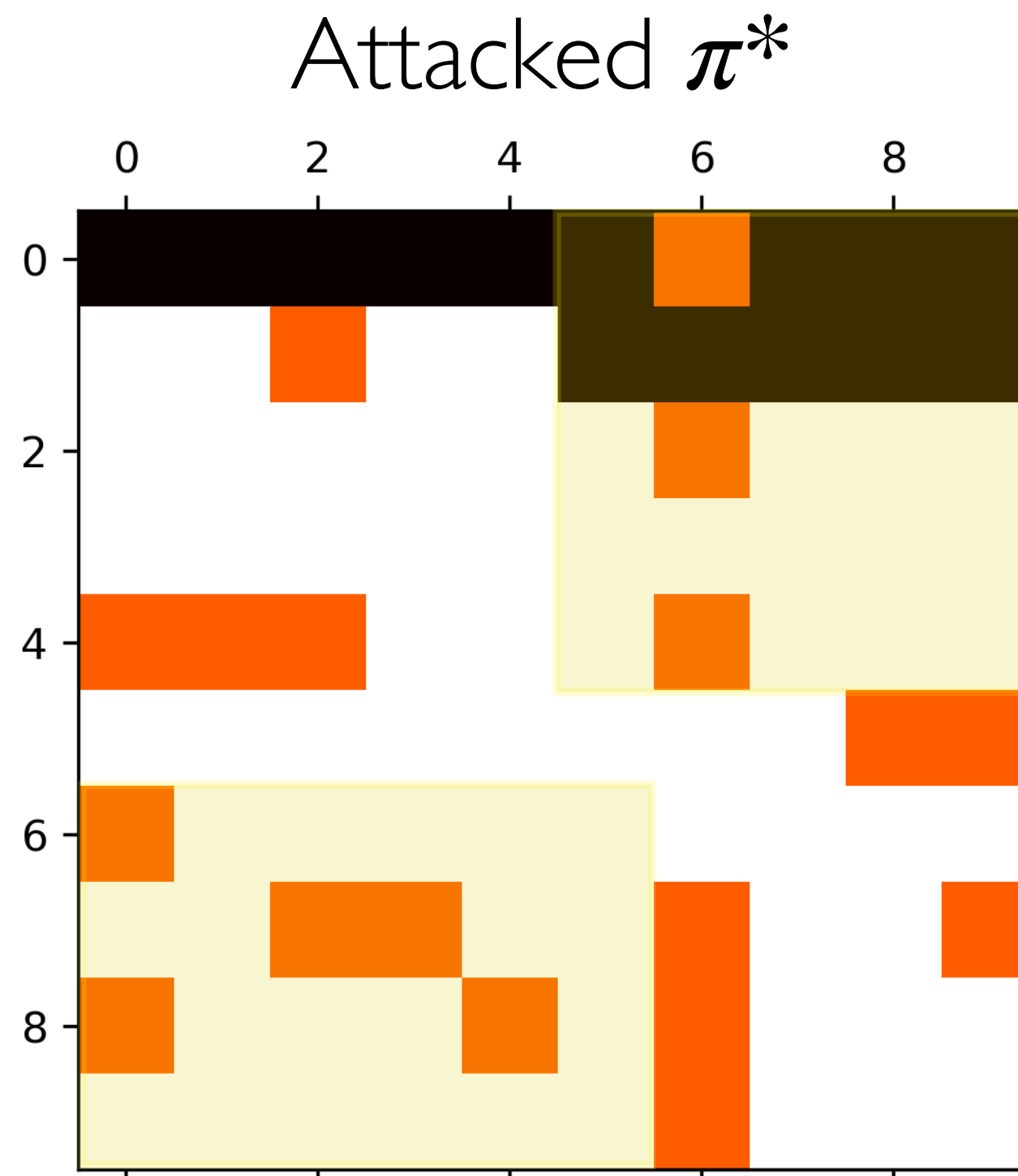
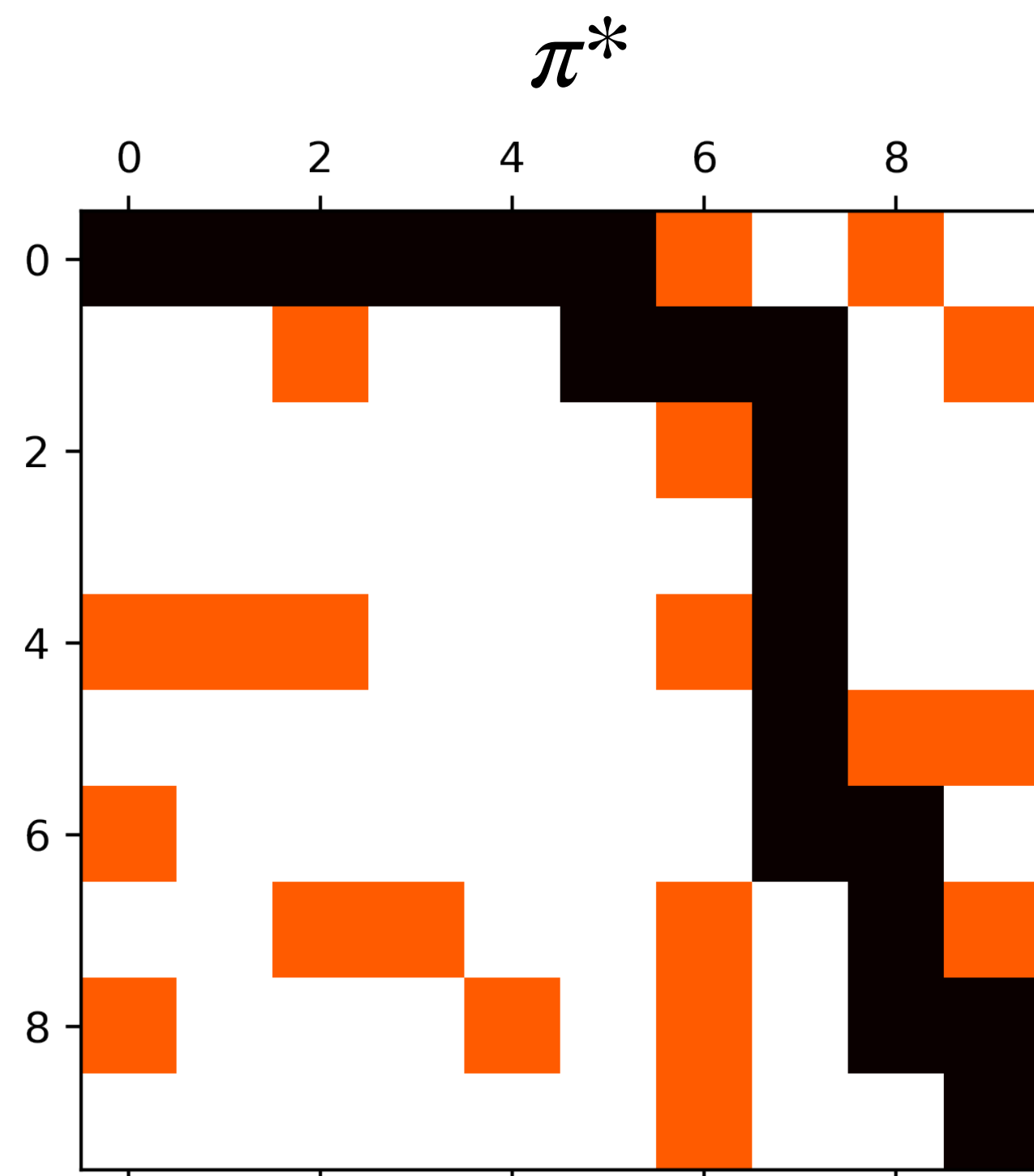
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# What's known?

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## Optimal Observation Attacks

### Robust Deep Reinforcement Learning against Adversarial Perturbations on State Observations

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## [Training-time] Action and Reward Attacks

### Understanding the Limits of Poisoning Attacks in Episodic Reinforcement Learning

**Anshuka Rangi<sup>1</sup>, Haifeng Xu<sup>2</sup>, Long Tran-Thanh<sup>3</sup>, Massimo Franceschetti<sup>1</sup>**

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## Defense against a specific reward attack algorithm

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### Defense Against Reward Poisoning Attacks in Reinforcement Learning

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Adish Singla  
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Goran Radanovic  
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Not robust! Attacker can change its algorithm later.

# The Attack Problem



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Attacker has its own reward  $g(s_t, a_t, r_t)$  that depends on the victim's.

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**Definition 1** (Attack Problem). For any  $\pi$ , the attacker's seeks a policy  $\nu^* \in N$  that maximizes its expected reward from the victim-attacker- $M$  interaction:

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# Adversarial Decomposition

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We decompose the attacked  $\pi$ -M interaction based on the *flow of information*.

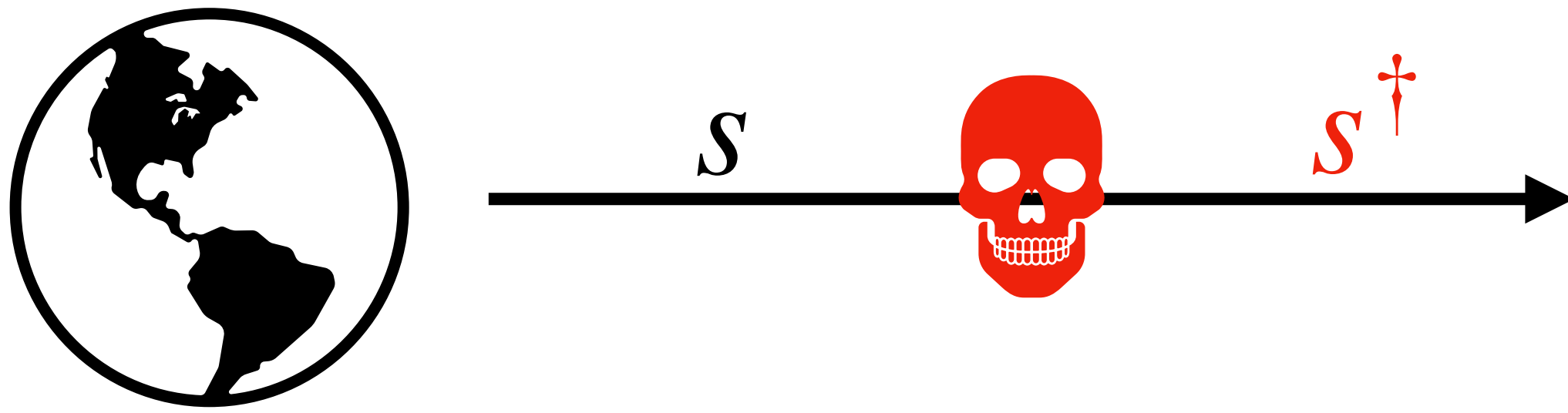
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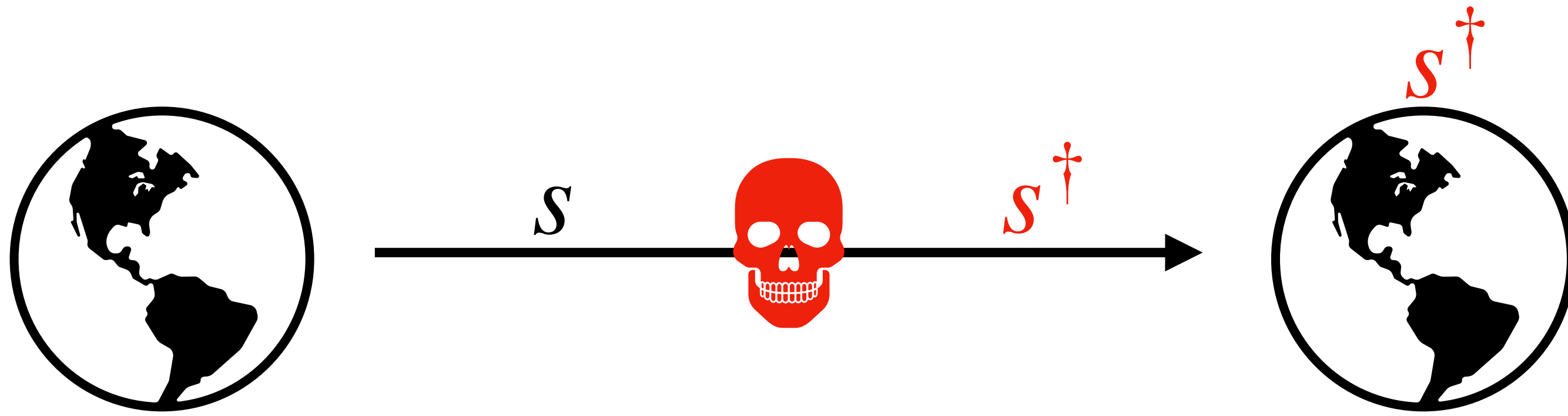
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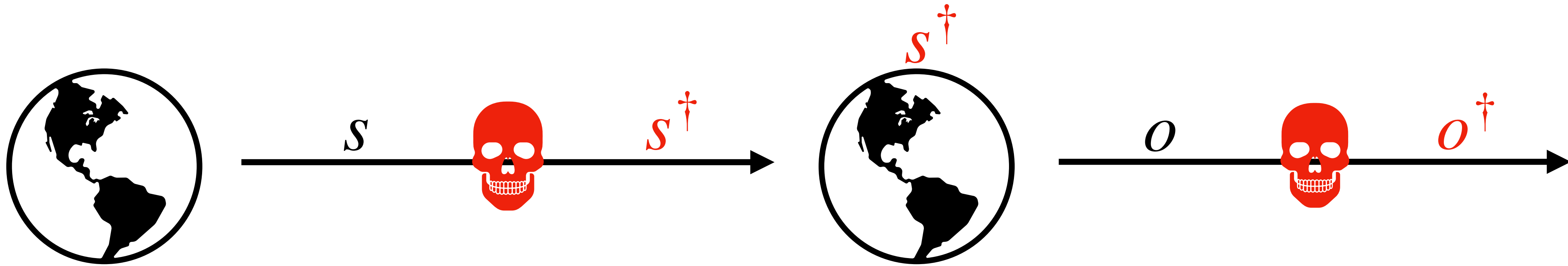
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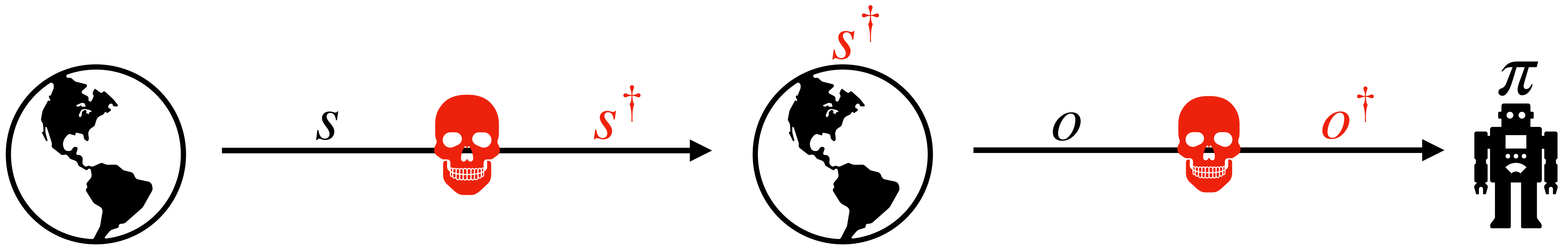
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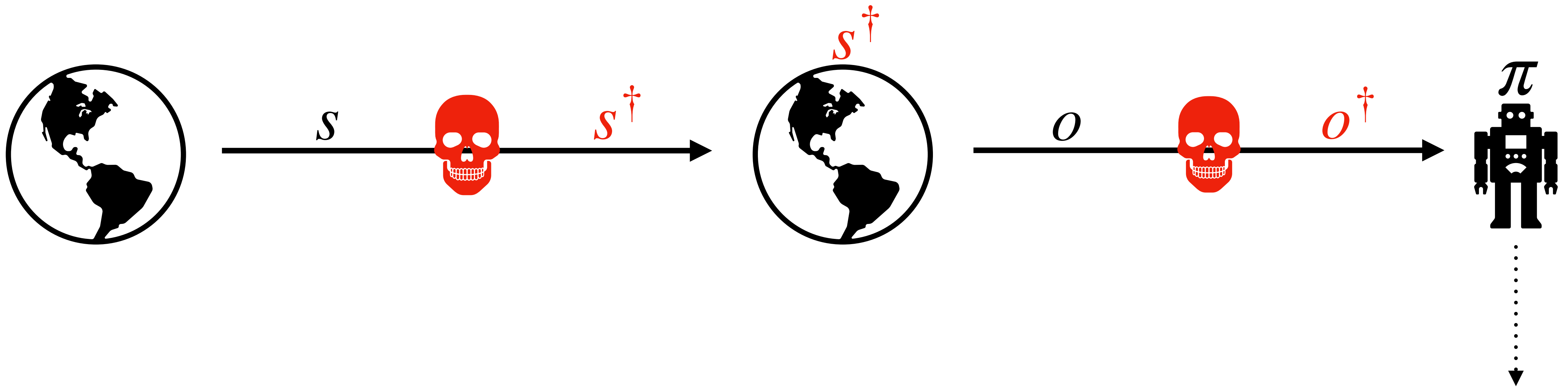
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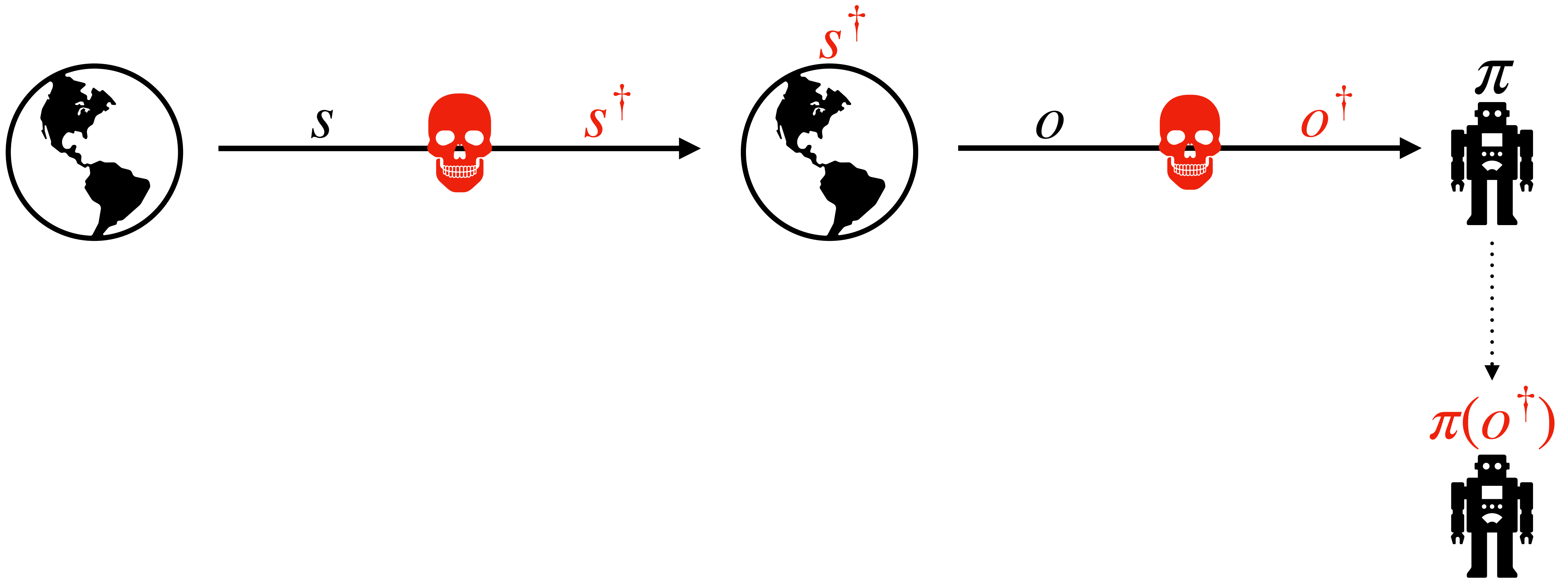
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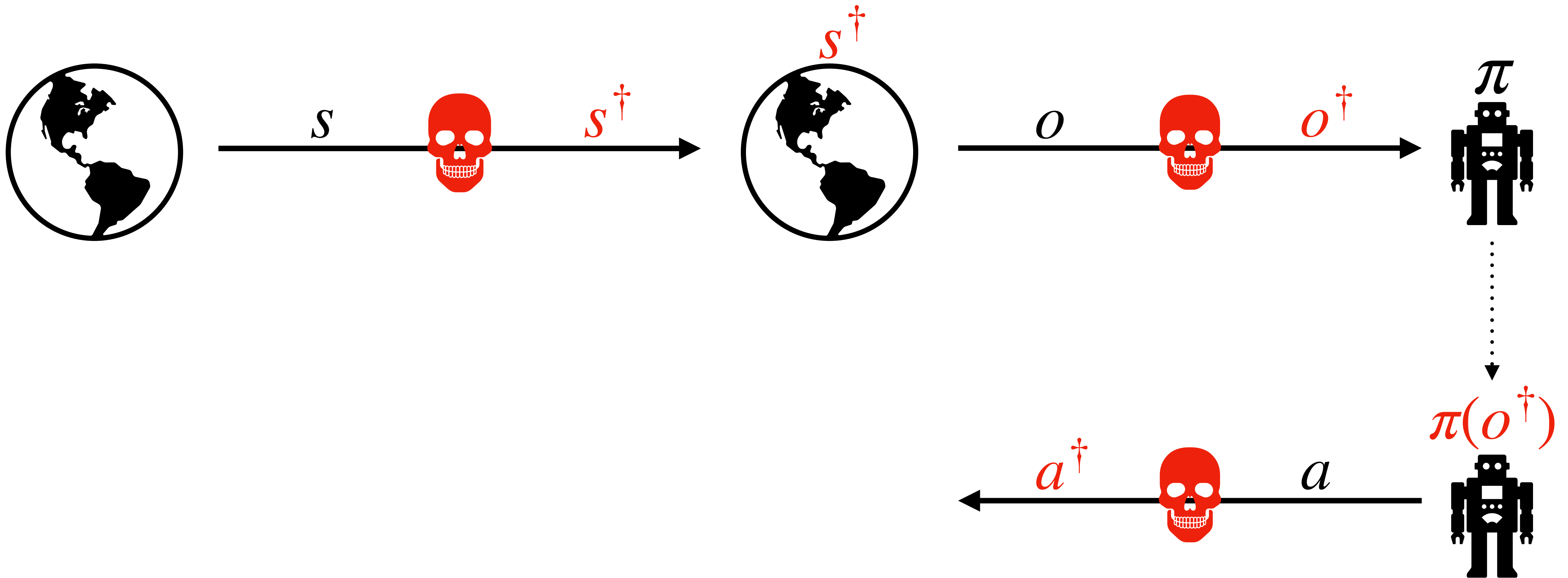
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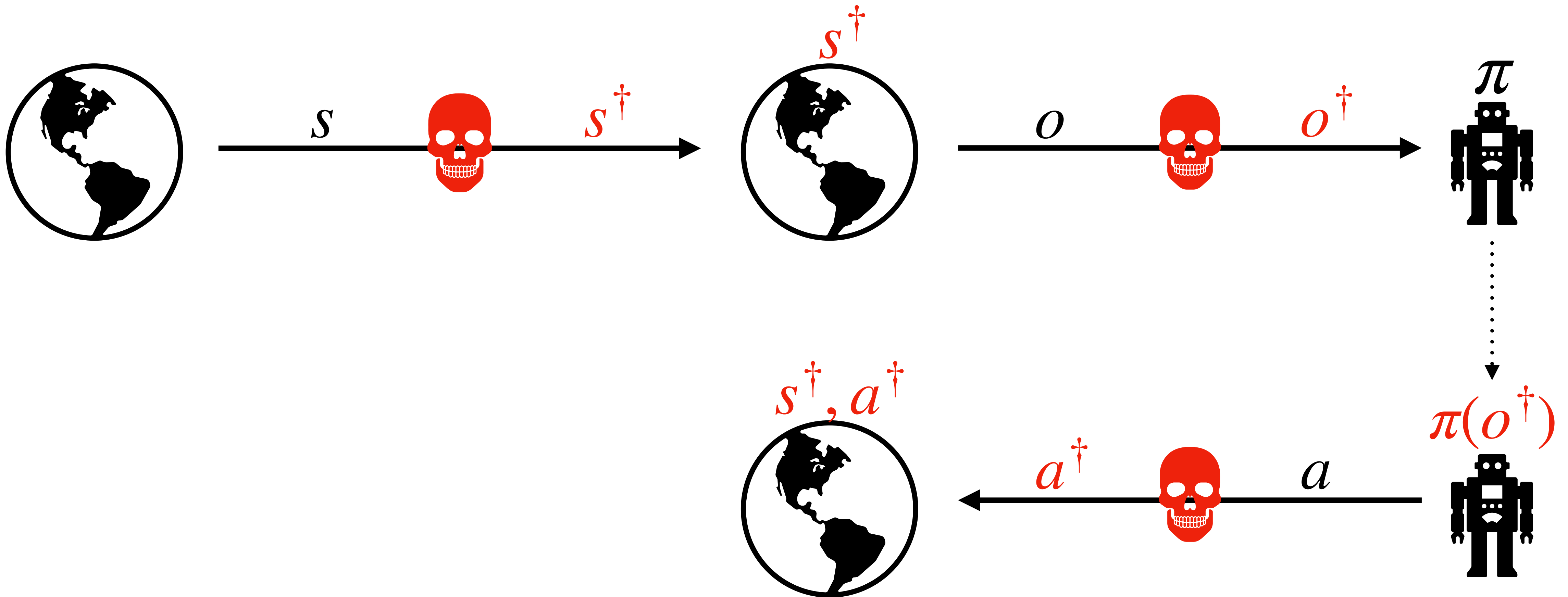
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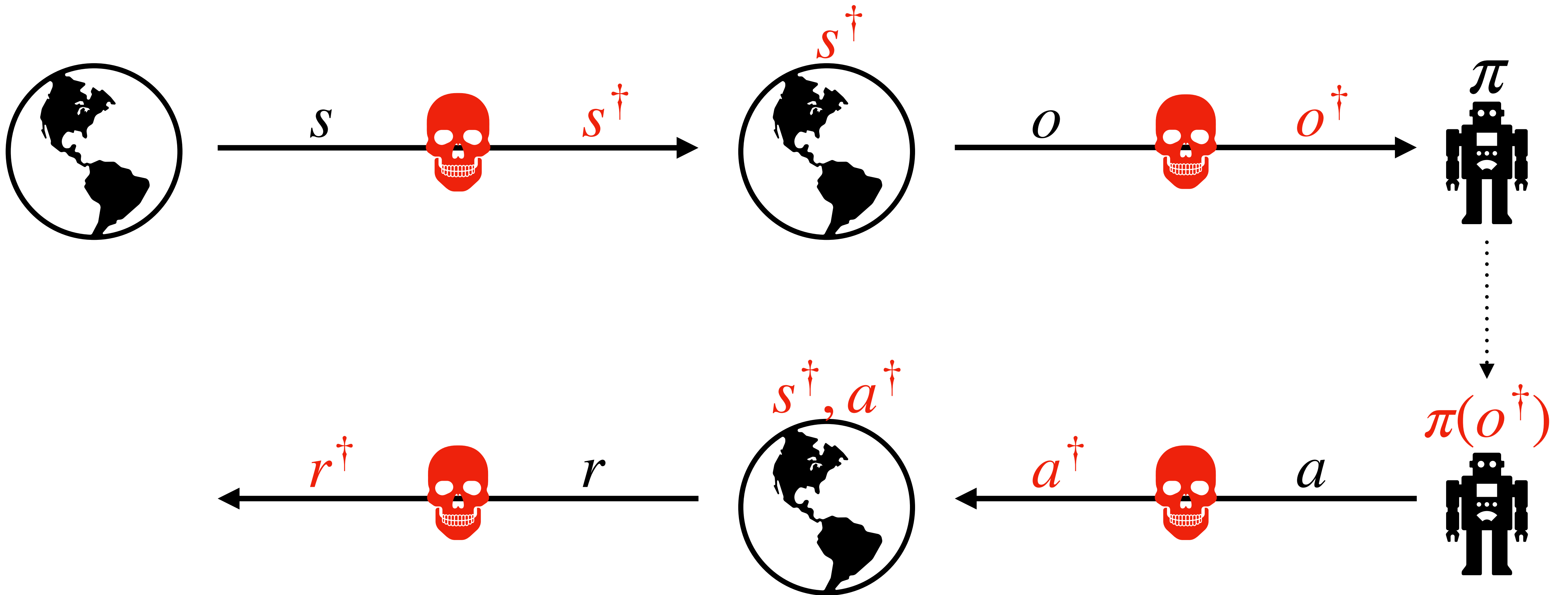
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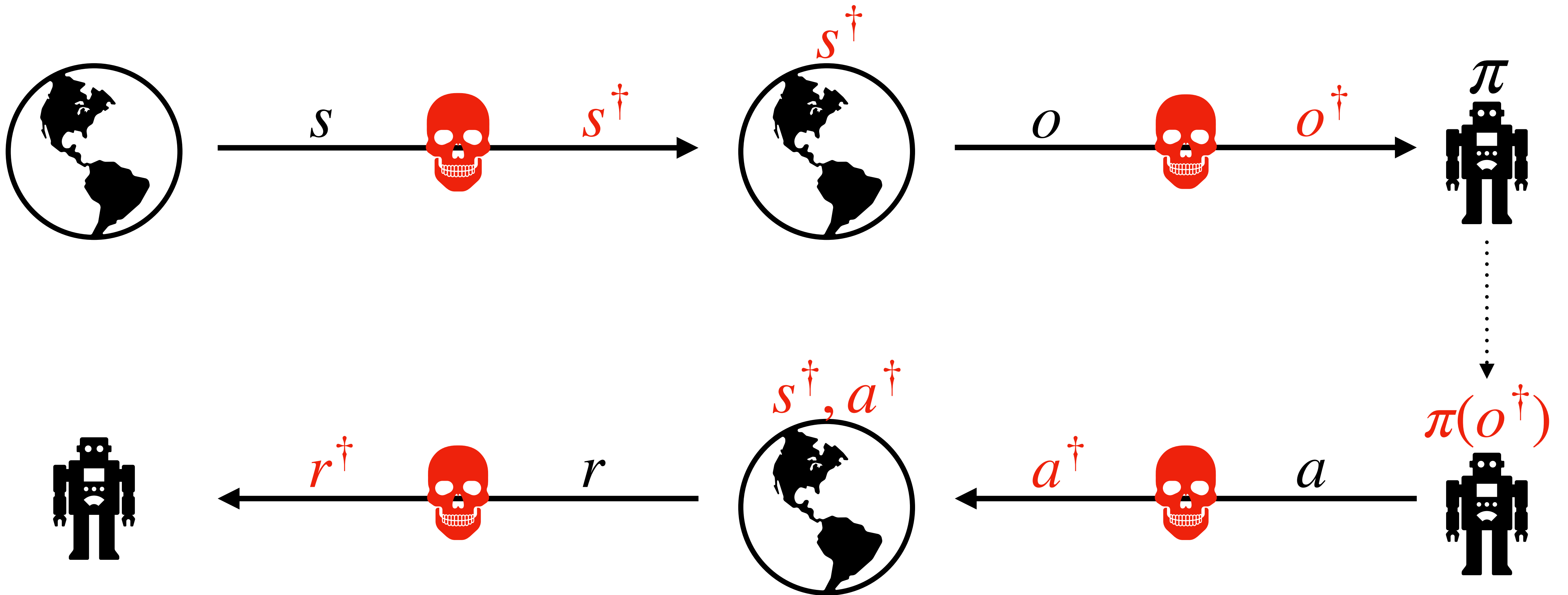
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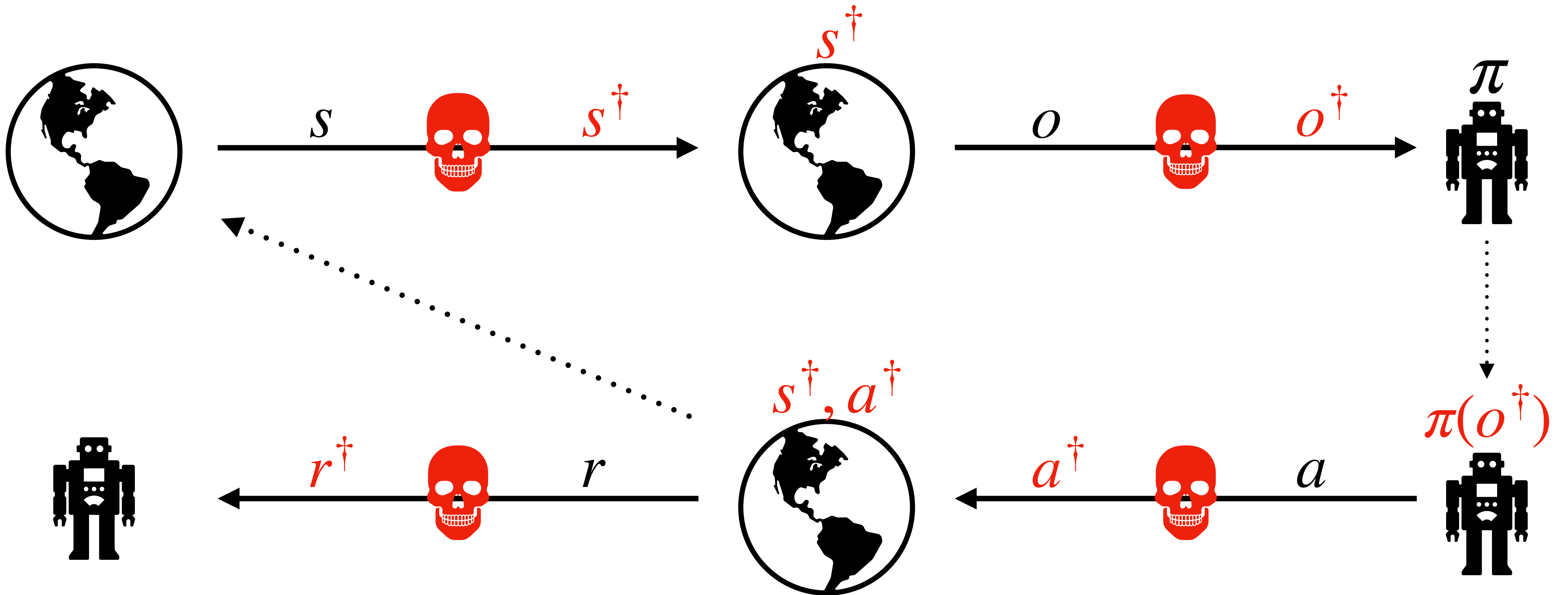
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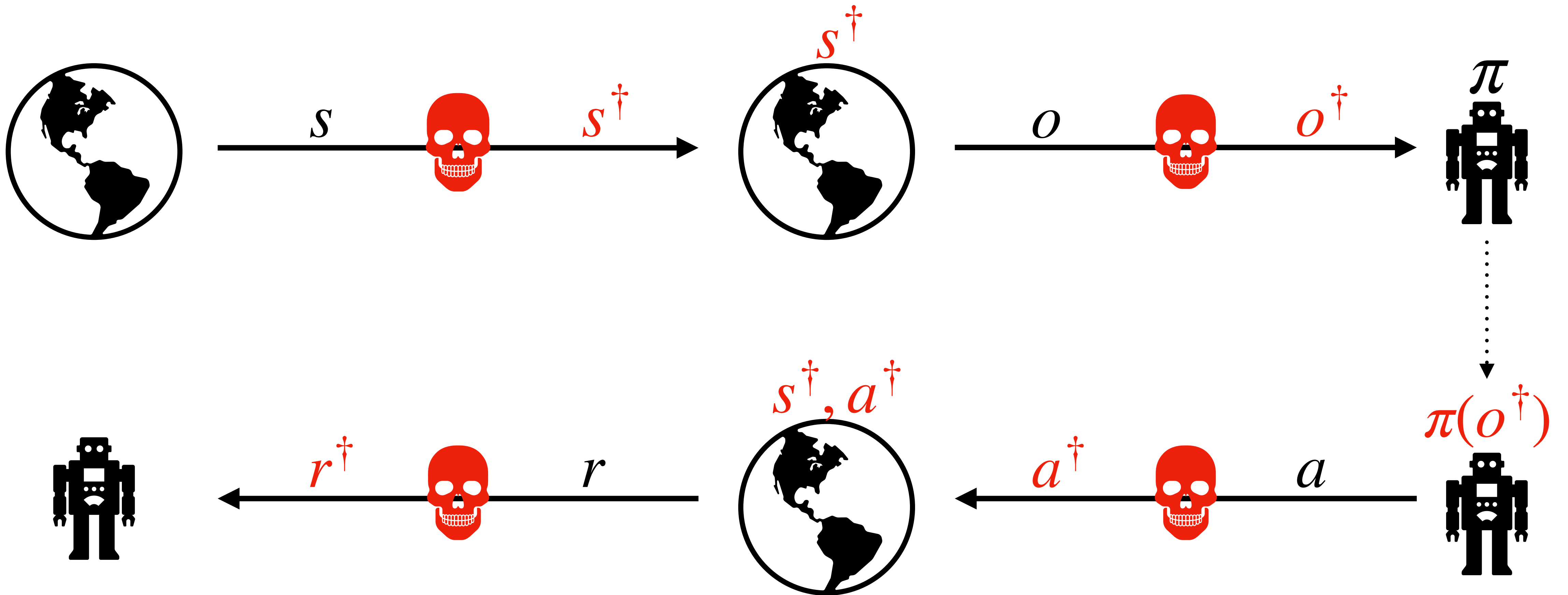
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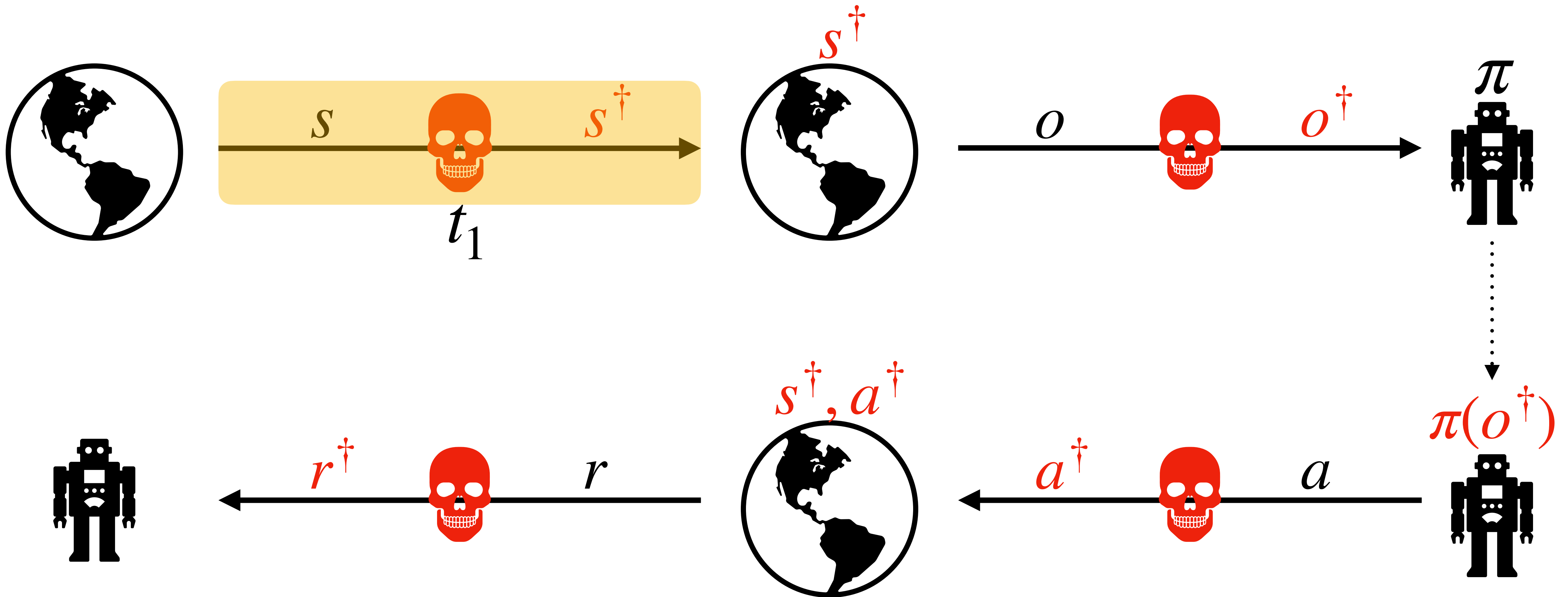
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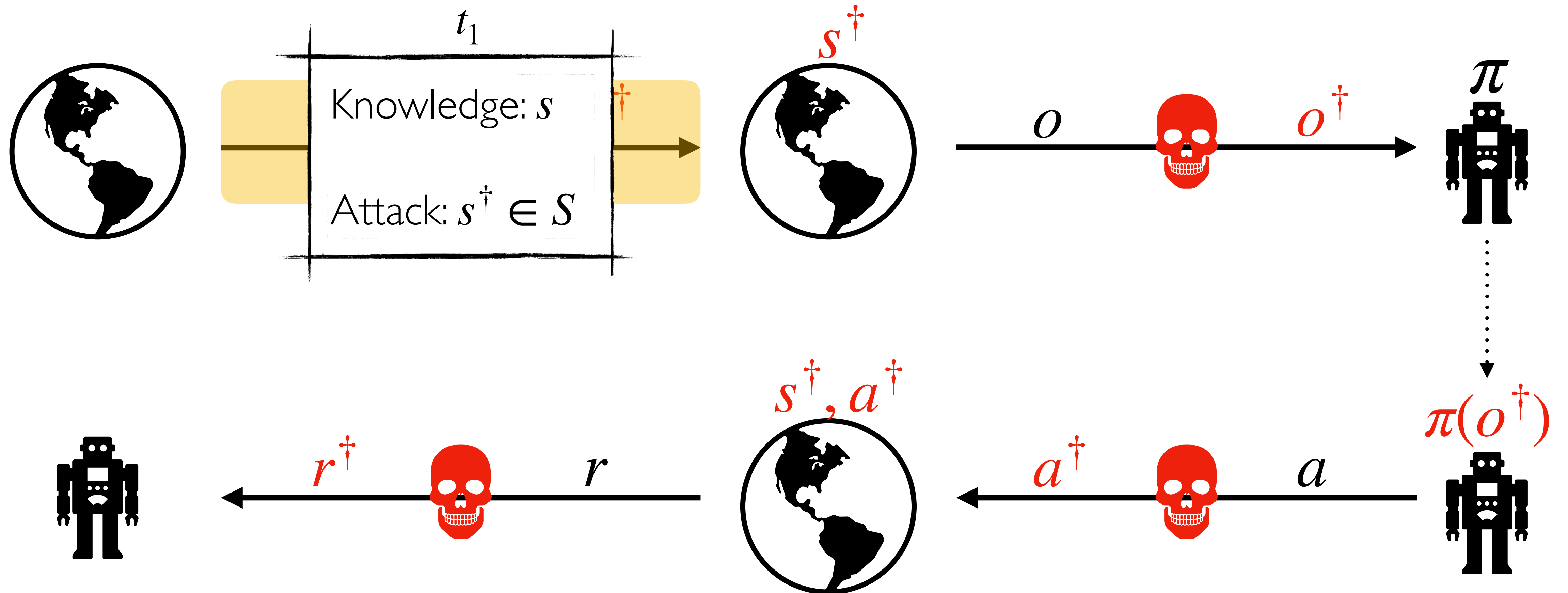
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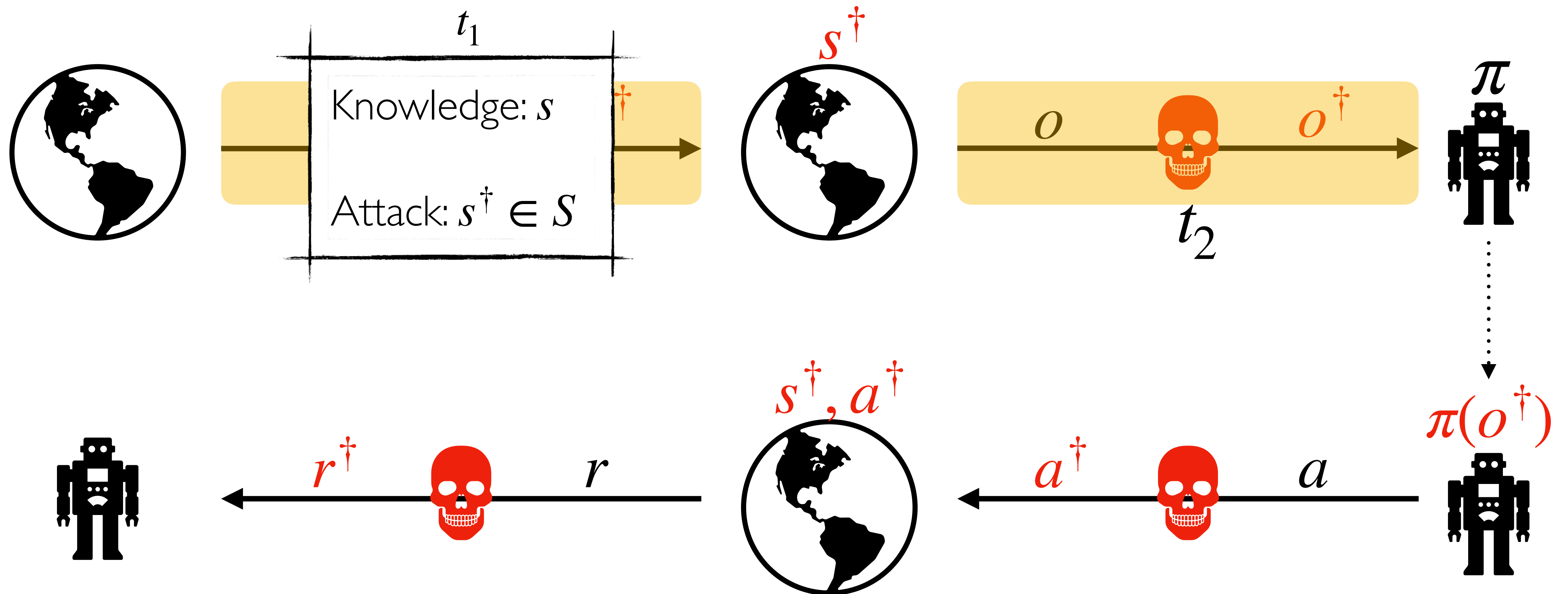
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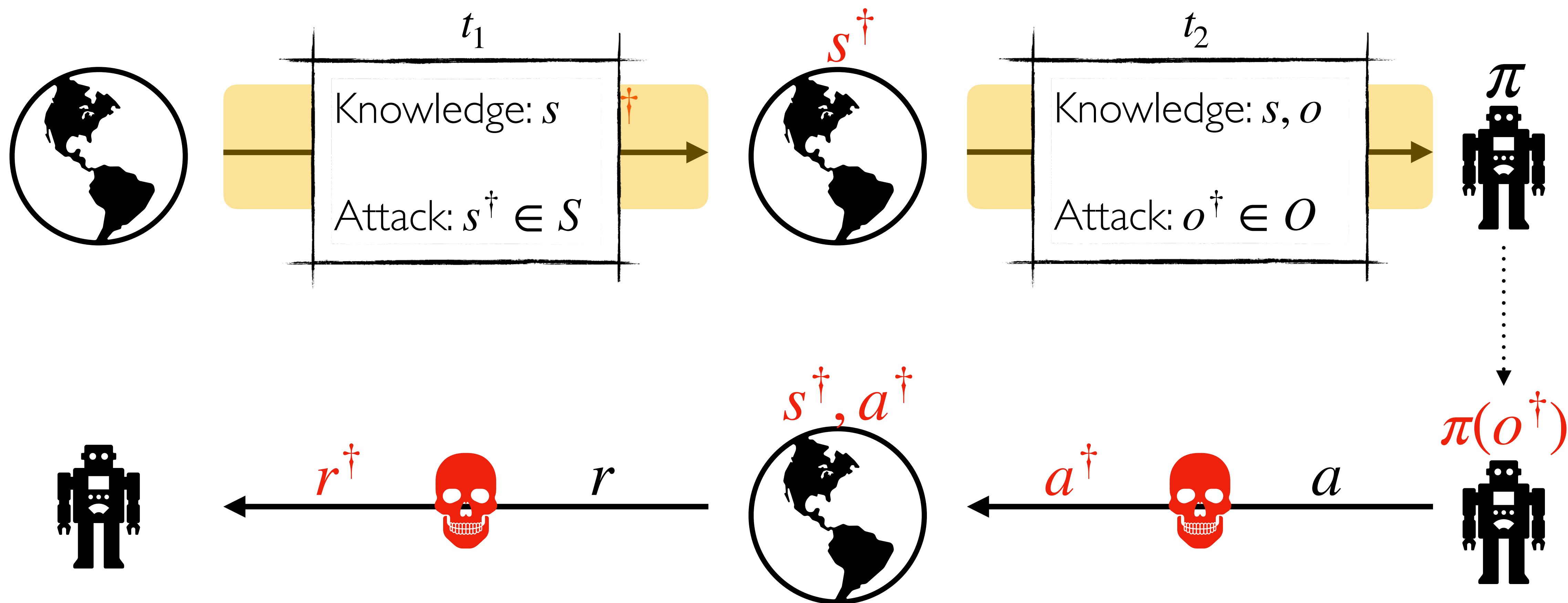
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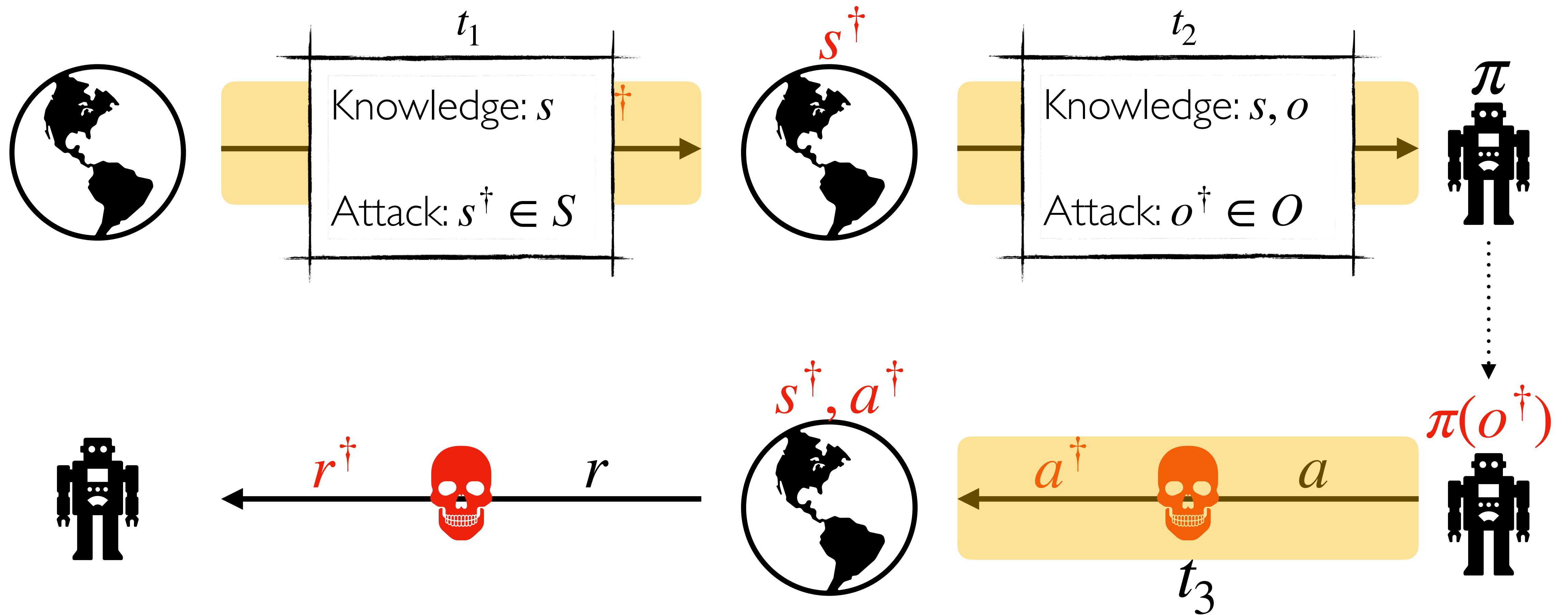
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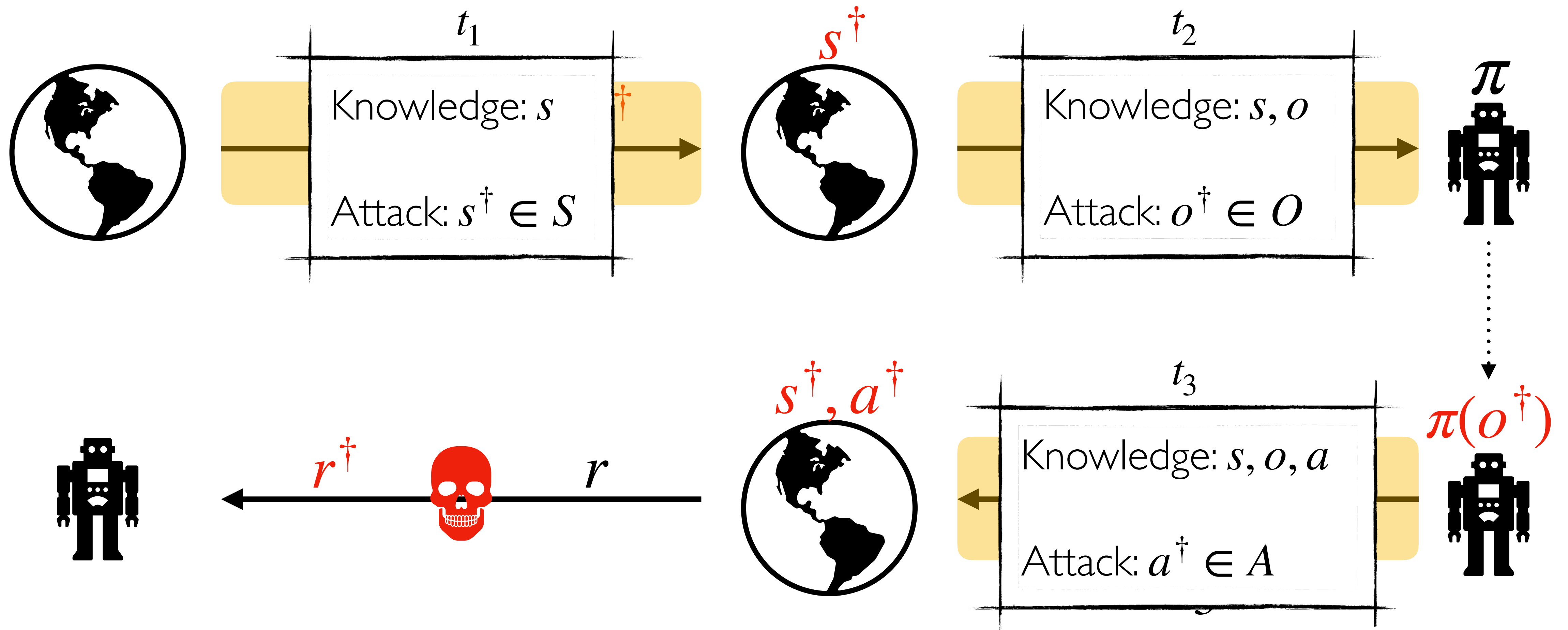
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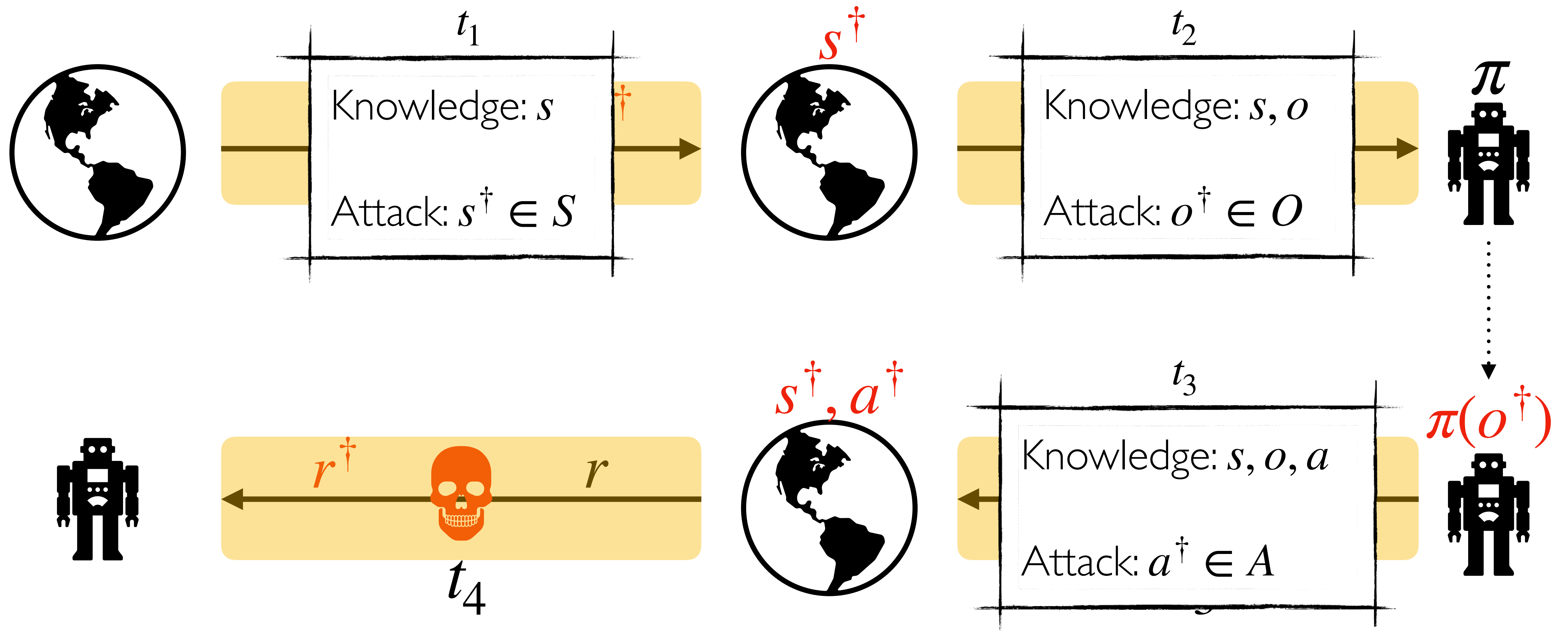
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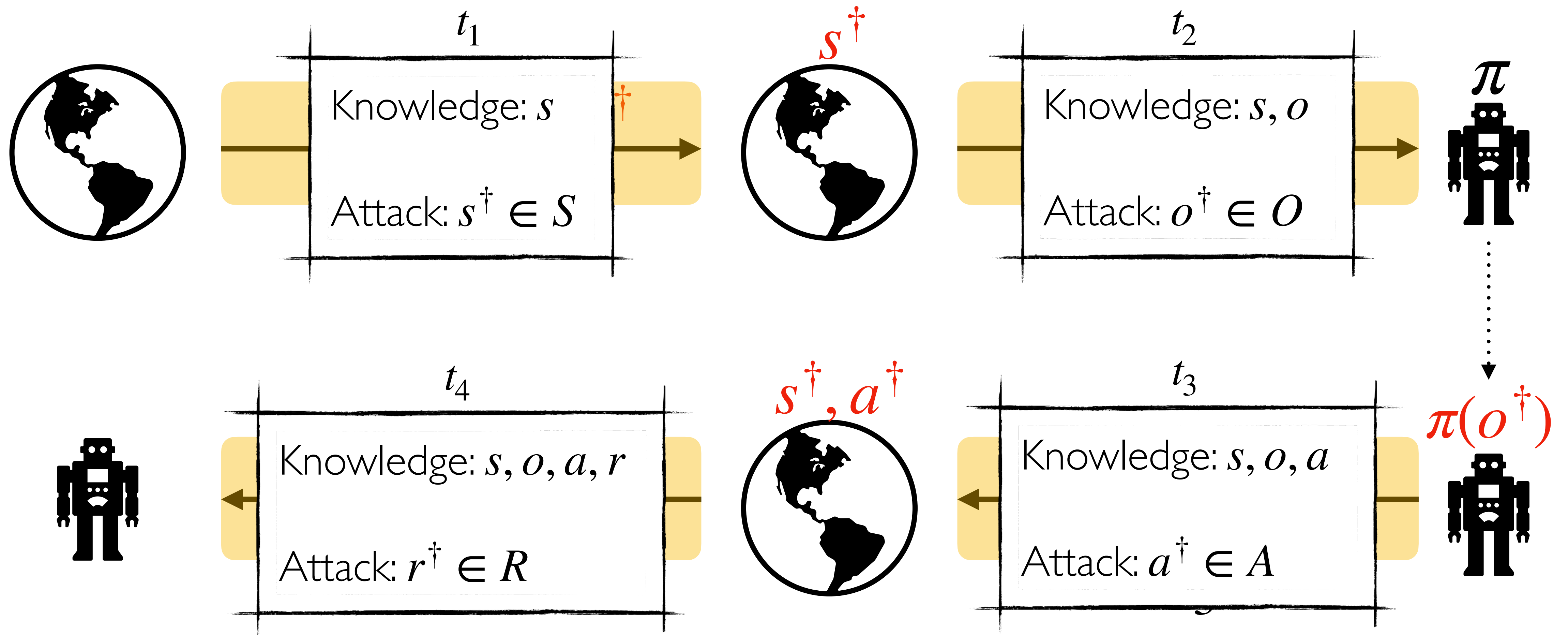
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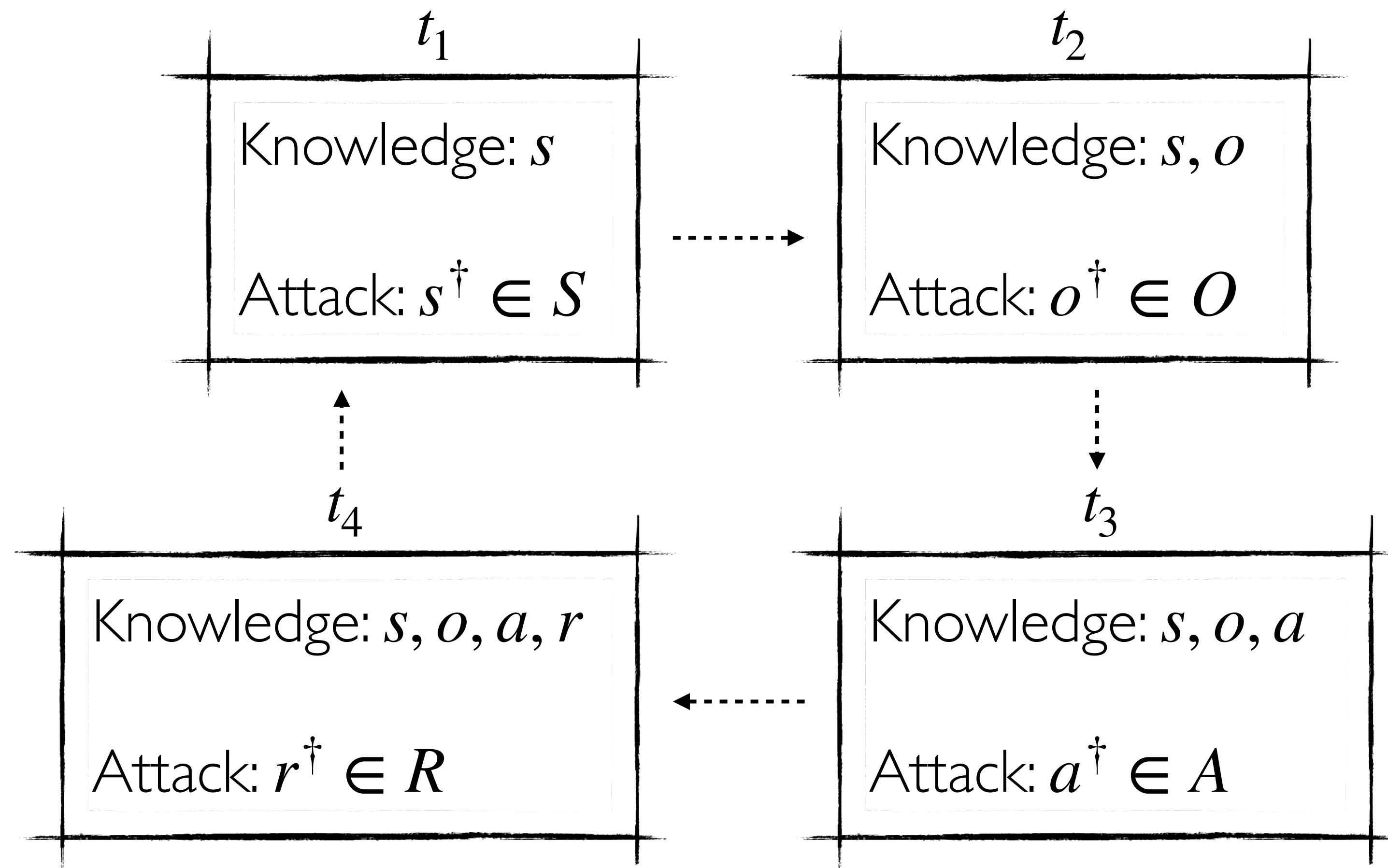


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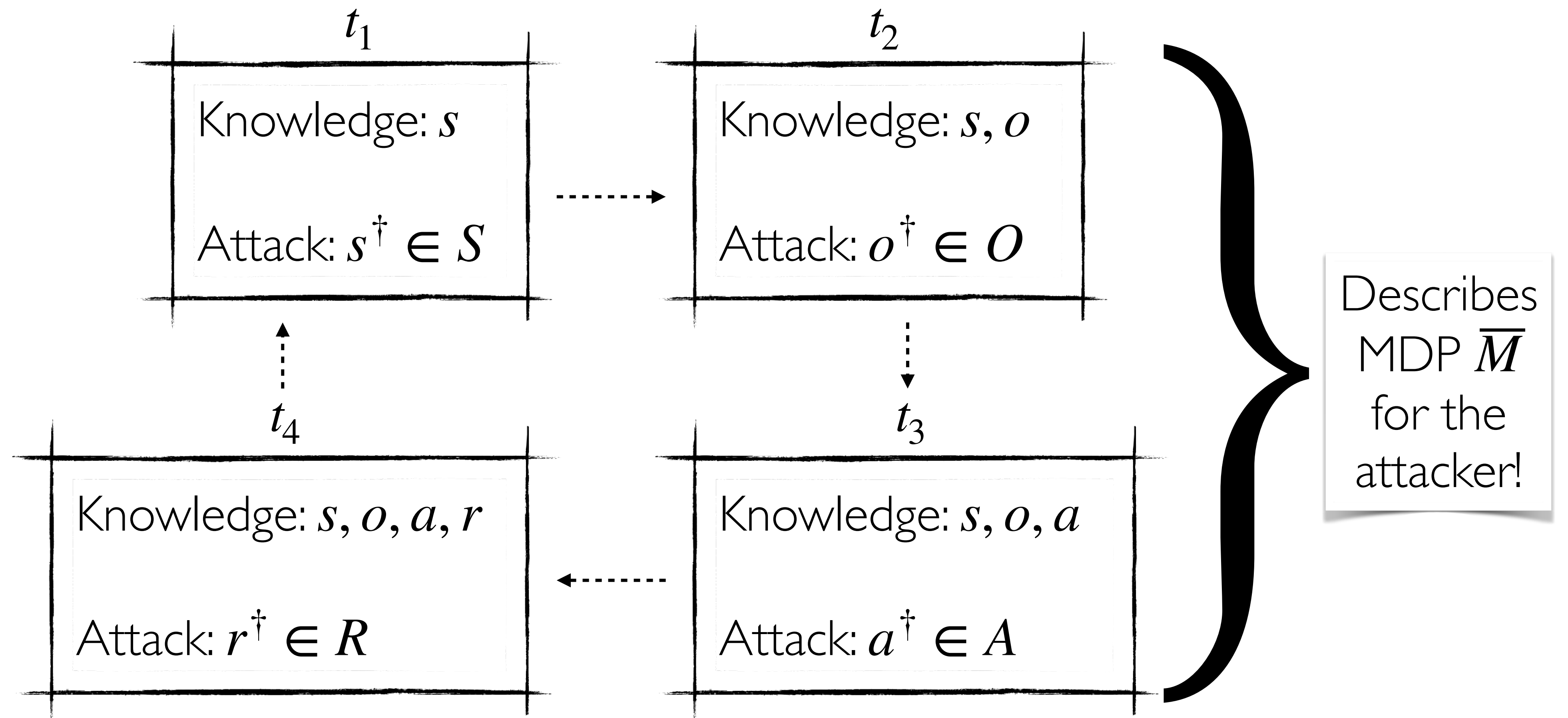
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# Attacker's Perspective



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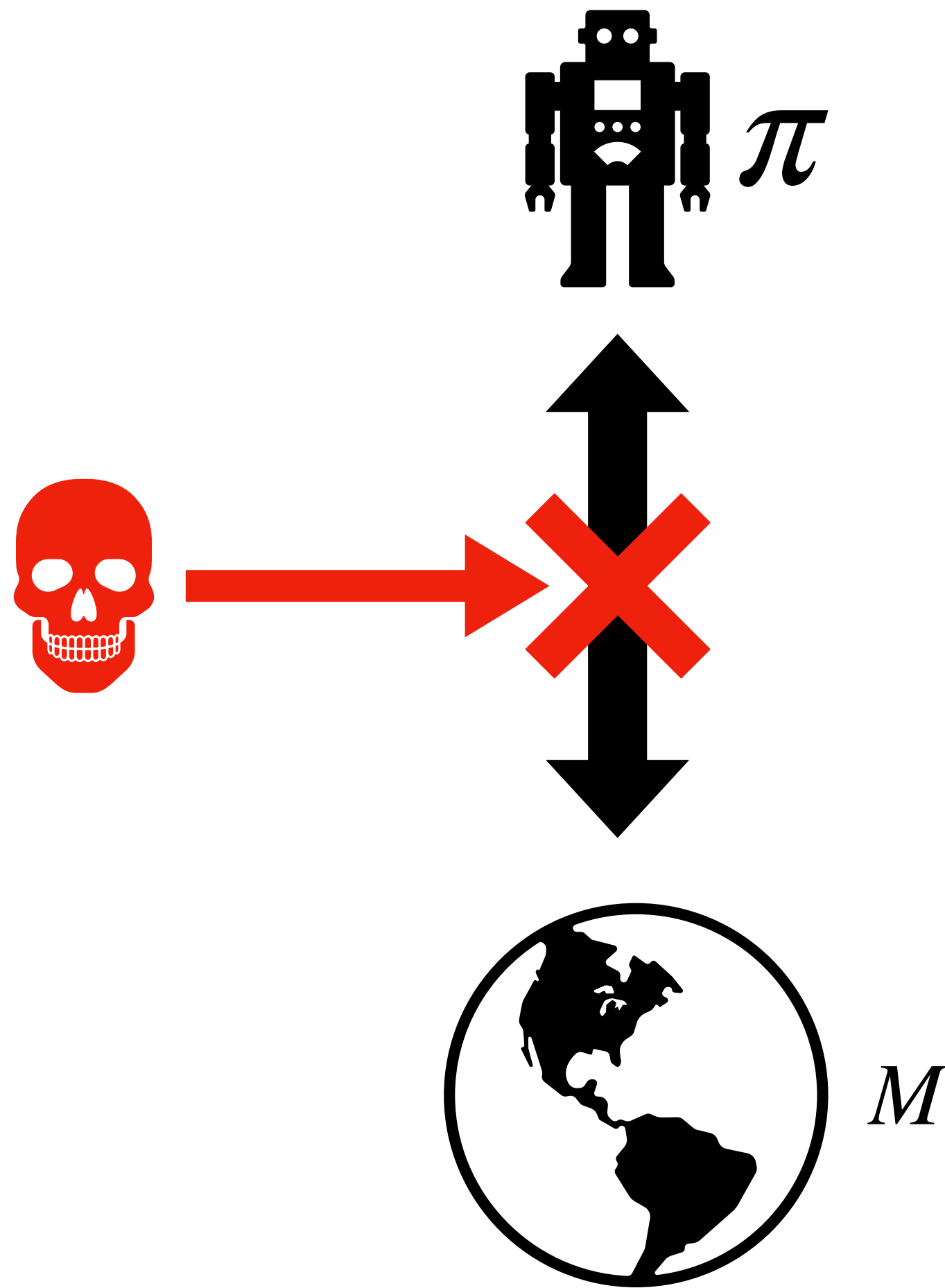
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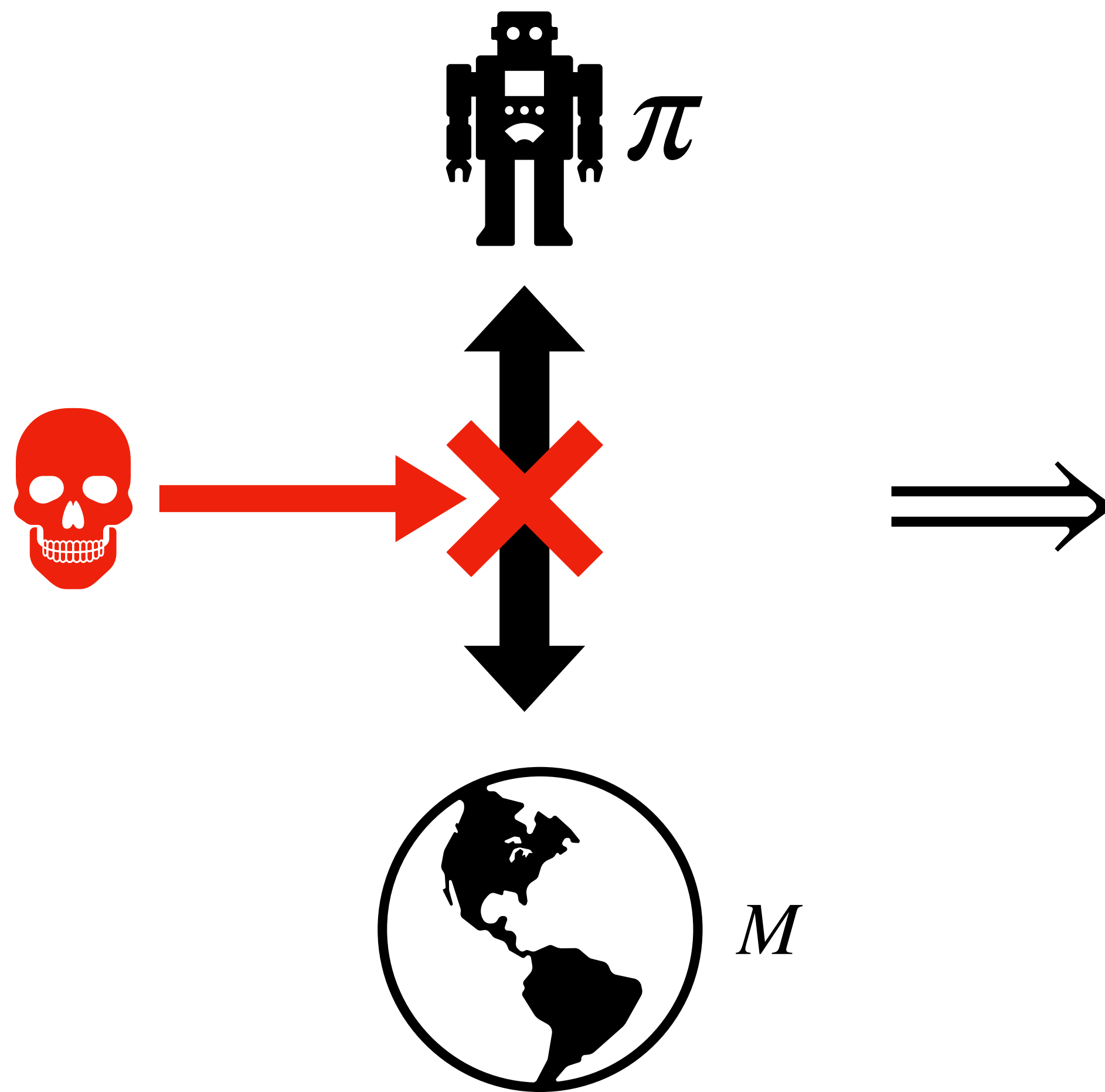
**Proposition:** Any optimal policy for  $\bar{M}$  is an optimal attack policy.

# Reduction to RL

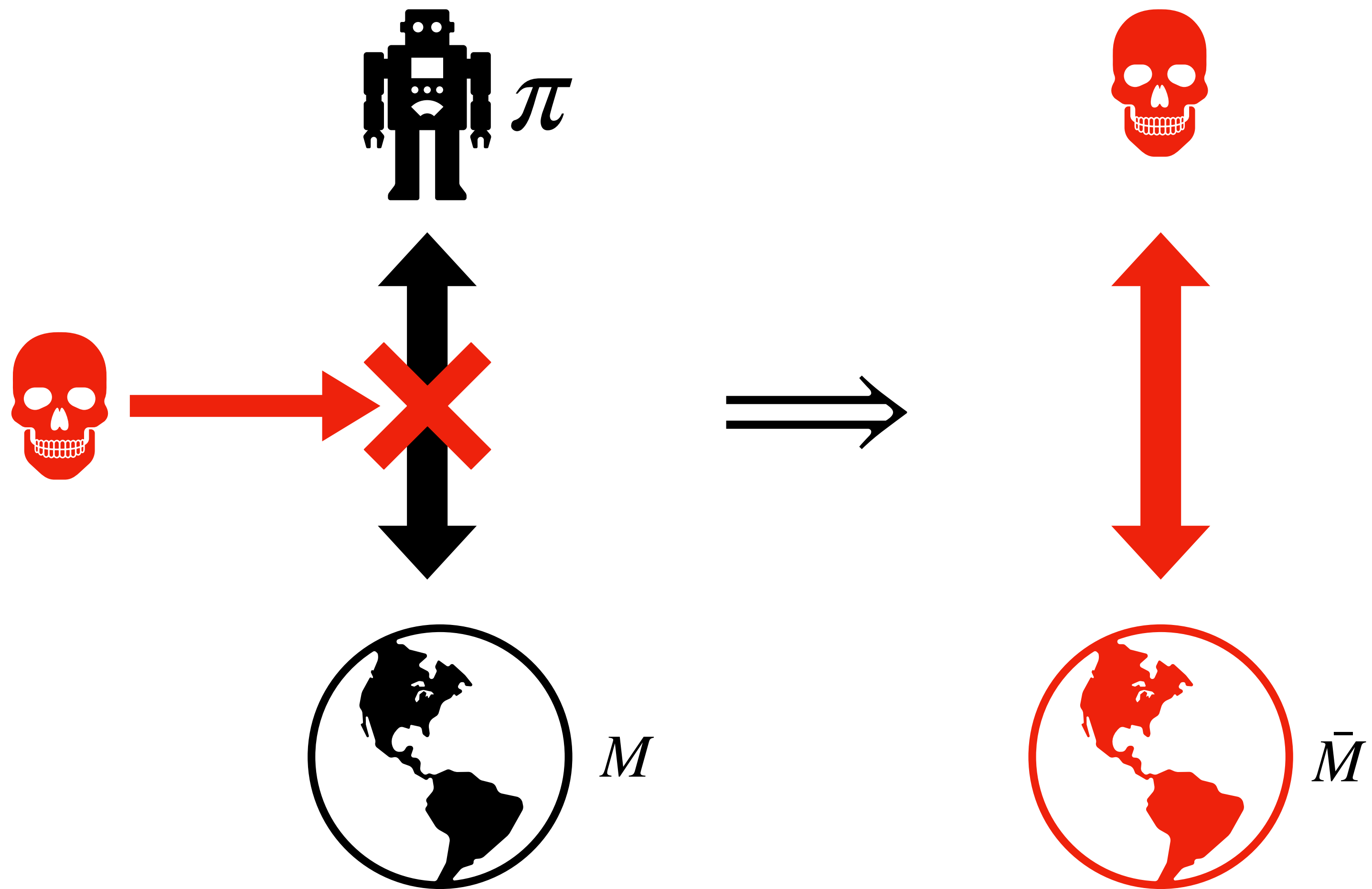
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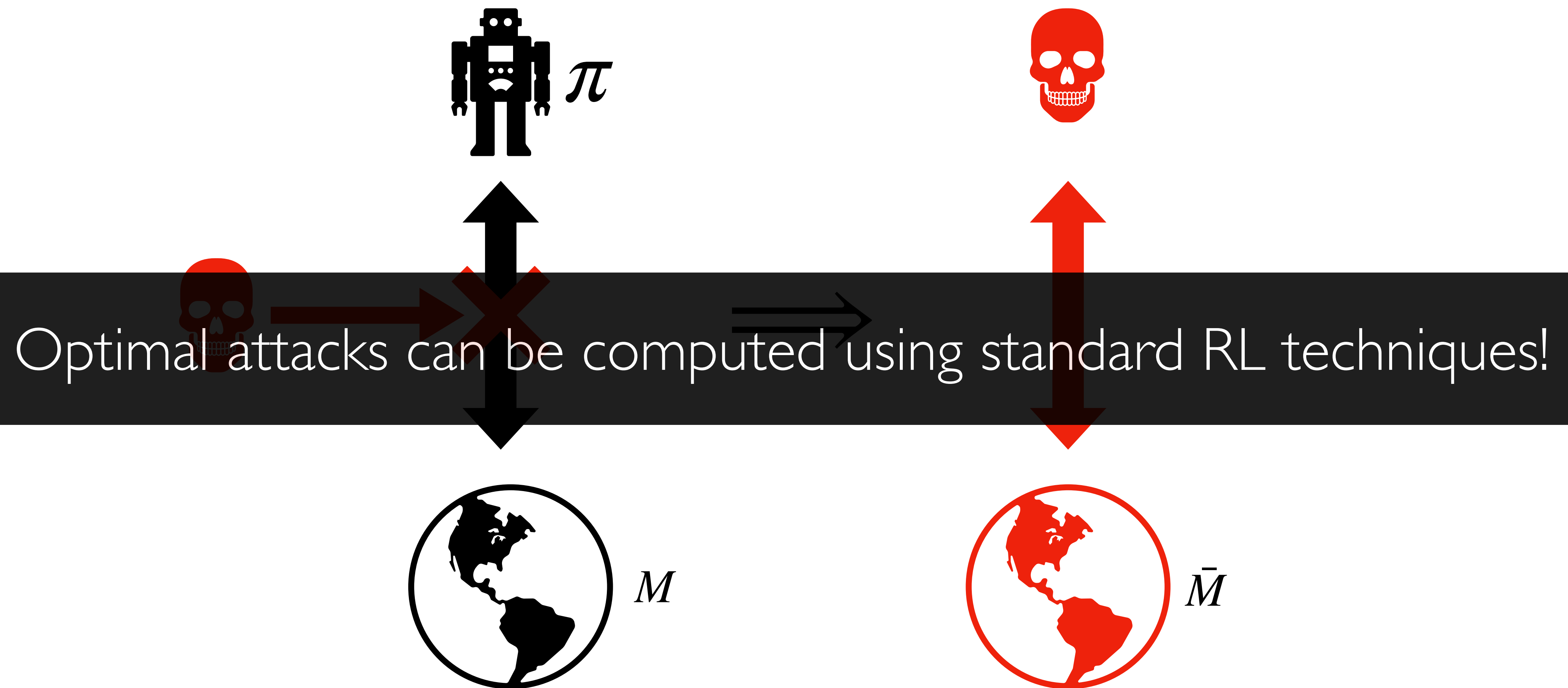
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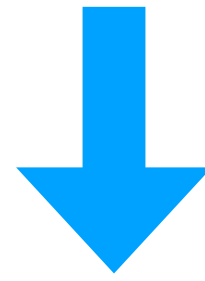
# Computational Efficiency

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$$|\bar{S}| \leq SOAR \quad \text{and} \quad |\bar{A}| \leq S + O + A + R$$

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Attacking RL *efficiently* reduces to RL!

$\bar{M}$  has only polynomially larger state and action space than  $M$ .

Can we defend against attacks?

Defense

# The Defense Problem

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Let  $(V_1^{\pi, \nu}, V_2^{\pi, \nu})$  denote the victim's and attacker's value, respectively.



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**Definition 2** (Defense Problem). The victim seeks a policy  $\pi^*$  that maximizes its expected reward from the victim-attacker- $M$  interaction under the worst-case attack:

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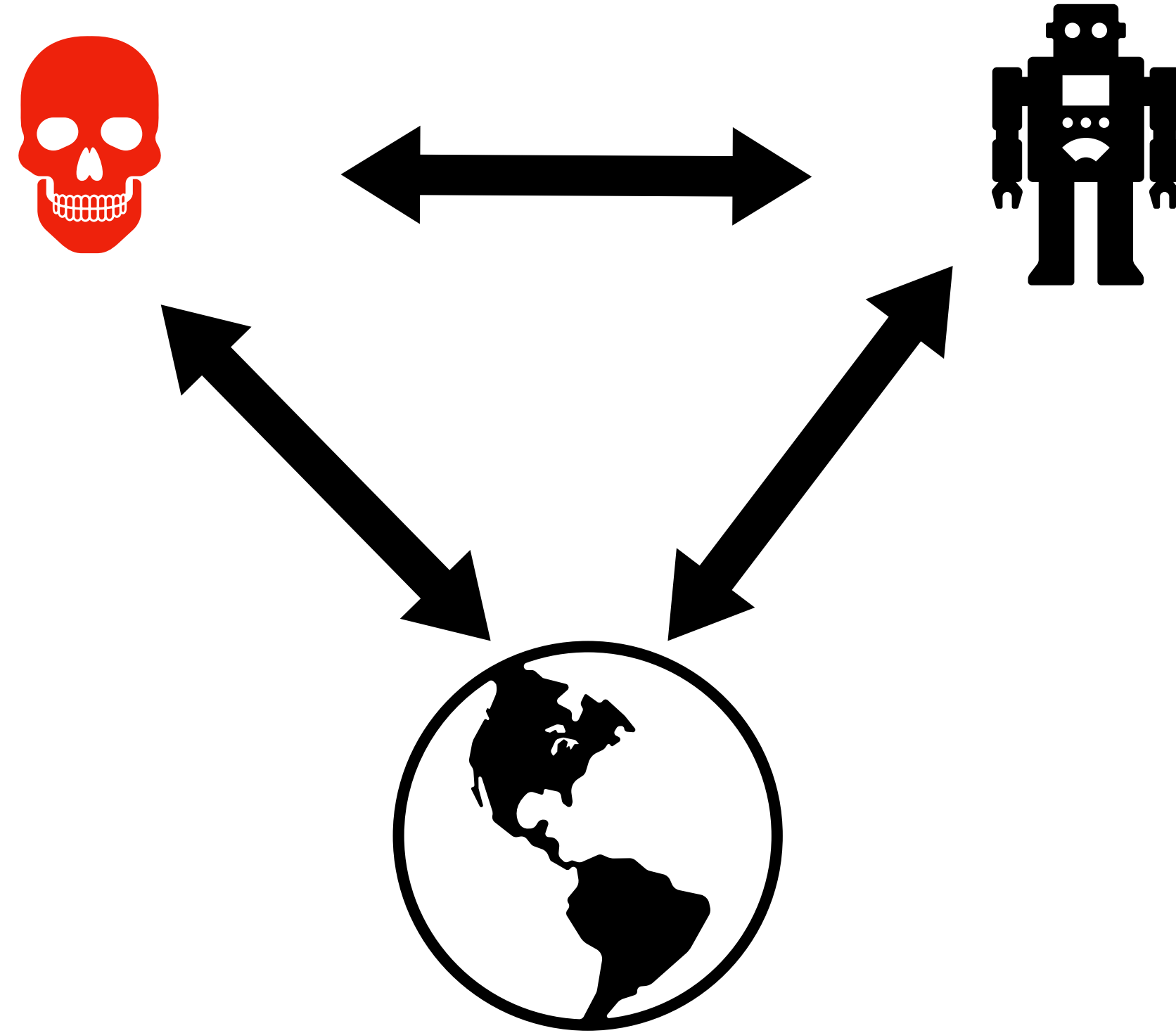
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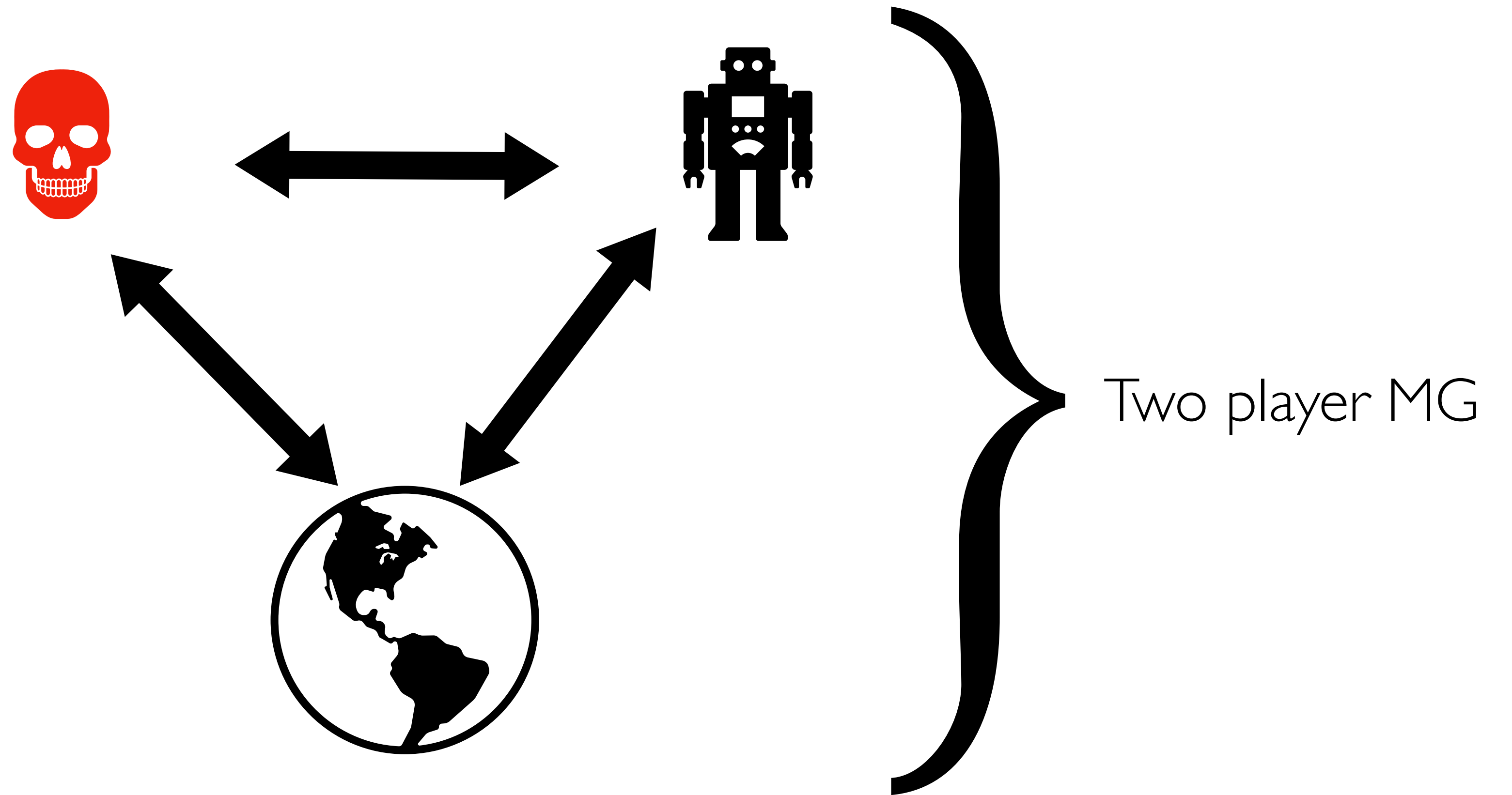

Avoids Cat and Mouse Game!

# Reduction to MARL

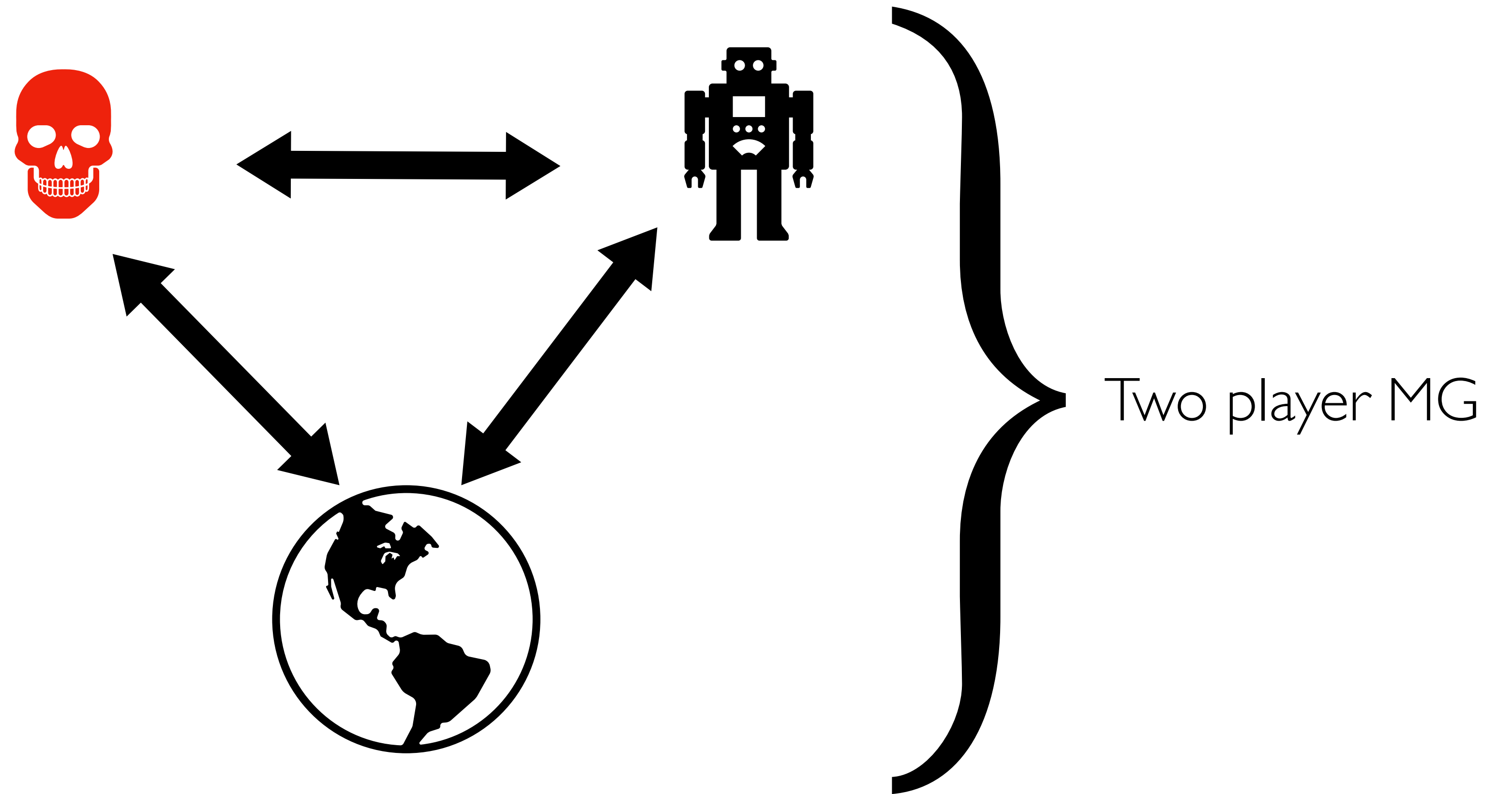
# Reduction to MARL



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Defense corresponds to a Weak Stackelberg Equilibrium (WSE).

# Challenges



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**Proposition:** The defense problem is as hard as solving POMDPs.

Thus, the defense problem is NP-hard to even approximate.

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Key: restrict observation attacks.

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P2

*S*

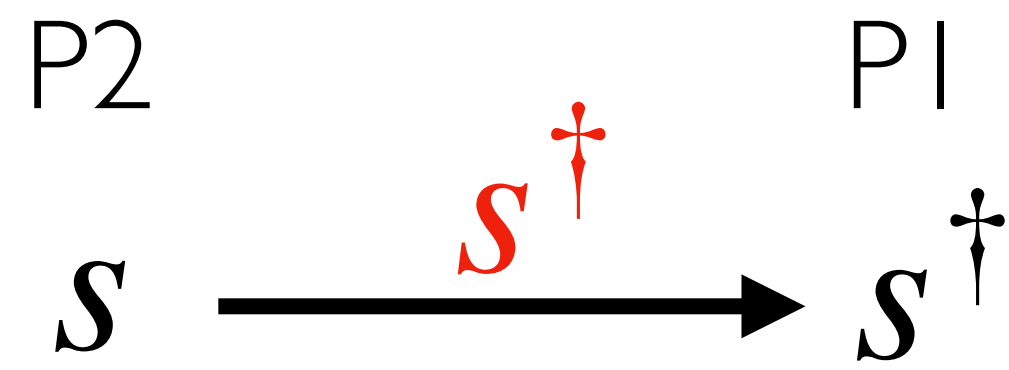
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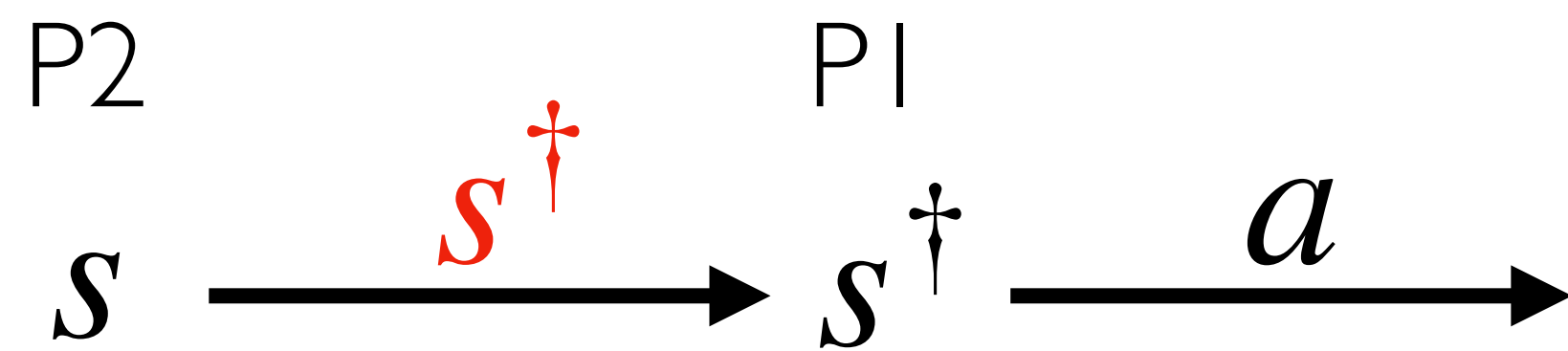
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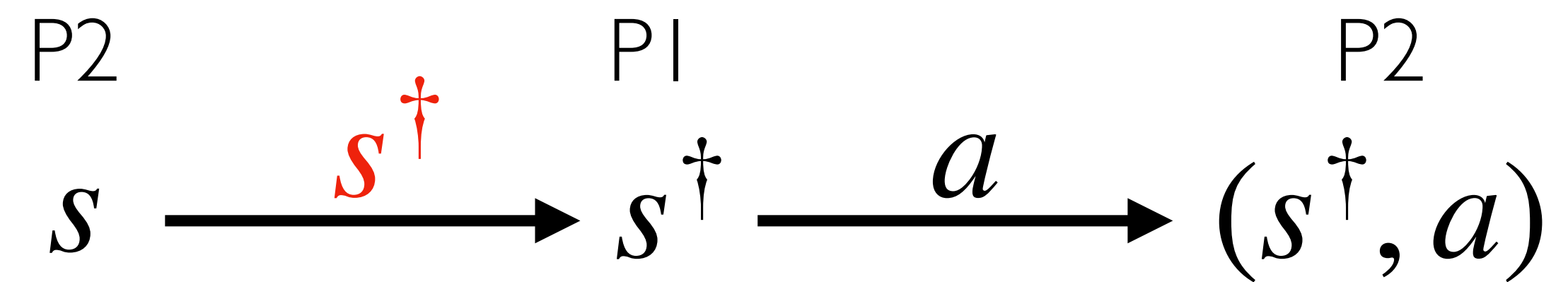
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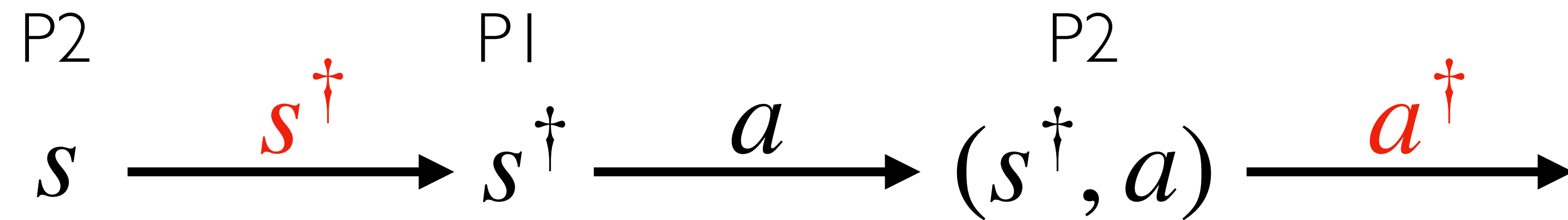
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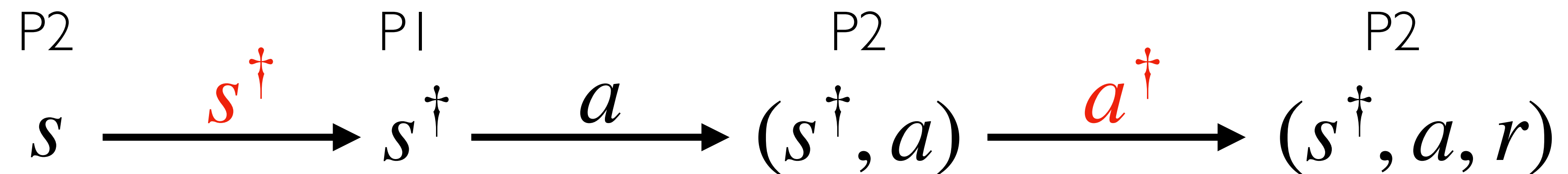
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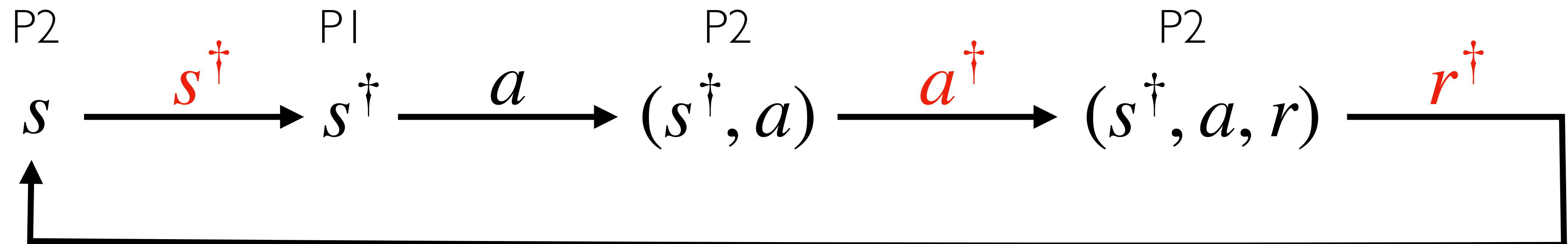
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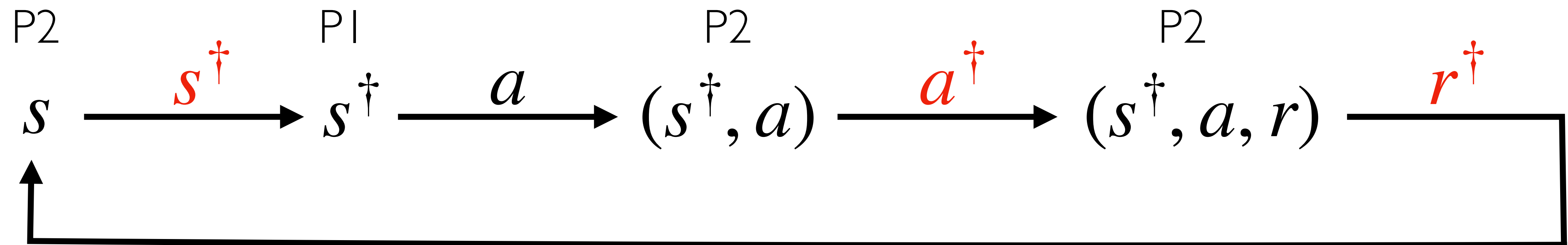
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Game evolves like a *turn-based* Markov game  $\overline{G}$ .

# Meta Turn-based Markov Game

1.  $\bar{\mathcal{S}}$  records the player's information at any subperiod:

$$\bar{\mathcal{S}}_1 = \mathcal{S} \quad \text{and} \quad \bar{\mathcal{S}}_2 = \mathcal{S} \cup (\mathcal{S} \cup \mathcal{A}) \cup (\mathcal{S} \cup \mathcal{A} \cup \mathcal{R})$$

2.  $\bar{\mathcal{A}}$  captures the actions available at any subperiod:

$$\bar{\mathcal{A}}_1 = \mathcal{A} \quad \text{and} \quad \bar{\mathcal{A}}_2(s) \subseteq \mathcal{S}, \bar{\mathcal{A}}_2(s, a) \subseteq \mathcal{A}, \bar{\mathcal{A}}_2(s, a, r) \subseteq \mathbb{R}$$

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**Proposition:** Any WSE for  $\bar{\mathcal{G}}$  is an optimal defense policy.



# *Efficient* Reduction to MARL

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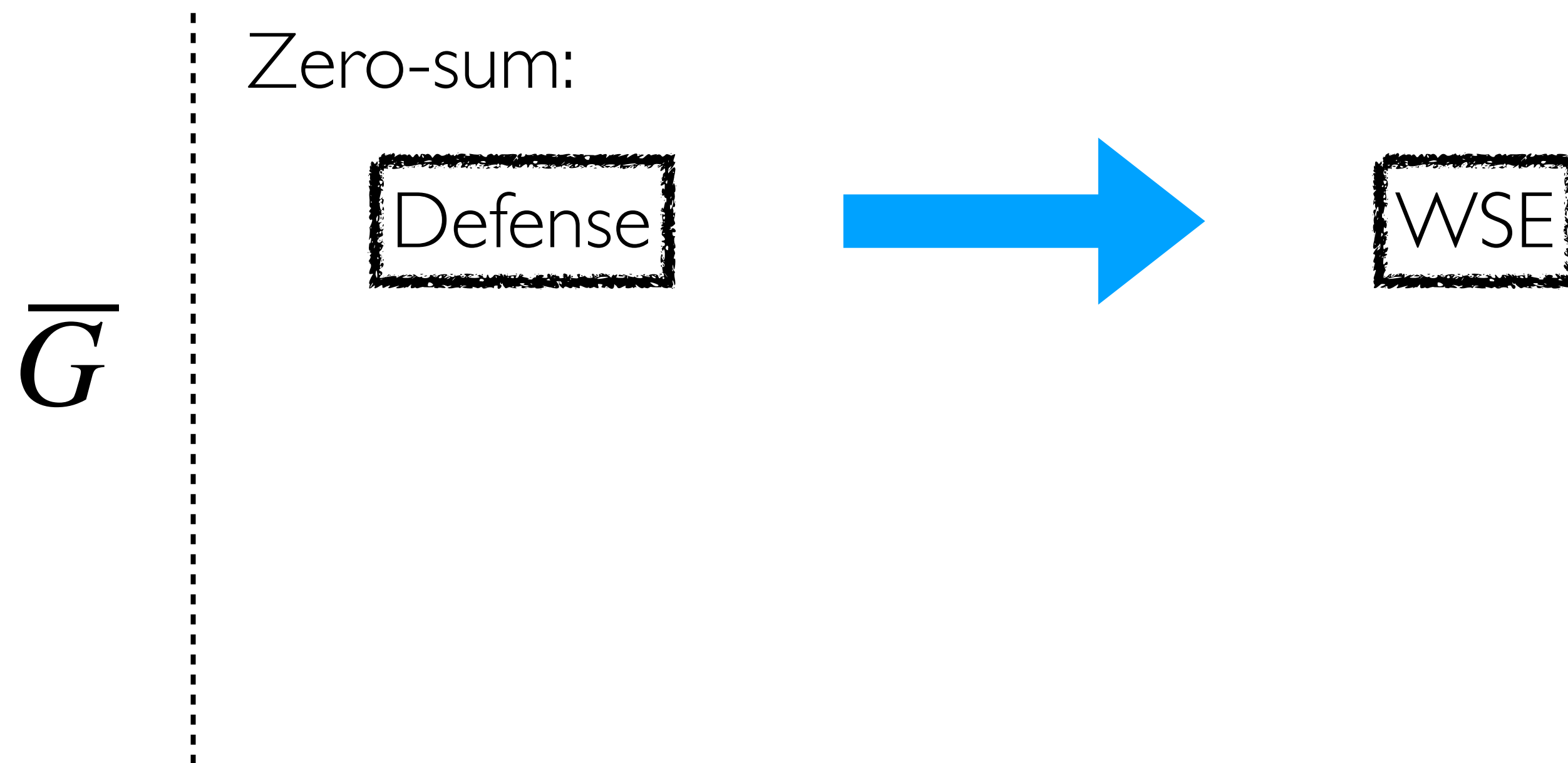
$\overline{G}$



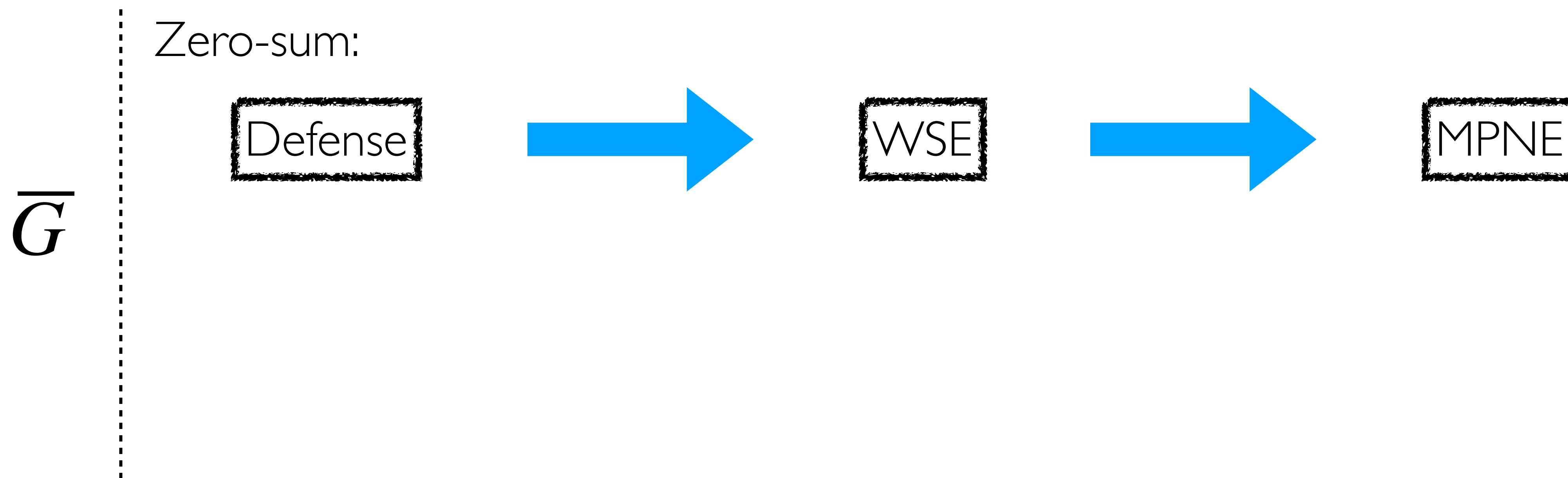
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$\overline{G}$  | Zero-sum:

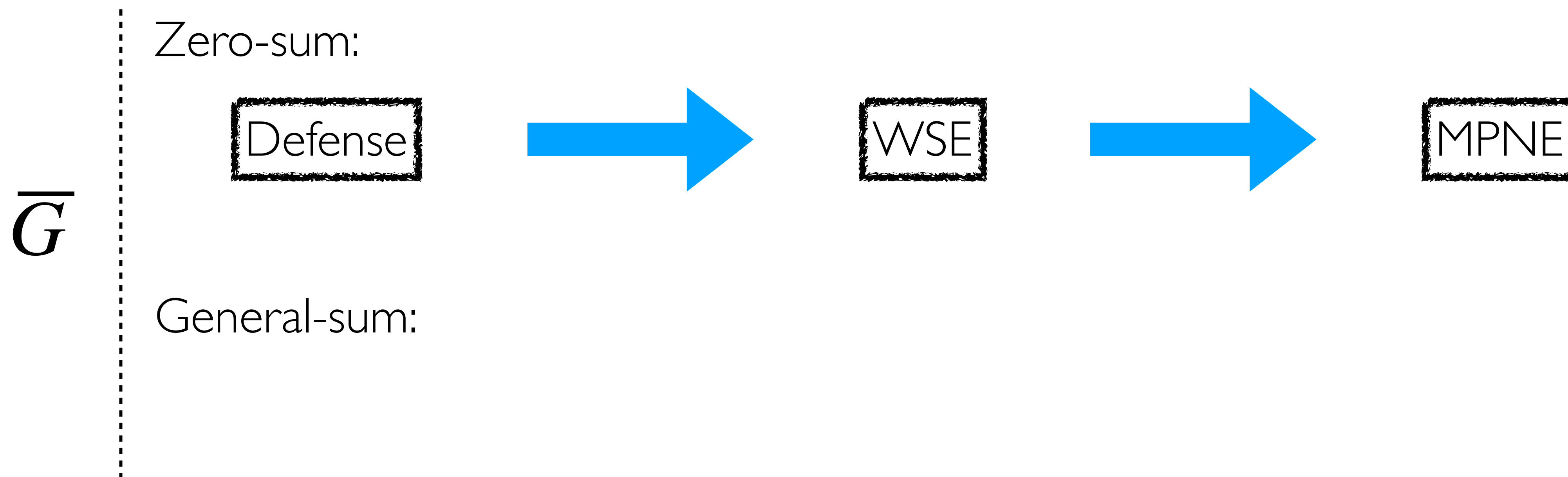
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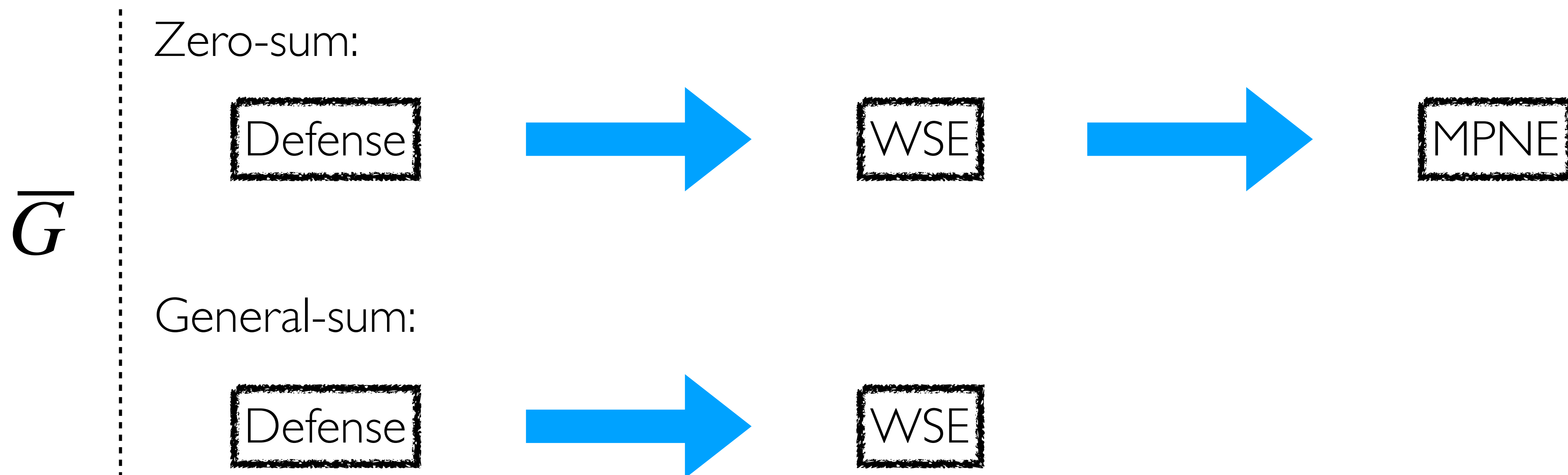
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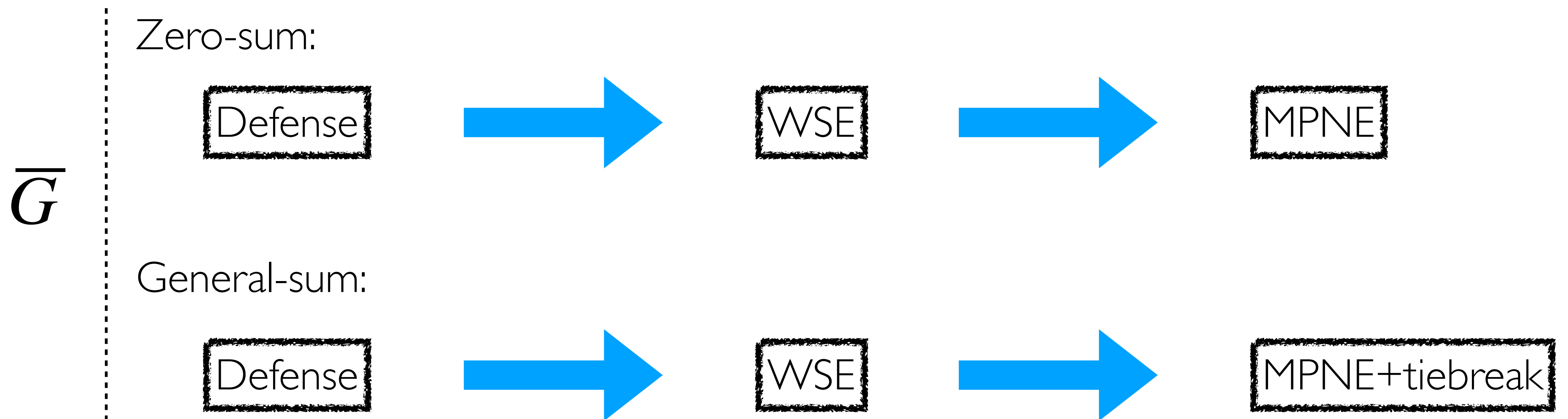
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# Rollback Algorithm

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Special Case: Action Attacks

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## Special Case: Action Attacks

I. Victim determines Attacker's best response to any action  $a$ :

$$BR_h(s, a) = \arg \max_{a^\dagger \in \bar{\mathcal{A}}(s, a)} \left[ g_h(s, a, r_h(s, a)) + \mathbb{E}_{s' \sim P_h(s, a^\dagger)} \left[ V_{h+1,2}^*(s', \pi_{h+1}^*(s')) \right] \right]$$

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2. Victim picks  $a$  based on the worst-case best-response:

$$V_{h,1}^*(s) = \max_{a \in \mathcal{A}} \min_{a^\dagger \in BR_h(s, a)} \left[ r_h(s, a^\dagger) + \mathbb{E}_{s' \sim P_h(s, a^\dagger)} \left[ V_{h+1,1}^*(s') \right] \right]$$

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Complete characterization: hard  $\iff$  observation attacks!

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